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[*Continued on page (III) of Cover.*

THE CONTROL OF VOLTAGE AND POWER FACTOR ON INTERCONNECTED SYSTEMS.

By OLIVER HOWARTH, Member.

(*Paper first received* 22*nd May*, 1934, *and in final form* 28*th February*, 1935; *read before the* TRANSMISSION SECTION 19*th December*, 1934, *also before the* TEES-SIDE SUB-CENTRE 21*st January*, *before the* NORTH-EASTERN CENTRE 11*th March*, *and before the* NORTH-WESTERN CENTRE 19*th March*, 1935.)

SUMMARY.

The paper describes the principles governing the parallel operation of power stations and gives the results of practical tests made to confirm the formula for the synchronous capacity required in an interconnector between power stations. The various types of apparatus available to regulate the interconnector current are described, and the requirements for satisfactory voltage and power factor control at the power stations are considered. The effect of operating interconnectors in parallel is examined in detail and it is shown that full commercial advantage can be obtained only if certain conditions are fulfilled. It is concluded that fine adjustment of the boost voltage in an interconnector is required and that interconnectors cannot be indiscriminately paralleled.

INTRODUCTION.

When two turbo-alternators operate in parallel the sharing of load between them will be governed by the steaming conditions and not by any characteristics of the electrical connection between them. It follows, therefore, that the electrical characteristics of the interconnection between them must be such that all transient and steady-load interchange conditions are adequately catered for without undue disturbance of normal running conditions. In other words, when the machines, or groups of machines, are in two interconnected stations it must be possible to control the voltage and power factor of each station, which necessitates controlling the power factor of the load on the interconnector. If load is suddenly thrown on or off the busbars of one of the stations the power transferred along the interconnector must change rapidly, the magnitude of the change depending upon the steam governors and not upon the electrical circuit. Amperes, volts, and power factor, must adjust themselves to steaming conditions. A change of load on the interconnector involves a change of power factor, not only on the interconnector, but on the generators in the two stations. Readjustment of power factor on the generators and the interconnector is necessary in order to restore normal running conditions.

It will, perhaps, help us to understand the electrical interconnector better if we imagine two turbo-alternators, each feeding into a separate network and mechanically coupled by a shaft of considerable length. The shaft must be sufficiently strong to withstand the torsion due to the maximum load it may have to transmit. It must also be sufficiently elastic to withstand without fracture the sudden change of torsion due to a sudden change of load on the network connected to one of the generators. A quality the shaft *must* possess is elasticity. When load is thrown on the network connected to one of the machines, that machine commences to slow down and the angular displacement between the machines due to twist of the shaft is altered. The torque in the shaft, and therefore the load delivered, are altered; thus the two machines share the load. The reactance of an electrical interconnector is analogous in its function to the elasticity of the interconnecting shaft. The vector diagram, Fig. 1, shows what happens if additional load is thrown on to the busbars of the receiving station. It is assumed that the interconnector has reactance but no resistance, which is equivalent to an interconnecting shaft having elasticity but no friction loss. The voltage vectors, V_a and V_b, represent the voltages of the two stations, A and B, and the difference voltage, V_a-V_b, applied to the interconnector results in a current, I, flowing, which will transmit power at almost unity

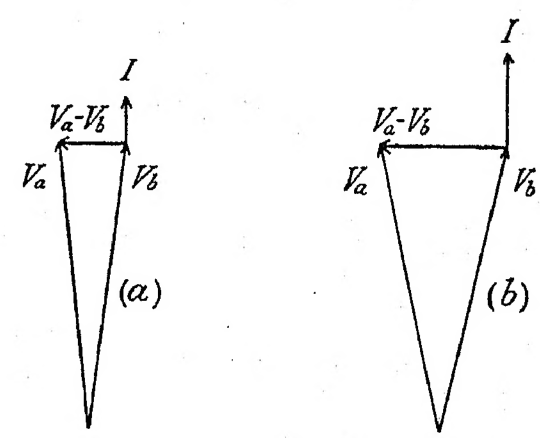

FIG. 1.

power factor, as shown in Fig. 1(*a*). If additional load is thrown on the busbars of station B, the machines in that station commence to slow down and the angular displacement between V_a and V_b increases as shown in Fig. 1(*b*), the voltage V_a-V_b, and in consequence the current I, increase, with the result that more power is transmitted from A to B, still at approximately unity power factor. If the angular displacement between V_a and V_b is 90° the power factor of the load on the interconnector will be 0·707 lag at station A and 0·707 lead at station B (see Fig. 2*a*). Any increase of load on the busbars of B will cause the angular difference between V_a and V_b to increase, but there will be a decrease in the amount of power transferred from A to B owing to the decrease in power factor overbalancing the increase in current. This condition is illustrated by Fig. 2(*b*), and is analogous to exceeding the elastic limit of the shaft referred to previously. When this limit is reached, any increase in twist of the shaft results in strain and not in an increase of power transmitted.

Let us suppose the two stations to be interconnected by a circuit having resistance but no reactance. This

would be analogous to two turbo-alternators coupled by a shaft having no elasticity; any change in the angular displacement between the machines would result in strain and, ultimately, fracture of the shaft. Fig. 3(*a*) shows the conditions when stations A and B are connected by a circuit having resistance but no reactance, and when power is passing from A to B at unity power factor. V_a and V_b are in phase and V_a is greater than V_b. If additional load is thrown on the busbars of station B it begins to fall back in relation to A and the angular

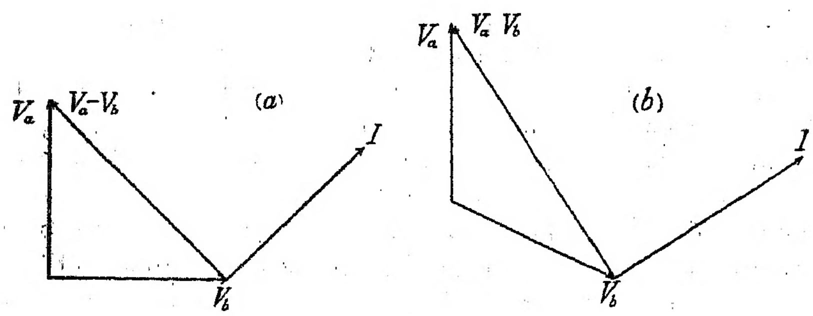

FIG. 2.

displacement between V_a and V_b increases, with the result that $V_a - V_b$ increases slightly, but at the same time its angle, and consequently the angle of I, alter considerably. The result is that slightly less power is received by station B from station A and the former continues to fall back and the stations will go out of synchronism, which is analogous to fracture of the shaft.

In the foregoing arguments the internal resistance and reactance of the machines have been ignored. When these are small compared with the interconnector

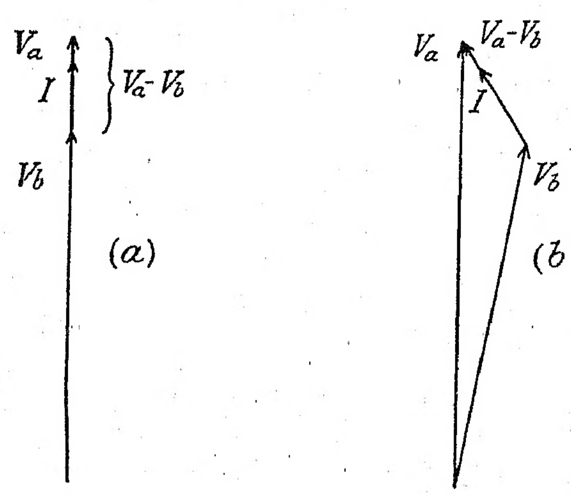

FIG. 3.

resistance and reactance their effect may be ignored: if it is desired to include them the voltages V_a and V_b in Figs. 1, 2, and 3, must represent the generated voltages and not the terminal voltages of the generators. Obviously when two generators are connected to the same busbars there is a negligible amount of reactance between the terminals of the machines, but satisfactory synchronous running is made possible by the internal reactance of the generators.

In the normal case where the interconnector has both resistance and reactance the conditions are as represented by Fig. 4(*a*) when power is transferred along the interconnector from station A to station B. V_a is higher than V_b and the current I in the interconnector is lagging. If additional load is thrown on to the busbars of station B it begins to fall back in phase and the angle between V_a and V_b increases; the voltage difference $V_a - V_b$ also increases and advances in phase; in consequence I increases and comes more nearly into phase with V_a and V_b, as shown in Fig. 4(*b*), and more power is transferred from station A to station B. Thus the two stations share the increase in load in accordance with their governor characteristics, and the interconnector current adjusts itself to the steaming conditions. If load is thrown off the busbars of station B, or on the busbars of station A, V_b will swing forward in phase compared with V_a and the difference voltage, $V_a - V_b$, will be retarded in phase; in consequence I will be reduced slightly and retarded in phase and thus less power will be transferred from station A to station B along the interconnector.

An inspection of Fig. 4 will show that for a given transfer of load at a given power factor a certain difference

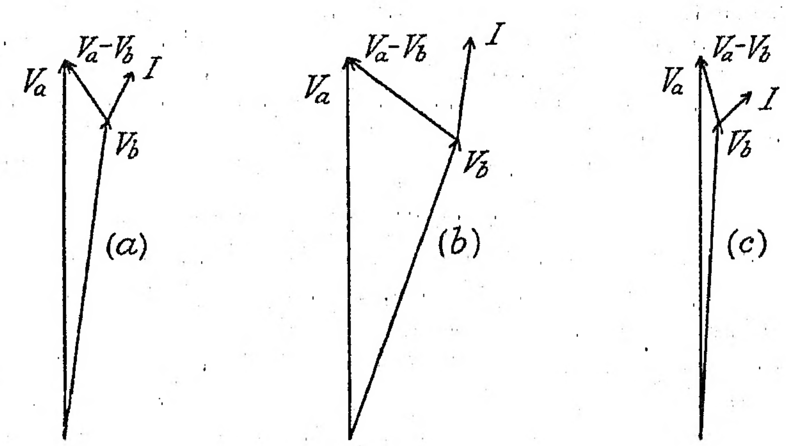

FIG. 4.

in the values of V_a and V_b must exist. A change in the value of V_a or V_b or both without any change of load or steaming conditions will result in a readjustment of the phase displacement between V_a and V_b, with a consequent change in the phase angle between I and V_b such that the power transferred remains unaltered.

It is necessary that the characteristics of the interconnector be such that all load-changes likely to occur on the combined system be adequately catered for without too rapid or too great a change in the angular relations of V_a and V_b, otherwise severe hunting or even loss of synchronism may take place between the stations. The requirements of a satisfactory interconnection were dealt with in papers by Messrs. Romero and Palmer * and Clough,† respectively.

If we describe the synchronous capacity of an interconnector as the rate of change of kilowatts transmitted per radian rate of change of angular displacement of the voltages of the two stations, the formula given in Appendix II of the paper by Messrs. Romero and Palmer may be simplified to

$$\frac{E^2 X}{1\,000 Z^2} \text{ kilowatts,}$$

where E = line voltage of the stations,
X = reactance of interconnector, ohms,
Z = impedance of interconnector, ohms.

* *Journal I.E.E.*, 1922, vol. 60, p. 287. † *Ibid.*, 1927, vol. 65, p. 653.

Mr. E. C. Stone in a paper on "The Operation of Power Plants in Parallel"* says that if the total capacity of the plant in the smaller station does not exceed the figure given by the above formula the interconnector will hold the stations in step in a satisfactory manner.

Some years ago the Lancashire Electric Power Co. had an interconnection between their Radcliffe power station and the Back o' th' Bank station of the Bolton Corporation. The formula showed the interconnector to be good for 15 000 kW, and a test was made with 14 000 kW of plant running in the smaller station under normal conditions of system load. The two stations were held in synchronism without any indication of hunting. Later, the Barton station of the Manchester Corporation was interconnected with the Radcliffe power station, and with the interconnector circuits arranged to give the minimum synchronous capacity of 29 000 kW no evidence of hunting was observed with 30 000 kW of plant running in the smaller station. These tests confirm the figure given by Mr. Stone.

Apparatus for Control of Interconnector.

In the previous Section it has been shown that the load on an interconnector between two power stations is determined by steaming conditions, and that the power factor of the load on the interconnector can be controlled by adjustment of the voltage in the interconnector circuit. This can readily be done by adjustment of the busbar voltages at the two stations, but it is rarely practicable to adjust station busbar voltages for this purpose, nor is it usually practicable to transmit power from one station to another at an abnormal power factor. We can, however, install apparatus to provide a voltage in the interconnector circuit which can be adjusted at will. This permits the interconnector power factor to be adjusted without alteration of the busbar voltages at the stations.

Perhaps the most desirable characteristic of boosting equipment installed in an interconnecting circuit is that the boost voltage should be smoothly variable from zero to full boost or buck. The only apparatus available until recently which fulfilled this requirement was the induction regulator; the moving-coil regulator, which also has this characteristic, has been put on the market recently. To allow of satisfactory paralleling with other circuits twin induction regulators arranged to boost without alteration of the phase relation between input and output voltages are desirable. A few years ago, before the advent of on-load tap-changing transformers, the choice of a booster lay between the induction regulator and the step-by-step booster. If a considerable range of adjustment was required and coarse steps were not desirable, induction regulators were competitive and were installed in many cases. The advent of the remote-controlled on-load tap-changing transformer has cut out the induction regulator where the interconnector circuit includes transformers, owing to the lower overall cost.

The apparatus available falls into four classes:—

- Induction regulators.
- Step-by-step boosters.
- On-load tap-changing transformers.
- Moving-coil regulators.

There is the further possibility of phase control, that is, briefly, to vary the voltage-drop in the interconnector by varying the kilovar loading. Synchronous condensers are used for this purpose, but as they are rarely an economic proposition except on very long lines they have been used very little, if at all, in this country up to the present time.

It may be assumed that the busbar voltage in a generating station must not vary more than 0·5 per cent from the schedule value if a satisfactory voltage is to be obtained at the consumer's terminals with the minimum expenditure on voltage regulators on the distribution system. If two stations, shown in Fig. 5, are interconnected by an overhead line and transformer banks for the purpose of transmitting power from the larger to the smaller station at a minimum power factor of 0·8, a voltage variation of 10 per cent will be required to deal with the drop in the interconnector. To cover possible changes of busbar voltages in the stations, and other contingencies, it will probably be desirable to provide for a voltage range of about 15 per cent. If an on-load tap-change transformer is provided it will be necessary to have steps not greater than 1 per cent in order to get within 0·5 per cent of the scheduled busbar voltage. This is due to the interconnector being large in relation to the size of the receiver station. Voltages intermediate between the taps can only be obtained by a change of power factor on the interconnector, and it will be necessary to change the kVAR loading of the interconnector by 2 000 kVAR to change the delivery voltage 0·5 per cent. This will necessitate a change of power factor from 0·8 to 0·94, or to 0·65 on the generating plant at the smaller station when operating at full load. If, however, there is 25 000 kW of plant at the smaller station it will only require a change of power factor from 0·8 to 0·83 or to 0·77 to change the busbar voltage 0·5 per cent with the plant fully loaded.

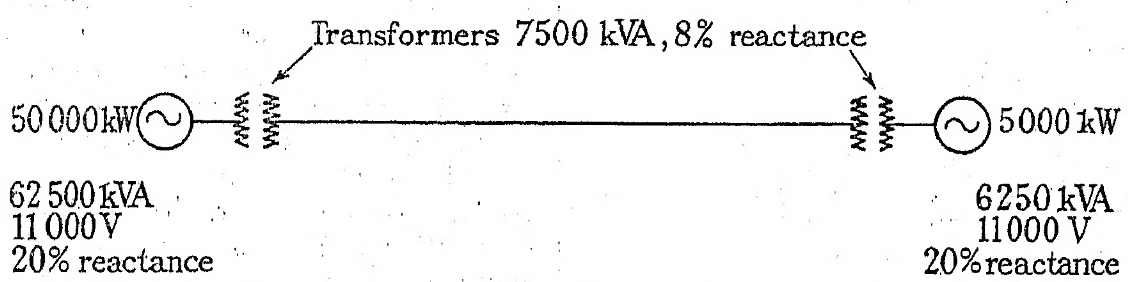

Fig. 5.

* *Transactions of the American I.E.E.*, 1919, vol. 38, p. 1651.

If the interconnector is considerably longer, or smaller in capacity, voltage regulation by power-factor change will be more practicable. For instance, suppose the transformers are smaller so that the total reactance is doubled, then to change the loading of the interconnector by 1 000 kVAR will alter the delivery voltage 0·5 per cent. This will require a change of power factor from 0·8 to 0·88 or to 0·73 on a generator operating at 5 000 kW. It is evident, therefore, that whilst steps of $1\frac{1}{2}$ or even 2 per cent may be quite satisfactory on an interconnector which is relatively light in size compared with the plant running, steps of not more than 1 per cent

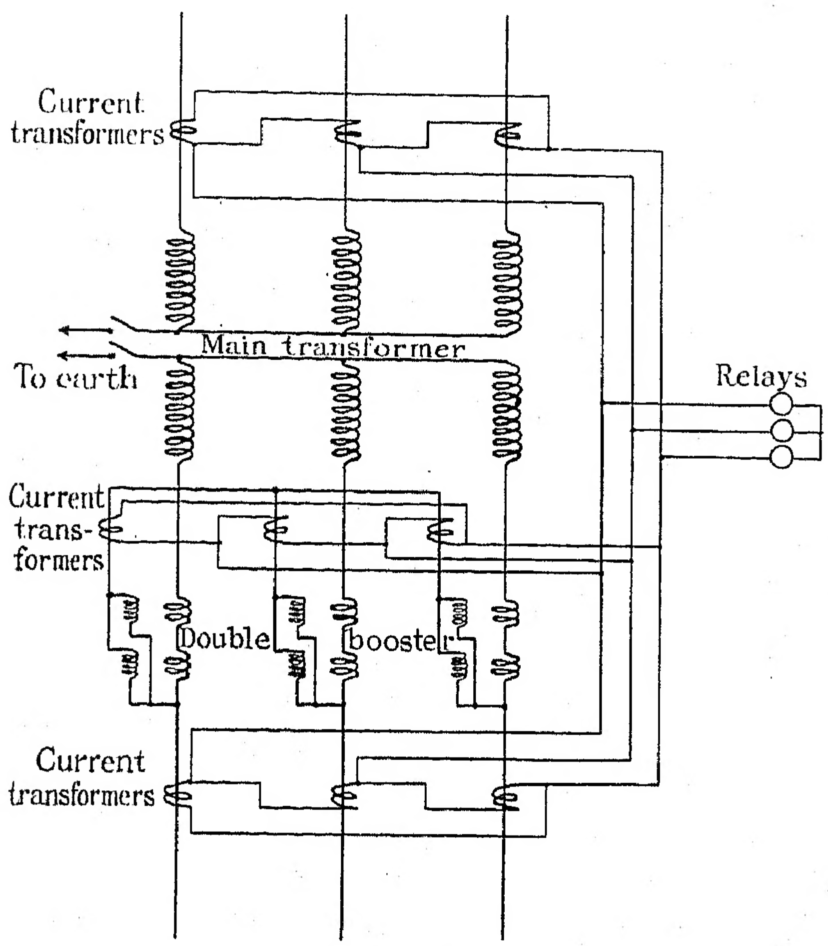

FIG. 6.

might be essential when the interconnector capacity is large compared with the plant at the point of delivery. As light-load periods, when little or no plant is running at the smaller station, must be considered, it may be said that when the voltage regulation in an interconnector is step-by-step the steps should not exceed 1 per cent for satisfactory voltage control of the interconnected systems.

An interconnecting circuit in a generating station ranks with a generator in importance, and rapid clearance of a faulty interconnector circuit is as important as rapid clearance of a faulty generator. The ordinary Merz–Price circulating-current protection can be made to give good service on a fixed-ratio transformer, but the introduction of variable ratio compels the use of some alternative form of protection. The core-balance system is quite good, provided the phases are completely segregated within the protected zone. A method of protecting a combined transformer bank and twin induction regulator used by the company with which the author is connected is shown in Fig. 6. This system can also be used with a self-contained step-by-step booster. The star-point connections of the booster primaries must be available for insertion of the protective current transformers, but the additional cost of doing this has proved to be justified by the satisfactory service given by this protection.

TYPES OF INTERCONNECTING CIRCUIT.

These may conveniently be divided into three types:—

(1) Single interconnector, with or without transformer.
(2) High-voltage interconnector with transformers, operating in parallel with lower-voltage interconnector.
(3) Interconnection by the grid of two stations already interconnected by the undertaker's own transmission system.

The first type has been dealt with in the previous Section.

The second type is a natural development when two stations which are interconnected by a cable operating at the busbar voltages grow in size so that it becomes necessary to increase the capacity of the interconnector between them. This type has two subdivisions. One is where there is a purely interconnecting cable with booster for transfer of load from the busbars of one station to the busbars of the other. The other is where two stations some distance apart are feeding into a common network and it becomes necessary to transfer load in bulk from the busbars of one station to the busbars of the other. In both cases the new interconnection will usually be at a higher voltage and have step-up transformers at each end.

Fig. 7 represents two stations which have been originally interconnected by an 11 000-volt underground cable, (*a*), and a 33 000-volt overhead line with transformer banks, (*b*), has been added. Typical resistance and reactance values for the two circuits are shown on the figure. The equivalent 11 000-volt figures are given for circuit (*b*), including the transformer banks. The angle of lag of the current behind the voltage applied to the circuit is shown for each line. It will be noted that there is a difference of 62° in the phase of the currents when the same voltage is applied to each circuit, i.e. if no boost voltage is applied in either of the circuits. Fig. 8 shows the power factor at which 10 000 kW would be transferred from one station to the other on either interconnector alone with various boost voltages from nothing to 14 per cent. It will be observed that the power-factor range on the lower-voltage circuit is from 0·45 lead to 0·59 lag, and on the higher-voltage circuit from 1·0 to 0·71 lag. It is also interesting to note that the kilovar curves are straight lines, that is, the variation in kilovars is directly proportional to the variation in boost voltage. The curves show that either of the circuits will operate satisfactorily as an interconnector for load transfers up to 10 000 kW.

The booster range required would not be the same in each circuit and would, of course, depend upon the load and power factor desired on the interconnector. In this and subsequent examples a load of 10 000 kW and a boost range of 14 per cent has been chosen, and

the station busbar voltages have been assumed to be equal in order to make the examples comparable. In all the examples the value of the quantities delivered to the receiver station is shown. Two factors which have little influence have been ignored; one is the variation

kVA delivered. The 10 000 kW is delivered at a power factor of 0·98 lag, and the total kVA delivered is again only 44 per cent in excess of the kVA in the low-voltage circuit. Fig. 9(c) shows the effect of boosting the voltage in the high-voltage interconnector by 10 per cent and in

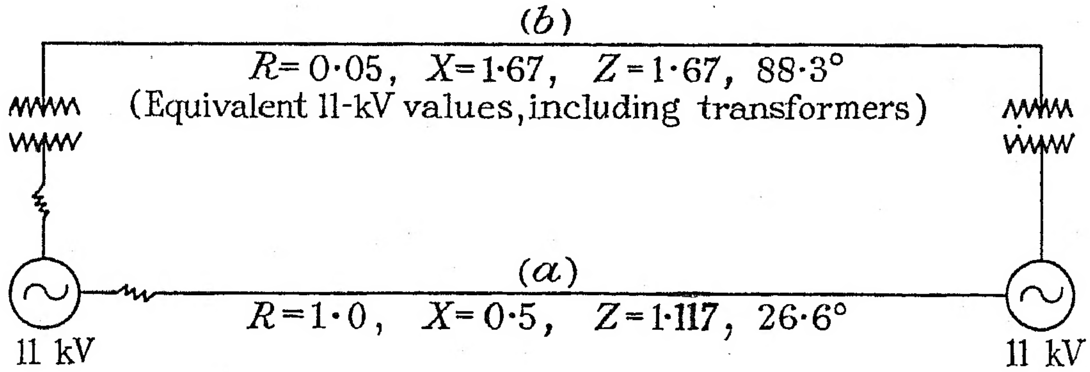

FIG. 7.

of the booster impedance (or transformer, if tap-change) with ratio, and the other is that the voltage due to phase displacement between the stations has been assumed to be 90° different from the receiver station voltage.

If the addition of the high-voltage circuit between the stations is to be a commercial advantage it must be possible to adjust the relative loads so that it carries at least 75 per cent of the total kVA transferred. Fig. 9 (a to e) shows the vector diagrams for the transfer of 10 000 kW from station A to station B under various conditions of voltage boost in the interconnectors. Fig. 9(a) shows the conditions when there is no boost voltage in either circuit; V is the busbar voltage of the receiver station, V_d the difference voltage between the stations, which sends the current along the interconnectors. I_a is the kVA in the low-voltage circuit (a), lagging 26·6° behind V_d, I_b is the kVA in the high-voltage circuit (b), lagging 88·3° behind V_d, and I_t is the vector sum of I_a and I_b, which is received on to the busbars of station B; it lags 50·7° behind V_d and therefore leads V by 39·3°. The load of 10 000 kW is therefore received by station B at a power factor of 0·77 lead. The vector sum of the kVA in the two circuits is only 44 per cent greater than the kVA in the low-voltage circuit. Delivery of 10 000 kW at a leading power factor would not be a normal condition, of course. The boosters in the two circuits would be used to adjust to a more suitable value the power factor of the load delivered. The boosters should also enable the load to be apportioned between the two circuits so that the full commercial advantage is obtained from the provision of the high-voltage interconnector. Fig. 9(b) shows the effect of boosting the voltage in each circuit by 5 per cent. OA shows the kVA in the low-voltage circuit and OB the kVA in the high-voltage circuit, due to the boost voltages; these represent a total of 4 950 kW. The voltage required to transmit the balance of 5 050 kW is provided by the station B falling back in phase until there is sufficient voltage difference to transmit the load, the kVA loading being represented by AI_a and BI_b in the low and high voltage circuits respectively. OI_a and OI_b represent the total kVA in the low- and high-voltage circuits respectively, and OI_t the total

the low-voltage interconnector by 5 per cent. Again, OA and OB represent the kVA in the low- and high-voltage circuits respectively due to the boost voltages, and AI_a and BI_b the kVA in the two circuits due to phase displacement between the stations; OI_a and OI_b represent the total kVA in each circuit and OI_t the

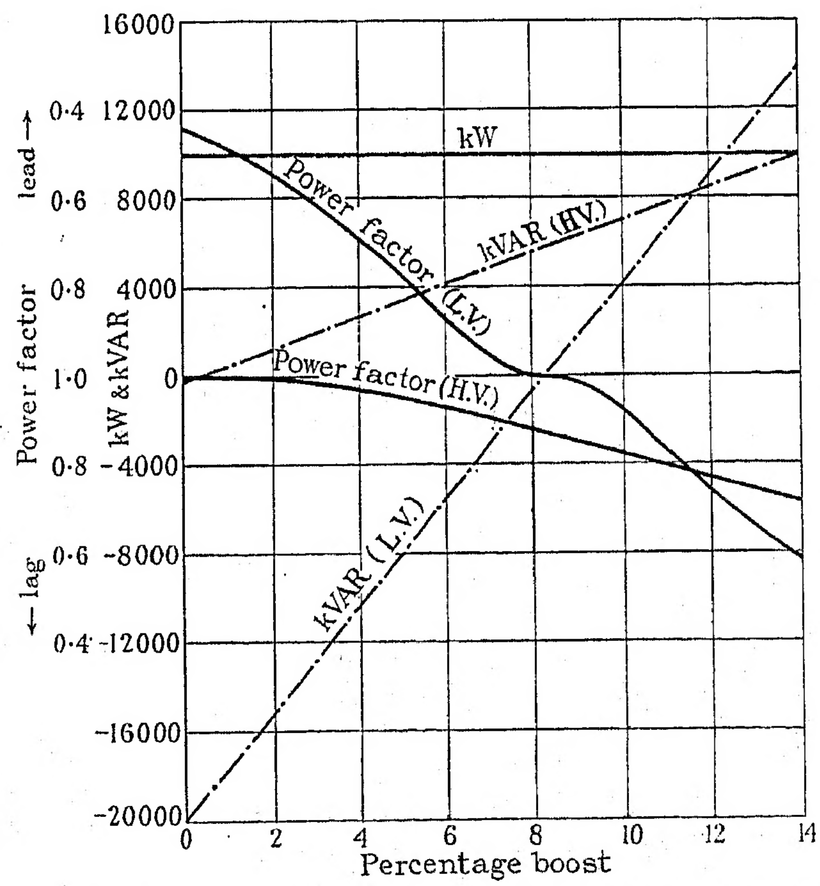

FIG. 8.—Results obtained with one interconnector only in use (see Fig. 7).

total kVA delivered. The 10 000 kW is delivered at a power of 0·87 lag, and although the total kVA is 64 per cent more than the kVA in the low-voltage circuit the effect of increasing the boost in the high-voltage circuit is to increase the angular displacement between I_a and I_b. The effect of varying the boost in the high-voltage circuit with a constant boost of 5 per cent in the low-voltage circuit is shown in Fig. 10. It will be observed that

varying the boost in the high-voltage circuit has very little effect upon the kW transmitted along the two circuits, or upon the kVAR loading of the low-voltage circuit. The kVAR loading of the high-voltage circuit of the low-voltage interconnector unless the high-voltage one were out of service for some reason, in which event the reactor could be cut out.

When a separate booster is installed in a 3-phase

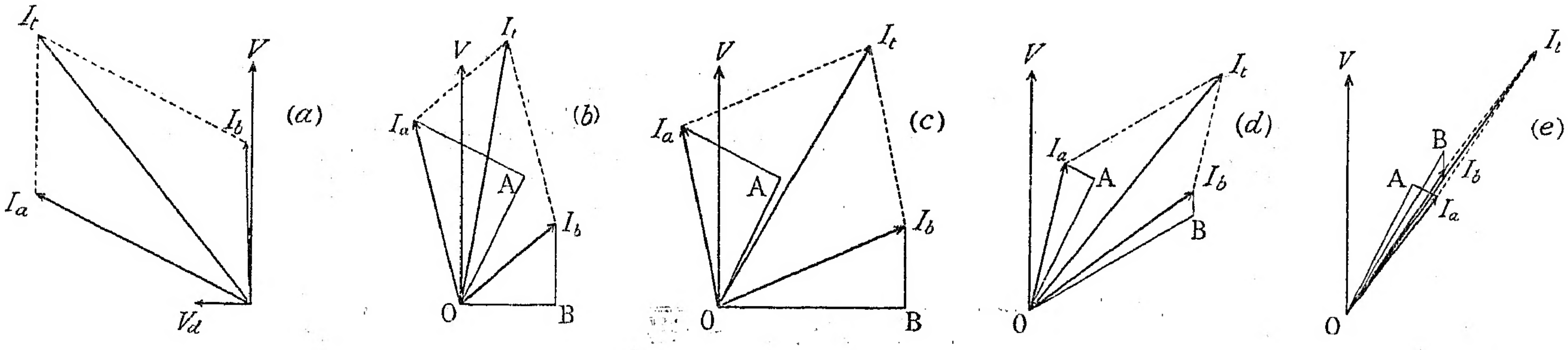

FIG. 9.

varies in proportion to the boost. The power factor curves show the angular displacements between the currents in the two circuits to increase with increase of boost, and as the total kVA loading is the arithmetical sum of the kVA in each circuit it is uneconomic for there to be much discrepancy between the arithmetic and the vector sum of the loads in the two interconnectors. In other words, there should not be much phase displacement between the currents in the two interconnectors. The upper curve in Fig. 10 shows the percentage excess of the arithmetic sum over the vector sum of the kVA in the two circuits. It will be seen that unless the load is delivered at about unity power factor the excess kVA will be undesirably high. At the same time the low-voltage interconnector will carry more than double the kVA carried by the high-voltage interconnector, which has sufficient copper (as shown by the equivalent resistance) to carry 10 times the load carried by the low-voltage interconnector. It is evident, therefore, that the provision of the high-voltage interconnection with booster, as shown in Fig. 7, would be quite uneconomic unless the low-voltage interconnector circuit was opened and the cable put to economic use in some other way.

Complete control of the relative loads and the power factors in the two interconnector circuits can be obtained by installing apparatus to vary the boost voltage and the phase of the boost voltage in one of the interconnector circuits. This might be done by providing an additional booster in the high-voltage circuit to inject a voltage leading the system voltage by 90°. This would be an expensive way of getting over the difficulty and would further complicate the work of the operating engineers. Another method would be to insert a reactor in the low-voltage interconnector circuit; later examples in the paper indicate that if this is sufficient to reduce the phase displacement between the currents in the two interconnectors to within a few degrees, when the same voltage is applied to each, satisfactory loadings can be obtained. This might involve increasing the boost voltage available in the low-voltage circuit, but it is doubtful, as the high-voltage circuit would probably be able to deal with any deficiency of load in the low-voltage circuit. It is unlikely that full use would be required

circuit such as the high-voltage interconnector in Fig. 7, it will usually be practicable to arrange for the phase of the boost voltage to be advanced 30° or 60°. Fig. 9(*d*) shows the effect of advancing the phase by 30°, with 5 per cent boost on the low-voltage interconnector and

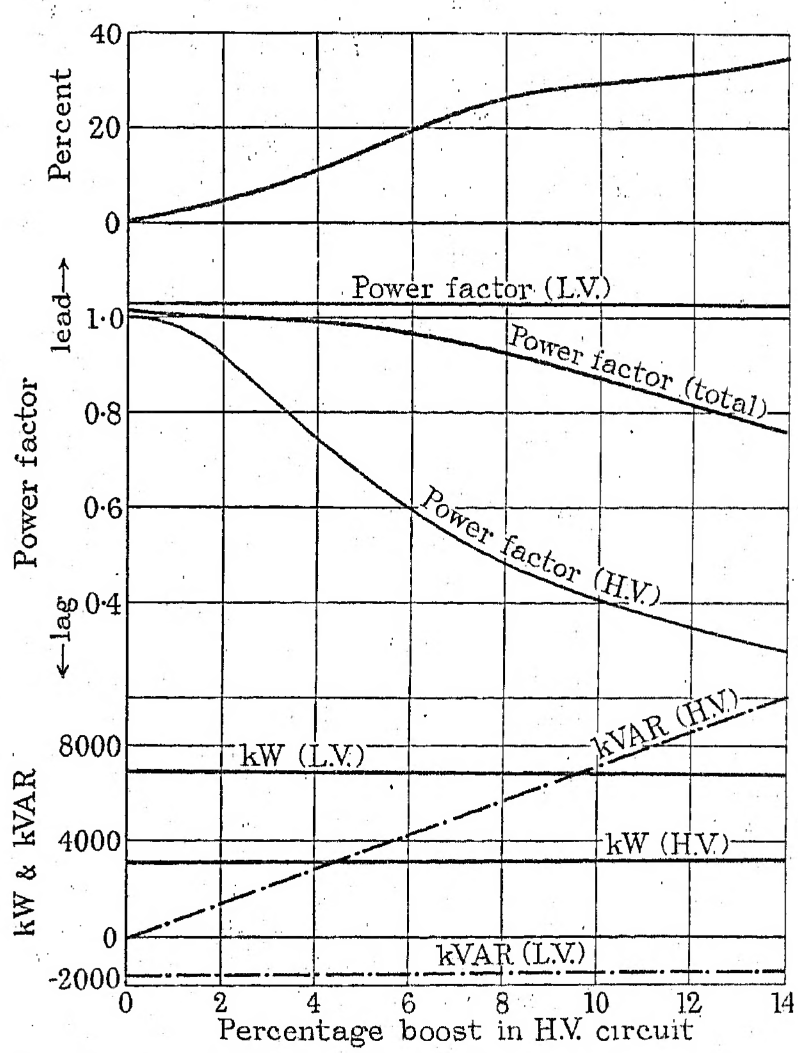

FIG. 10.—Results obtained with 5 per cent boost in low-voltage circuit of Fig. 7: boost in high-voltage circuit in phase.

10 per cent boost on the high-voltage interconnector. It will be seen that this is a considerable improvement on the conditions shown in Fig. 9(*c*). I_a and I_b are more nearly in phase, and I_t is 125 per cent in excess of I_a; the power factor of the delivered load has been

reduced to 0·8 lag. Fig. 11 shows the effect of varying the boost in the high-voltage circuit under the above conditions. It shows that more practicable conditions are obtained. An increase of boost in the high-voltage interconnector reduces the kVA loading of the low-voltage interconnector appreciably; the power factor of the delivered load can be reduced to 0·67 lag, against 0·76 lag with the boost in phase; and the sum of the kVA in each feeder does not exceed the delivered kVA by more than 7 per cent.

Fig. 9(*e*) shows the effect of advancing the phase of the boost voltage in the high-voltage interconnector by 60°, with 5 per cent boost in the low-voltage interconnector and 10 per cent boost in the high-voltage interconnector. This shows a further alteration in operating conditions. I_a and I_b are more nearly in phase, I_t is now 119 per cent in excess of I_a, and the power factor of the delivered load is 0·82. Fig. 12 shows the effect of varying the boost in the high-voltage interconnector under the above conditions. The three power-factor curves are nearer together and the sum of the kVA in the two interconnectors is not more than 2 per cent in excess of the kVA delivered. The kVA loading of the low-voltage interconnector with 14 per cent boost is higher than with 30° advance in the boost voltage, and the kVAR loading of the high-voltage interconnector is lower; the lowest power factor of the delivered load is 0·7 against 0·67 with 30° advance. A comparison of Fig. 8 with Figs. 11 and 12 shows that although 14 per cent boost in the high-voltage circuit enables 10 000 kW to be transmitted at a power factor of 0·7 lag, this boost does not prevent the low-voltage interconnector from taking an undue share of the load when they are in parallel, and it is evident that two such interconnectors, differing by 60° in their time constants, cannot be economically paralleled.

Where the two interconnectors have the same proportion of reactance to resistance the two can be economically operated in parallel. Fig. 13 shows such an arrangement, and it should be noted that the high-voltage interconnector is three times the capacity of the low-voltage interconnector as shown by the resistance figures, which are all 11 000-volt values. Even under these conditions the currents in the two interconnectors will not be in phase unless the percentage boost is the same in each circuit. Fig. 14 shows the effect of varying the boost in the high-voltage circuit with a boost of 10 per cent in the low-voltage circuit. There is no difficulty in loading the high-voltage circuit three times as heavily as the low-voltage circuit, and over a range of boost from 4 per cent to 14 per cent the sum of the kVA in the interconnectors is not more than 2 per cent in excess of the delivered kVA. Fig. 15 shows the effect of varying the boost in the low-voltage circuit with a constant boost of 10 per cent in the high-voltage circuit; the curves are very similar to those in Fig. 14, except that the power factor of the total load with 14 per cent boost is higher in Fig. 15 than in Fig. 14, as might be expected with the same percentage boost in a higher-impedance circuit. These two figures show that it is commercially practicable to operate two interconnectors in parallel if the ratio of reactance to resistance in the two circuits is approximately

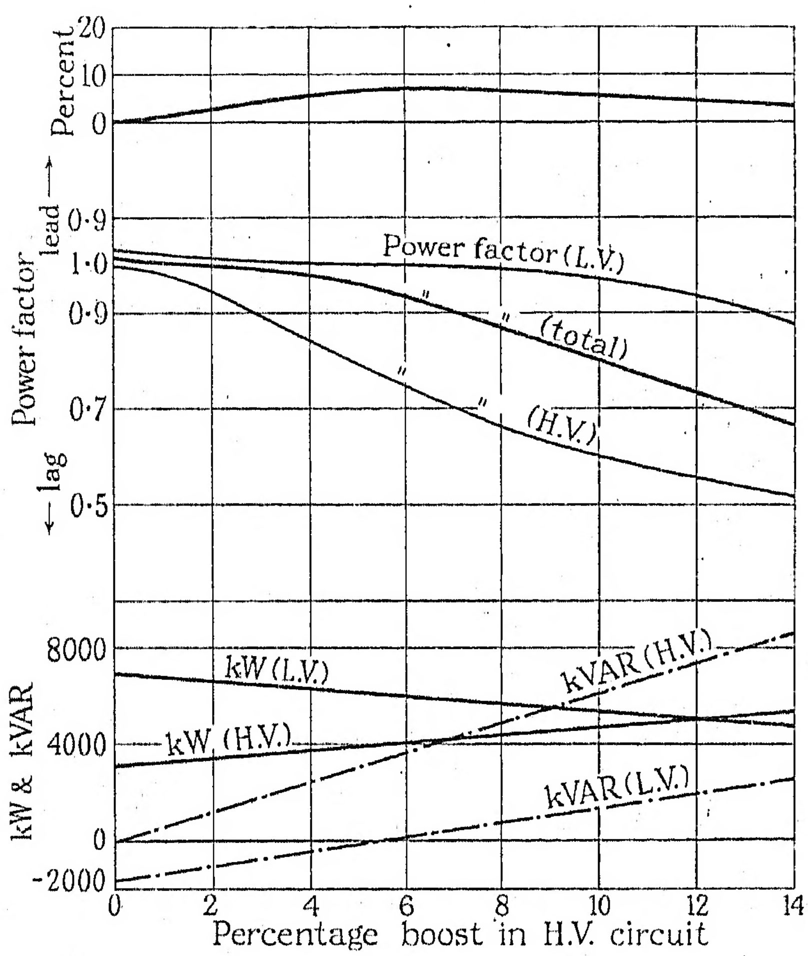

FIG. 11.—Results obtained with 5 per cent boost in low-voltage circuit of Fig. 7: phase of boost in high-voltage circuit = + 30°.

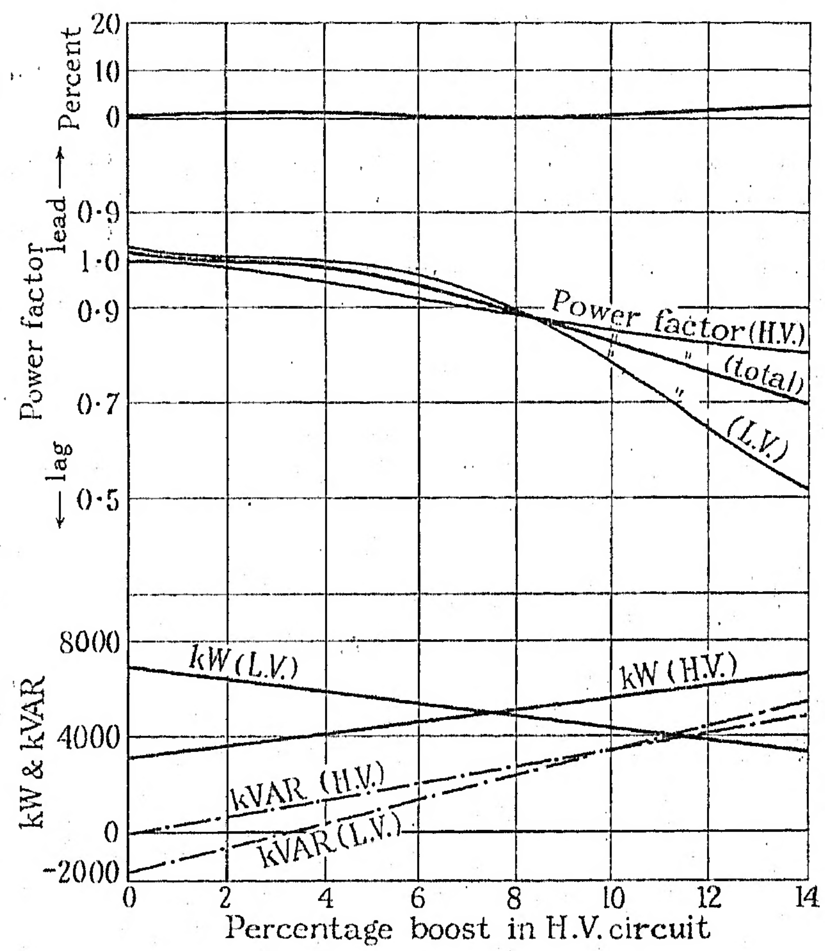

FIG. 12.—Results obtained with 5 per cent boost in low-voltage circuit of Fig. 7: phase of boost in high-voltage circuit = + 60°.

the same. A considerable variation in the proportion of the load carried by each interconnector can be made without introducing any objectionable conditions.

Interconnector circuits between power stations are not always used solely for the transfer of load from one station to the other, and inability to adjust the loading to the proportionate carrying capacity of the two interconnecting circuits may be quite unobjectionable. Such a condition has arisen in Lancashire, where the Kearsley and Padiham stations of the Lancashire Electric Power Co. have been operating in parallel for several years, connected by the Company's 33-kV feeders as shown in Fig. 16. Kearsley is a 33-kV station, and in addition to four 33-kV grid feeders there are two 30 000-kVA transformers which step up to 132 kV; these are connected to three 132-kV feeders, one to Barton, one via Bolton and Blackburn to Padiham, and the other via Rawtenstall to Padiham. Padiham is an 11-kV station and there are two 12 750-kVA 11/33-kV transformer banks each with twin induction regulators having a range of ±10 per cent, which are connected to the L.E.P. feeders. The transformer taps can be selected so that the boost range is, say, +15 to −5 per cent if desired. There are also two 15 000-kVA 11/33-kV transformers connected to the C.E.B. 33-kV busbars, to which are connected three 33-kV feeders and two 20 000-kVA 33/132-kV transformers connected to the 132-kV feeders. All the C.E.B. transformers have on-load tap-change gear in accordance with standard C.E.B. practice. The L.E.P. feeders are used to deliver loads up to 30 000 kW to Rochdale, and loads up to 5 000 kW to Stubbins. The

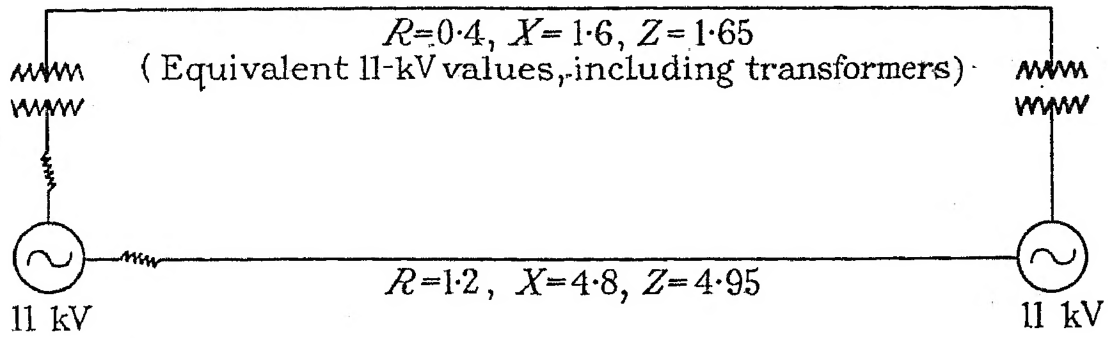

FIG. 13.

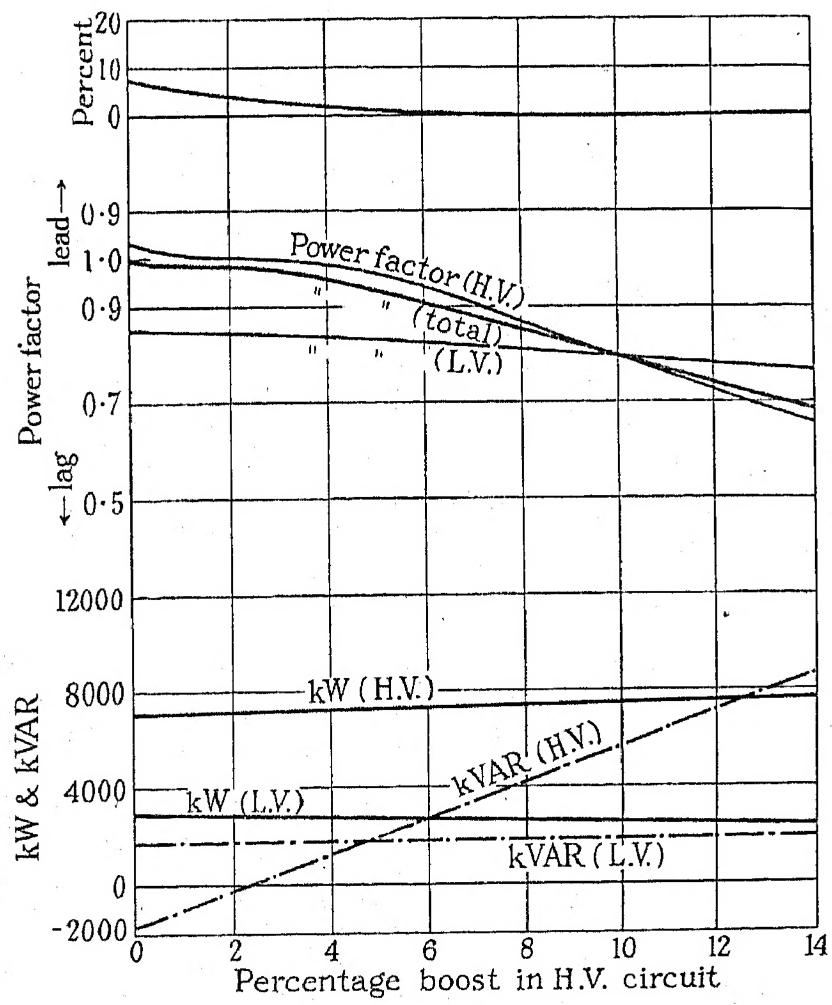

FIG. 14.—Results obtained with 10 per cent boost in low-voltage circuit of Fig. 13.

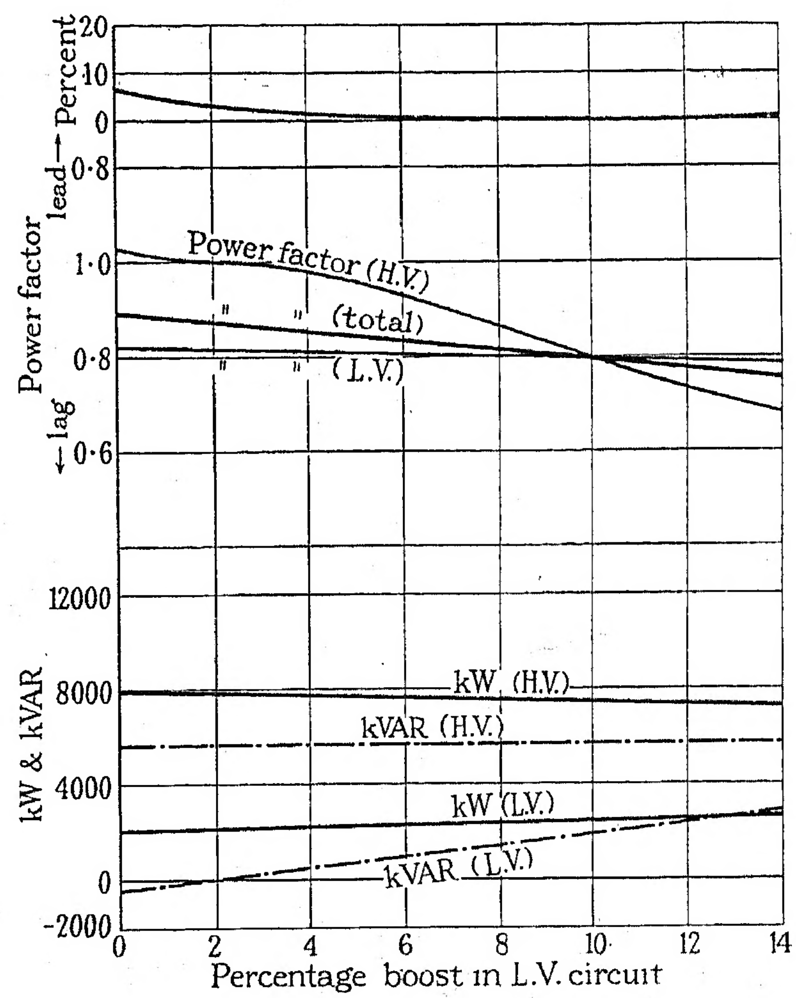

FIG. 15.—Results obtained with 10 per cent boost in high-voltage circuit of Fig. 13.

grid circuits are used for transfer of loads, generally including transfer between the 132-kV and the 33-kV grid circuits at Kearsley and at Padiham. The object of the grid connections at Kearsley and at Padiham is the usual one of bringing these selected stations into the general grid scheme. The object of the L.E.P. interconnections, which have been in use for several years, is to enable the Company to make the most economical use of the plant in the power stations. Under normal conditions loads up to 10 000 kW are fed into the Company's interconnectors at Padiham, and although this load is not sent to Kearsley it may conveniently be assumed to be in order to determine the relative loadings which can be obtained on the two interconnector circuits. Allowance for any load tapped off along the route or transferred over any portion of the grid circuit concerned can readily be taken into consideration by deducting the voltage-drop due to this from the boost voltage available.

Fig. 17 shows the interconnector circuits as two parallel circuits and gives the equivalent 33-kV values of resistance, reactance, and impedance. Fig. 18 shows the effect of varying the grid transformer taps with 7 per cent boost in the L.E.P. circuits, when a load of 10 000 kW is delivered at Kearsley from Padiham. The kW and kVA on the L.E.P. interconnectors and the kW on the grid interconnectors are approximately constant; the kVAR on the grid varies directly as the boost. Fig. 19 shows the effect of varying the boost in the L.E.P. interconnectors with 2 per cent boost in the grid circuit, and Fig. 20 shows the same with 7 per cent boost in the grid circuit. These show that the kW and kVAR on the L.E.P. interconnectors increase with increased boost; on the grid circuits the kW decrease and the kVAR remain approximately constant. The figures show that it is not possible to transfer all the load to either the grid or the L.E.P. interconnectors; although the kW on the grid circuits can be reduced almost to zero it is impossible to reduce both the kW and kVAR to zero simultaneously. Whilst it is not possible to

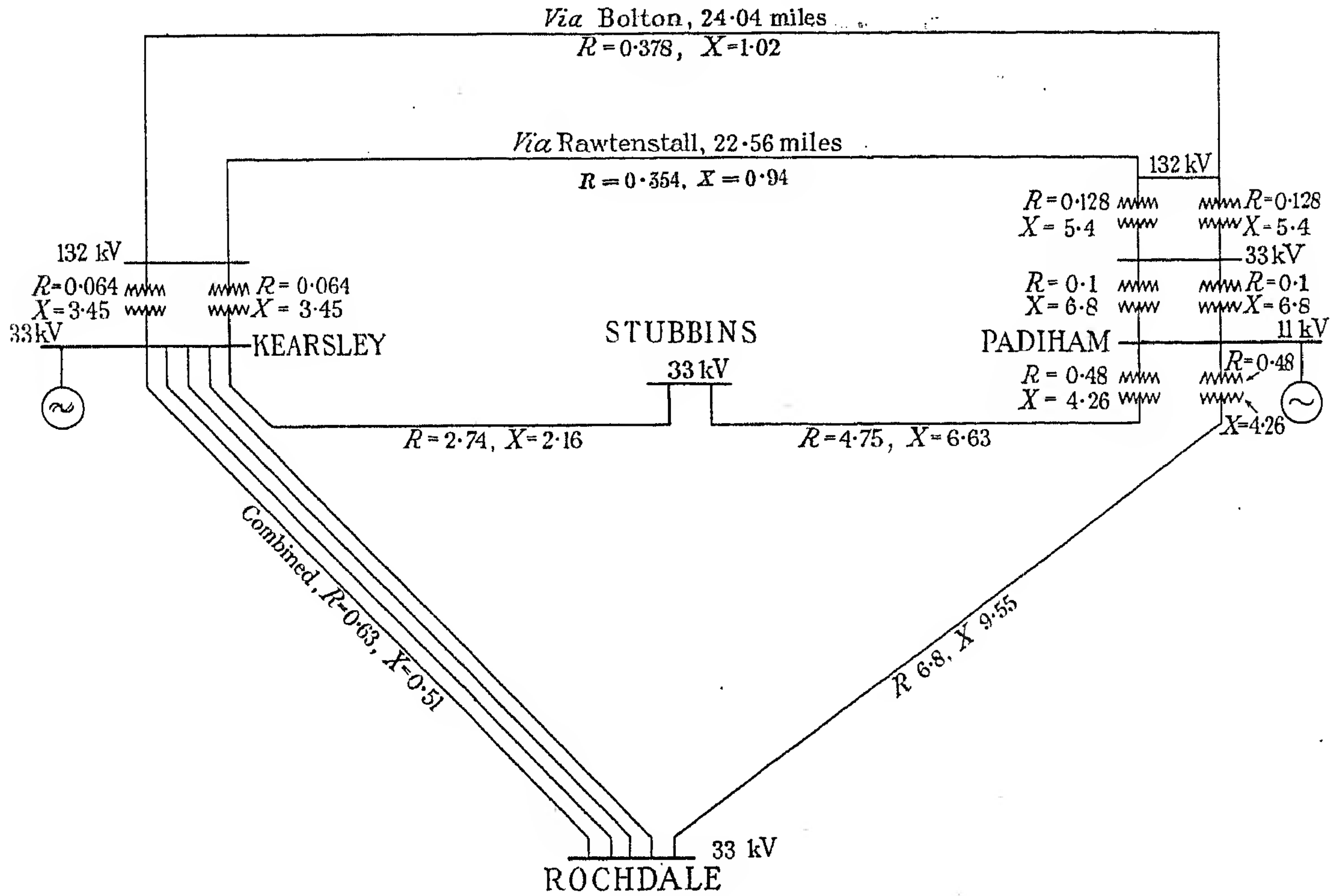

Fig. 16.

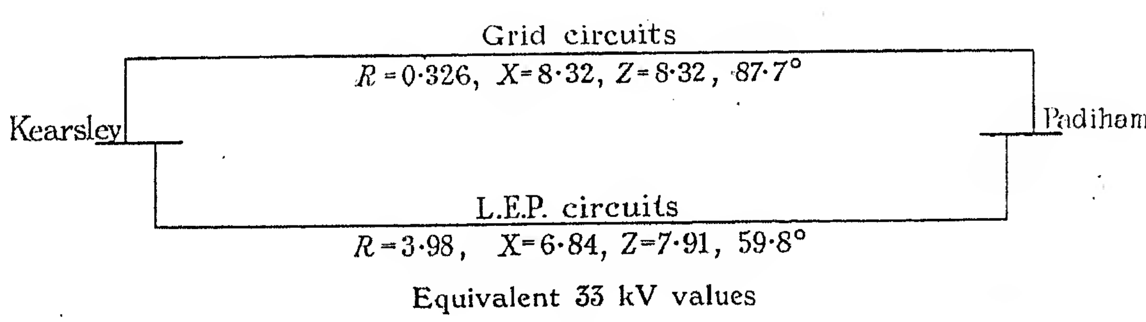

Fig. 17.

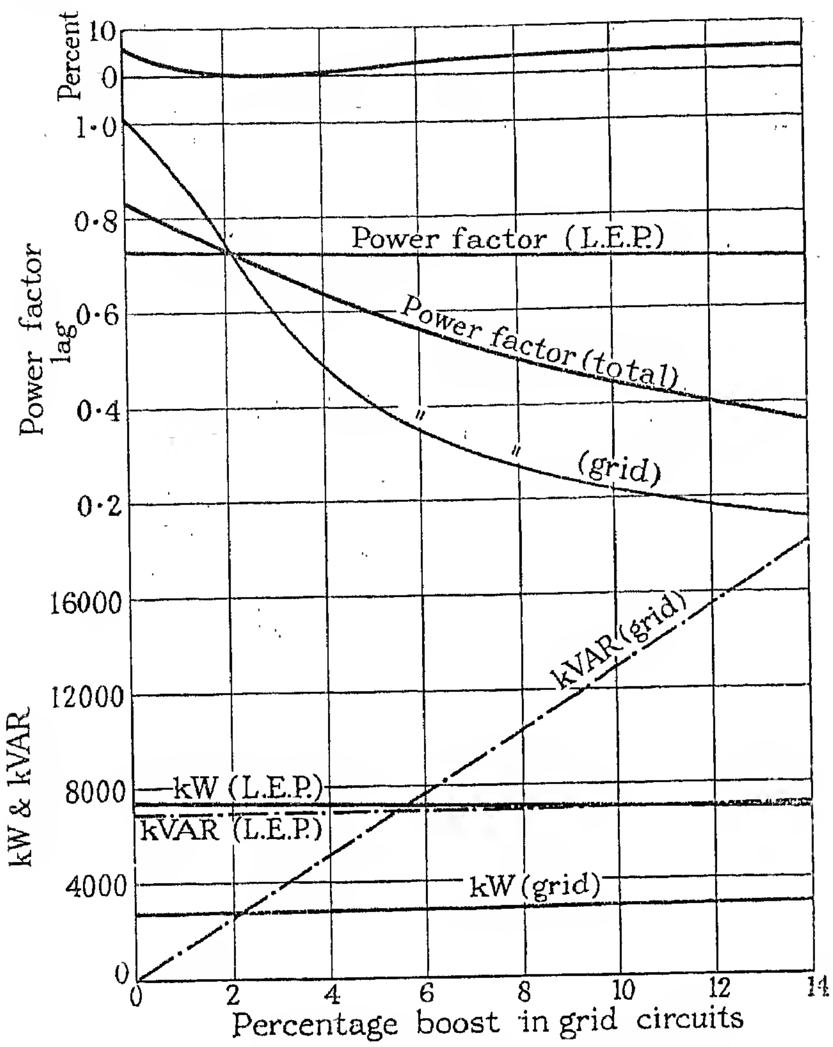

FIG. 18.—Results obtained with 7 per cent boost in Lancashire Electric Power Co.'s circuits.

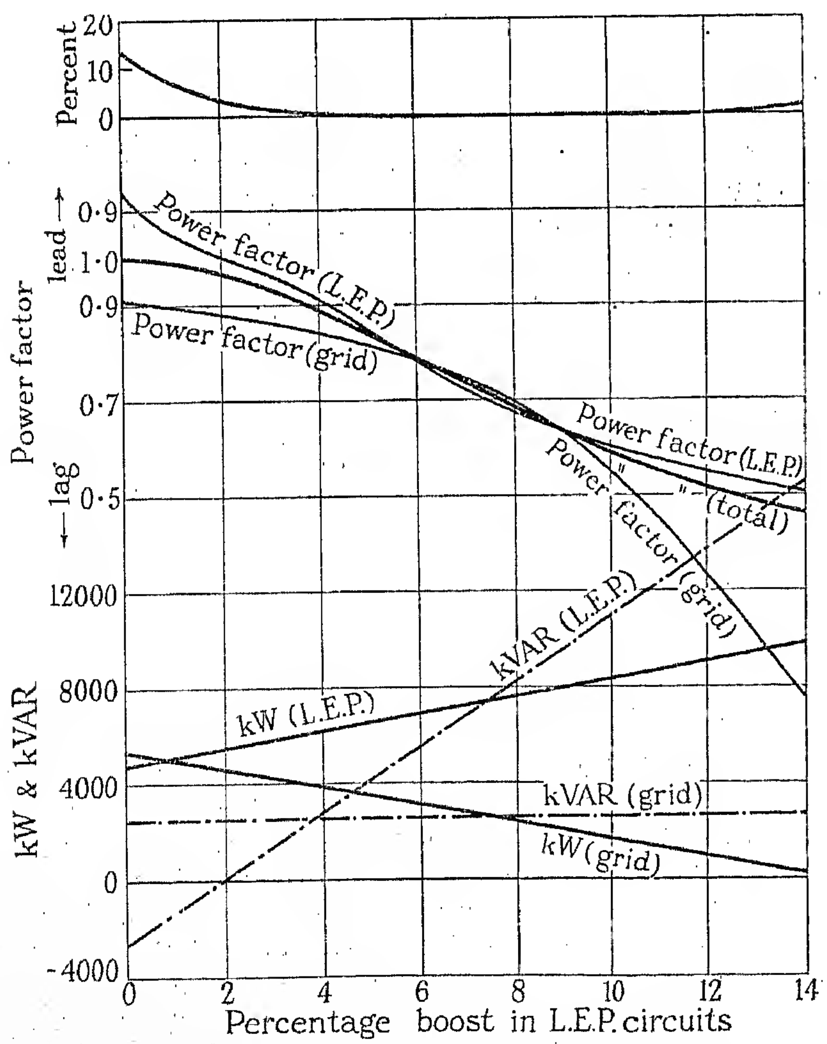

FIG. 19.—Results obtained with 2 per cent boost in grid circuits.

obtain the loading conditions desired, which are that loads up to 10 000 kW should be carried on the L.E.P. interconnections, as they were before the grid connections were made, the actual conditions approximate sufficiently closely to this to make operation with the double interconnection satisfactory. It should be noted that the double interconnection would not be satisfactory if it were necessary to transfer the load over the grid circuits, as the L.E.P. circuits would carry too large a proportion of the load.

The grid and L.E.P. circuits between Kearsley and Padiham were first paralleled on the 17th February,

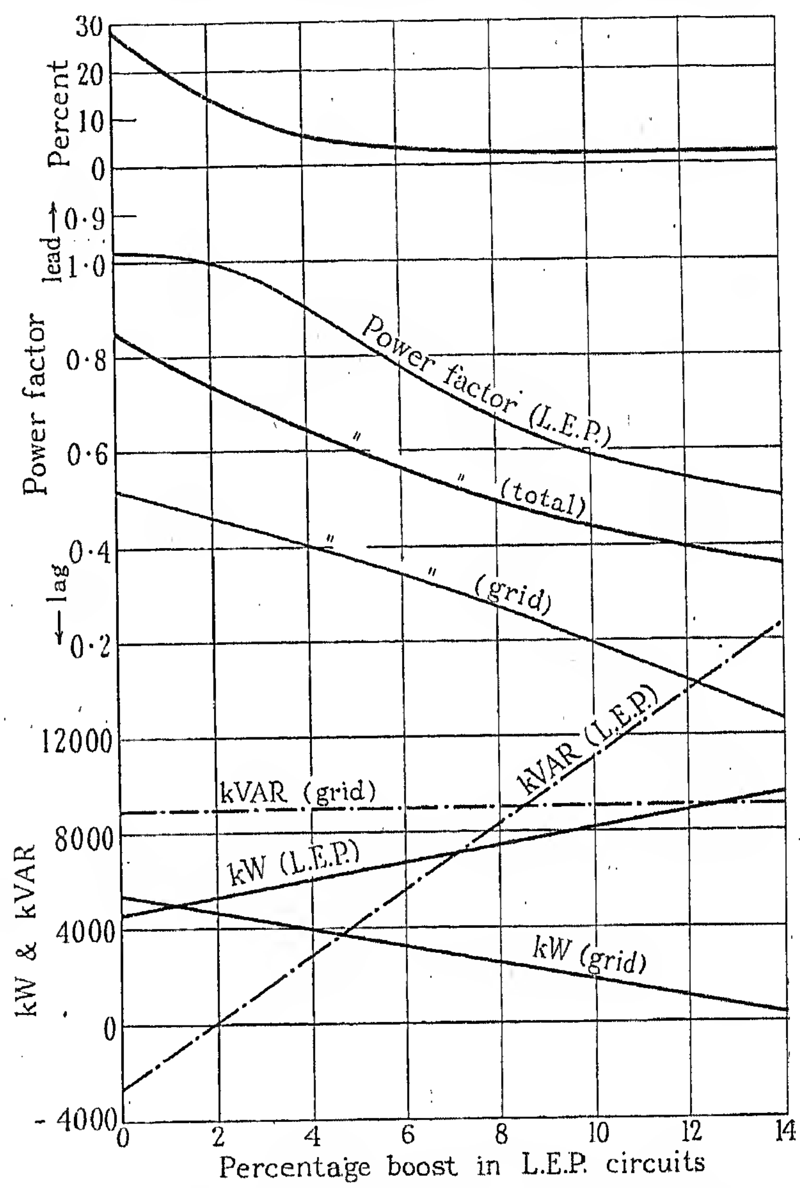

FIG. 20.—Results obtained with 7 per cent boost in grid circuits.

1934, and on the 20th February observations were made of the effect of regulating the boosters on the L.E.P. transformer circuits and the taps on the grid transformers at Padiham, with constant conditions at Kearsley. Both 33/11-kV transformer taps were operated simultaneously and also both 132/33-kV transformer taps. The results are shown in the Table. During the period a small amount of load (between 200 and 500 kW) was exported to the feeders connected to the grid 33-kV busbars: this has been ignored in the Table. A comparison of observations 1 and 2 shows that a reduction of boost in the L.E.P. circuits results in a reduction of kW in L.E.P. and an increase in grid circuits: the kVAR in the L.E.P. circuits is considerably reduced and remains

approximately the same on the grid circuits. Observations 3 and 4 show an increase of boost in the L.E.P. circuits to have the opposite effects. These confirm Figs. 19 and 20. Observations 2 and 3 show that an increase of boost in the grid circuits decreases the kW and power factor slightly in the L.E.P. circuits and increases the kVAR considerably in the grid circuits. Observations 4 and 5 show that a decrease of boost in the grid circuits causes a reduction (amounting to reversal) of the kVAR loading in the grid circuits, with little effect upon the L.E.P. circuits. These confirm Fig. 18.

A consideration of relative impedances of the 132-kV lines and the transformers in the grid circuits shown in Fig. 16 indicates that a considerable load could be transferred over the lines without appreciable interference with the characteristics shown in Figs. 18, 19, and 20. Load could be exported or imported, within limits, at each station simultaneously, but if an attempt were made to export at one station and import at the other this would be equivalent to transferring load, and the Table shows that under various tap conditions the transfer of loads of 10 000 kW and above involves carrying most of the load in the L.E.P. interconnectors.

Table.

C.E.B. 132/33-kV Transformers: Taps numbered 1–15, ratios 145/33 to 119/33, lowest tap, highest ratio.
C.E.B. 33/11-kV Transformers: Taps numbered 1–11, ratios 35·5/11 to 30·5/11, lowest tap, highest ratio.
All power factors are lagging. Kilovars are lagging when + and leading when —.

Observation number	Time	Padiham Generator			L.E.P. Circuits				C.E.B. Circuits			
					To Kearsley		To Rochdale		33/11 Tap number	132/33 Tap number	Export	
		kW	Power factor	Busbar voltage	kW	Power factor	kW	Power factor			kW	kVAR
1	11.30	13 400	0·76	10 600	3 300	0·65	3 200	0·6	3	9	3 000	+ 500
					Boosters adjusted							
2	11.40	13 300	0·94	10 700	2 700	0·94	2 600	0·86	3	9	4 200	+ 800
3	11.50	13 300	0·86	10 600	2 500	0·9	2 500	0·84	2	8	4 200	+3 400
					Boosters adjusted							
4	12.0	13 600	0·78	10 600	3 800	0·78	3 000	0·78	2	8	3 500	+3 300
5	12.5	13 400	0·84	10 650	4 100	0·7	3 100	0·7	4	10	3 100	—1 700
6	12.10	12 300	0·78	10 600	3 100	0·76	2 500	0·78	2	8	3 500	+2 800
7	12.20	6 000	0·82	10 550	100	—	200	0·96	2	8	2 000	+3 000
8	12.25	5 900	0·86	10 600	500	0·8	300	0·94	3	9	1 800	+ 700

Conclusions.

Satisfactory control of voltage and power factor on a system interconnected with other systems requires reasonably fine adjustment of boost in the interconnector circuit, more especially where the capacity of the interconnection is large in relation to the plant capacity of the station. When such is the case the exact adjustment of the busbar voltage at the station will depend upon the power-factor range over which the generating plant can be operated being sufficient to cater for the difference between steps on the step-by-step booster or on the on-load tap-change transformer.

Interconnector circuits, whether used solely for transfer of load between stations or not, cannot be indiscriminately paralleled. If the ratios of reactance to resistance in the circuits paralleled are not approximately equal they cannot be economically loaded unless an adjustable quadrature voltage is provided in addition to the normal in-phase boost. In cases met with in practice, however, it is usually possible to operate over a satisfactory range of conditions. In some cases, advancing the phase of the boost voltage 30° or 60°, or adding reactance to one of the circuits, will enable satisfactory results to be obtained. General rules cannot be laid down and each case must be considered on its merits.

The author wishes to express his thanks to Mr. C. D. Taite, engineer and manager of the Lancashire Electric Power Co., for permission to make use of data obtained in the course of his duties, and for the facilities granted to him during the preparation of the paper.

APPENDIX.

(*Received 28th February*, 1935.)

This appendix shows the method adopted to calculate the loading in parallel interconnecting circuits under various conditions of boost. The figures used in the example are based upon Fig. 7 in the paper, and the conditions of loading when 10 000 kW are transferred with (1) no boost, and (2) with 5 per cent boost in line (*a*), are calculated. The same method of calculation has been used for all the curves and for the vector diagrams in Fig. 9. In the calculations it has been assumed that the voltage due to phase displacement is 90° different from the receiver-station voltage.

A difference of V volts between two stations running at equal voltages will be approximately 90° different in

phase to each and will produce in lines (*a*) and (*b*) of Fig. 7 the following currents:—

Line (*a*): $V/1\cdot 117$ amperes.
Line (*b*): $V/1\cdot 67$ amperes.

The components of the currents which are in phase with the station voltages will be:—

$$\text{Line } (a): \frac{X_a}{Z_a} \times \frac{V}{1\cdot 117} = \frac{0\cdot 5V}{(1\cdot 117)^2} = 0\cdot 401V$$

$$\text{Line } (b): \frac{X_b}{Z_b} \times \frac{V}{1\cdot 67} = \frac{1\cdot 67V}{(1\cdot 67)^2} = 0\cdot 599V$$

The proportion of kW carried by each line with equal busbar voltages is therefore:—

$$\text{Line } (a): \frac{0\cdot 401}{0\cdot 401 + 0\cdot 599} = 0\cdot 401$$

$$\text{Line } (b): \frac{0\cdot 599}{0\cdot 599 + 0\cdot 401} = 0\cdot 599$$

The relation between the kVAR and the kW in each line is:—

$$\text{Line } (a): \frac{R_a}{X_a} = \frac{1\cdot 0}{0\cdot 5} = 2\cdot 0$$

$$\text{Line } (b): \frac{R_b}{X_b} = \frac{0\cdot 05}{1\cdot 67} = 0\cdot 03$$

and the kVAR will be leading.

If we assume 10 000 kW transferred from one station to the other with equal busbar voltages and without boost, the kW loading of the lines will be:—

Line (*a*): $0\cdot 401 \times 10\,000 = 4\,010$ kW
Line (*b*): $0\cdot 599 \times 10\,000 = 5\,990$ kW

The kVAR loading of the lines will be:—

Line (*a*): $2\cdot 0 \times 4\,010 = 8\,020$ kVAR (leading)
Line (*b*): $0\cdot 03 \times 5\,990 = 180$ kVAR (leading)

a total of 8 200 kVA (leading).

If a boost of 5 per cent is applied in line (*a*) the load transferred due to this boost will be as follows:—

$$\text{Boost voltage per phase} = \frac{0\cdot 05 \times 11\,000}{\sqrt{3}} = 317.$$

$$\text{Load transferred} = \frac{317}{Z_a} \times \frac{R_a}{Z_a} \times \frac{11\,000 \times \sqrt{3}}{1\,000}$$
$$= \frac{317 \times 1\cdot 0}{(1\cdot 117)^2} \times \frac{11\,000 \times \sqrt{3}}{1\,000} = 4\,840 \text{ kW}$$

And the kVAR transferred due to this boost voltage will be

$$\frac{317}{Z_a} \times \frac{X_a}{Z_a} \times \frac{11\,000 \times \sqrt{3}}{1\,000} = \frac{317 \times 0\cdot 5}{(1\cdot 117)^2}$$
$$\times \frac{11\,000 \times \sqrt{3}}{1\,000} = 2\,420 \text{ kVAR (lagging)}.$$

If a total of 10 000 kW are to be transferred and the boost voltage in line (*a*) will cause 4 840 kW to be transferred, then the balance of 5 160 kW must be transferred by phase swing. The kW in each line due to phase swing will be:—

Line (*a*): $0\cdot 401 \times 5\,160 = 2\,070$ kW
Line (*b*): $0\cdot 599 \times 5\,160 = 3\,090$ kW

And the kVAR loading due to phase swing will be:—

Line (*a*): $2\cdot 0 \times 2\,070 = 4\,140$ kVAR (leading)
Line (*b*): $0\cdot 03 \times 3\,090 = 93$ kVAR (leading)

The total load in line (*a*) will be

$$4\,840 + 2\,070 = 6\,910 \text{ kW}$$

and

2 420 (lagging) + 4 140 (leading) = 1 720 kVAR (leading).

The load in line (*b*) as shown above is 3 090 kW and 93 kVAR (leading).

The total load on the two lines is

$$6\,910 + 3\,090 = 10\,000 \text{ kW}$$

and 1 720 + 93 = 1 813 kVAR (leading).

Discussion before the Transmission Section, 19th December, 1934.

Mr. J. R. Beard: The most interesting part of the paper to me is that in which the author deals with the difficulties involved in the use of parallel interconnectors with widely different characteristics. This is not a new problem; in fact it was met quite early on in the pioneer development of interconnection of the North-East Coast, when the 20-kV system was superimposed on the original 5 750-volt system, and again at a later stage when the 66-kV system was superimposed on the 20-kV system. To-day, of course, there is, in addition, the 132-kV grid system superimposed on the 66-kV system of the North-Eastern Electric Supply Co., and for the moment it looks as if a certain amount of finality has been reached; although, if electricity supply develops as we hope, it may not be long before even 132 kV may seem inadequate for our requirements.

No doubt similar problems have also arisen as other interconnected systems have developed, but it is perhaps the advent of the grid which has brought them more forcibly before our notice, as this has provided parallel interconnections to most of the main interconnections which originally existed between different undertakings. On the whole, however, the operating difficulties which have been encountered have been much less than might have been anticipated.

If one comes down to fundamentals it is, as the author says in his Conclusions, impossible to load with the maximum economy parallel circuits of unequal characteristics unless an adjustable quadrature voltage is provided in addition to the normal in-phase boost. I think this indicates that there is very considerable scope for providing improved voltage-control systems which will help to meet such conditions, although, as the author says, in many instances fairly satisfactory results can be obtained by advancing the phase of the boost voltage 30° or 60°. It may be of interest to record that this

latter method was adopted some considerable number of years ago on the 66-kV interconnectors between the Tees and the Tyne, which were paralleled by the existing 20-kV system. Such methods have, however, the disadvantage that while a system can be adjusted for one set of loading conditions it will require to be readjusted every time the loading varies.

My own experience has been that the problem we are considering is usually a transitory one which tends to crop up each time a step up is taken in transmission voltage. Gradually, as the new higher-voltage system expands, it assumes relatively greater importance, and the original high-voltage system tends to subside into mere distribution networks; its importance for interconnection becomes negligible, and little difficulty is experienced in breaking various interconnections and forming a series of separate networks fed from the new higher-voltage system. This is the course things took in my own experience on Tyneside, where the Carville and Dunston stations originally fed into a common 5 750-volt network on which was superimposed a 20-kV network. In time the lower-voltage network was split into two, one based on Carville and one on Dunston. A similar sort of development is at present taking place as between the 66-kV system on the North-East Coast and the original 20-kV system.

On the grid system itself, quite a considerable amount of trouble has been taken to avoid the secondary distribution networks forming parallel paths to the main 132-kV grid, and experience has shown that this has conduced to simplification of the operation of the system. In many instances too where existing networks have paralleled grid points, arrangements have been mutually made whereby the lower-voltage interconnectors are split and converted into distribution feeders. This is a tendency which will tend to grow, but at the same time the problem presented by the paper is one which I think will always be with us to some extent and will well repay detailed study.

These problems raise somewhat urgently the importance of the development of network calculating tables which are not subject to the limitations of the majority of the present tables. The latter cannot take account of the reactances and resistances of the various sections of a network, but only of the reactance or resistance alone. Such tables are obviously quite incapable of dealing with such problems as those referred to in the paper.

Mr. W. Fennell: A number of the illustrations included in the paper refer to grid circuits operating in parallel with existing circuits; in this connection I should like to point out that the Central Electricity Board will probably not allow much freedom for what I will call "private" interconnections between points connected to the grid. Apart altogether from the question of distribution of the load, there is the question of control. It is, of course, almost certain that, in view of the very large capacity of the grid between any two points compared with the size of almost any of the private interconnections, the power companies will not want to make such interconnections. They will use their pre-grid interconnectors as local high-voltage distributors to points of their areas. As base-load power stations grow, the transference of 10 000 kW from one station to another will not—normally, at any rate—be a matter of great importance. In most areas there is a reluctance on the part of the Central Electricity Board to giving more than one connection to each undertaking, however large. In the N.W. England Area, the North Wales Power Co. is a case in point; their huge area has only one grid connection. With regard to the author's suggestion that he has confirmed the figures of the kilowatts that can be passed without instability, I notice that he stopped just below the limiting value to which the formula stated matters could go. I suggest that he has only confirmed that power can be transmitted up to that limit, but that he is not quite sure that it could not be transmitted beyond.

Throughout the paper the author refers to transmitting power from the larger to the smaller station; not once does he mention getting power from the smaller to the larger. Under certain conditions there is a temporary need to bolster up the largest power station by getting all the small ones around it to feed into its section. Does the direction in which the power is transmitted make any difference to the problem? If so, trouble will arise when a fault develops and sections get isolated; it is then necessary to rely on even quite small stations until conditions have improved.

Mr. C. W. Marshall: In contradistinction to the previous speakers, I am of the opinion that phase control is an essential part of the development of interconnected systems, and that it is impossible to attain maximum economy without it, despite the continual splitting-up of voltage systems and the process of associating them with still higher voltages.

With regard to the author's theoretical treatment of the subject, I am unable to see why one should bring the term "power factor" into consideration. The problem seems to me purely a question of balancing kilowatts and kilovars, and of taking care of the load capacities of plant by considerations of maximum current. I should like to see the term "power factor" eliminated. Mr. Fennell mentioned the shortcomings of the stability test which the author made; what we really want to know is whether any change in conditions was made at the time of maximum transfer, to determine whether the transfer was in fact stable.

With regard to the types of phase-control apparatus which the author mentions, there is no doubt that the induction regulator is the most elegant one, but it is unfortunately costly and difficult to safeguard against short-circuit forces. There seems to me to be no escape from the use of tap-change equipment with appropriate phase displacement. Regarding the fineness of the steps, I fail to see how it is practicable to get much below the $1\frac{1}{2}$ per cent steps which have been standardized on the grid, without unduly increasing the cost of control equipment. The author introduces a detail point in connection with the protective equipment for an induction regulator. I should be interested to know what steps he takes to ensure stability of the protection against inadvertent tripping on transients, such as are due to switching.

He naturally confines his paper to steady conditions, but I think it should be observed that in the problem

of dynamic stability great benefit is to be derived from phase-regulating equipment of interconnectors, in that it enables the alternators to operate at the condition of maximum stability. So far, our experience in regard to stability has been very good, but there is no doubt that there are definite possibilities of serious instability occurring—the more so as the load increases. I can instance* the case of a German experiment on instability where two large alternators were arranged to work in parallel but at different power factors, supplying load on the same busbars; when the load was switched out the two alternators ran out of step, although they were identical machines in all respects. This illustrates clearly the necessity for keeping similar conditions on the electrical side and on the steam side so as to ensure transient stability.

Mr. E. T. Norris: It is important to bear in mind the very varied uses of interconnectors. The necessity for a wide range of types of regulating gear depends entirely upon those uses. Interconnectors may be used for controlling the load distribution, the wattless-load distribution, or even, on occasion, the voltage itself. The functions of the control of load may be either to utilize more efficiently the capacity of the stations available or to run them more economically. There is a further use for regulators (see Fig. 7) which has nothing to do with control of load distribution between stations or between different parts of a system, namely, the control of load distribution between the interconnectors themselves where these are of different lengths or different impedance values.

With regard to means of regulation, on-load tap-changing on the main transformers is always the cheapest and generally the most satisfactory method, and it also has the advantage of giving the least losses. In many cases where main transformers are not available, boosting transformers with on-load tap-changing gear are the most economical proposition. The choice is partly determined by the question of size of step, with which the author has dealt very clearly. In many cases the interconnector is smaller, in comparison with the stations it is interconnecting, than his examples suggest; and a larger step is permissible. To use a very small step not only puts up the cost of on-load tap-changing gear but may even make it prohibitive, because it is not practicable to provide a large number of steps, bearing in mind that all these steps have to be suitable for permanent running, and not merely for use in the course of changing from one running position to another. Where the step has to be very small, either induction regulators or moving-coil regulators must be used.

The author showed two lantern slides of moving-coil regulators; the larger one of these is actually in use now on an interconnector. (Mr. Norris here exhibited a lantern slide.) This is a moving-coil regulator in a 10 000-kVA interconnector operating at 6 kV. Since there is no phase displacement, twin regulators are not required. (Mr. Norris here exhibited another lantern slide.) This shows a large on-load tap-changing booster with a phase angle which can be set at either 30° or 60°, giving 24 per cent voltage variation. Two 24 000-kVA transformers are employed, each controlling a 66-kV interconnector.

Mr. W. E. M. Ayres: The author's diagrams show the absolute necessity of the flexibility given by wattless current for the paralleling and satisfactory running of power stations together. This is one of the features which makes a.c. engineering possible, apart from being one of its difficulties. Mention is made on page 354 of the question of stability, and the necessity of avoiding a big angle of displacement between the voltages of the two stations. I saw some time ago a model representing two generators connected by a transmission line, consisting of various pulleys with weights and levers to represent the loads inclined at the angle of the alternator fields. These were connected together by a spring at the top, and it was easy to see how, with increasing weight, these angles opened out. When the levers reached an inclination of 90° they simply fell down, and ceased to support any more load. The model had several other interesting features. For instance, if it was set for a certain angle rather less than 90°, when a load was slowly applied in the centre of the spring the levers opened out to 90° and then fell out; yet when a much heavier load was applied with a sudden impulse, the levers did not fall out. That kind of phenomenon has actually been encountered in transmission-line circuits when the impulse has been represented by a fault which has been quickly cleared.

As regards the apparatus available, for the grid circuits I would not suggest anything but on-load tap-changing transformers. They are compact, efficient, and exactly the apparatus to use. The question of the size of the steps is important, particularly when one has different circuits in parallel. The booster transformer and the induction regulator are also suitable, but are more expensive. The booster is not always the better as regards price or losses: each case had to be looked into individually, and the choice depends largely on the size and the number of steps required.

I should like to refer to a 20 000-kW transmission line in Japan for interconnecting a steam station with a water-power station some 80 miles away in the hills. On account of the heavy rating of the line in comparison with the generating plants, it is necessary to have very small steps. The water-power station is run to its capacity, and has to receive power on its busbars during its heavy peaks. It also runs through the night to supply power the other way, so that the steam sets can be shut down. Twin induction regulators are used, with fully automatic control. The system has been in operation for a number of years, with very great success. One would have thought that a case like this might have been one for the synchronous condenser, but on account of the characteristics of the line a suitable synchronous condenser would require so large a kVA rating that it would cost as much as another power station; therefore, a much cheaper and more efficient way is to apply voltage control. In deciding what kind of regulator one must use, and also in making the calculations for predetermining its effect, one must take into account the impedance of the regulator, which must be lumped with that of the line; and, particularly if one is trying to forecast results, one must take into

* R. Rudenberg: "Electrical Switching Procedure" (Julius Springer), 3rd ed., p. 169.

account the differences in the impedance of the regulator at different positions.

The system of transformer protection shown on page 356, which summates the primary and secondary currents on one side against the primary current on the other, has been very successful. It was applied to a large number of regulators for the Sydney Municipal Council as far back as 1925. It has, however, only been applied to large units. Small and medium-size regulators are connected solid in the line without any protection.

The author refers on page 358 to the additional boosters required for quadrature control. From Figs. 10, 11, and 12, one can see that quadrature control would have greatly improved the circumstances. In the case illustrated in Fig. 16, in the connection between Padiham and Kearsley, quadrature control would certainly have controlled the power flow in the grid; but, as has been pointed out in previous papers (for instance, in the one I read in 1931)*, this effect can be obtained with infinitely-variable twin induction regulators, used for the dual purpose of voltage and phase-angle variation. Only slight modification to the operating gear is required.

The author also mentions the problem of the different characteristics of parallel lines. I should like to ask whether he has ever actually tried loading with reactance in order to make the characteristics more equal. In one case, a cable was reinforced with an overhead line. The two feeders were paralleled and put on to the line together, and everything went well until one branch was switched out, when the whole section tripped. It was quite impossible to get the section to stay in unless both branches were put in and made alive together. It was found that there was an appreciable difference in phase angle owing to the unequal impedance characteristics.

In conclusion, I should like to suggest that the value of the paper would be greatly increased if the author added an appendix giving the methods of arriving at his curves.

Mr. H. P. Young (*communicated*): A paper with so comprehensive a title might be expected to deal critically with the competing apparatus for the control of power factor and voltage, and it is rather disappointing that the author's wide experience in this direction has found so little expression. The conditions represented in Fig. 5 and analysed on pages 355 and 356 are those commonly met with in this country, and the author's analysis clearly demonstrates the necessity for apparatus which will give an infinitely variable boost voltage in the interconnector. It must be remembered that first cost is not everything, and in any case the cost of the boosting plant forms a very small proportion of the total. To install apparatus on an important interconnector by which the boost can only be varied in relatively coarse steps, and thereby sacrifice the economic control of current, does not seem to be inherently sound. It is probable that in the future the polyphase induction regulator will receive greater consideration, since it possesses advantages in addition to an infinitely variable boost; no switching of high-current or high-voltage circuits is necessary, with the result that maintenance charges are low and there is a high factor of safety against breakdown. Also, the large high-voltage interconnector transformers are left in their simplest form by the removal of the boosting tappings therefrom. Again, in the case of interconnectors which are frequently tapped along their routes for supplies of energy, it is possible that synchronous condensers may find a field of employment, particularly in the case of long and heavily-loaded interconnectors.

The author exhaustively analyses the conditions of power factor under which energy may be transferred between power stations; Fig. 9, which shows separately the kVA transferred due to a phase-difference between the busbar voltages and the kVA transferred due to an in-phase voltage boost, makes the analysis very clear. The diagrams merit careful study, showing as they do that interconnectors having widely different circuit constants cannot be paralleled without due consideration of the boosting requirements. This section of the paper would be rendered even more valuable if the author would add an appendix giving samples of his calculations relating to the various graphs.*

Mr. O. Howarth (*in reply*): Mr. Beard summarizes the developments which have taken place in the past on the North-East Coast in the paralleling of interconnectors. It would appear that the exact adjustment of loads on the interconnectors has not been essential, and in consequence it has not been necessary to provide a quadrature boost voltage in order to load the lines accurately. The advent of the grid interconnectors in parallel with undertakers' lines involves dual control and introduces complications due to metering. These factors do introduce additional difficulties in operation.

Mr. Fennell is correct in stating that the test referred to on page 355 only confirmed that the kilowatts of plant referred to could be held in synchronism without determining the limiting figure. It was not desirable to go farther, owing to the risk of interfering with the supply to the two systems. The conditions discussed in the latter part of the paper refer to the transfer of power from the smaller (Padiham) station to the larger (Kearsley) station. The problem is the same whichever the direction of flow of power.

It is interesting to note that Mr. Marshall is of the opinion that phase control will be essential in the development of interconnected systems. He would like to see the term "power factor" eliminated; no doubt engineers can get used to kilovars, but many are used to operating with 360° scale power-factor indicators, and they find the side-zero kilovar indicators with reversing switch, which have been installed in the North-Western Area, distinctly awkward compared with the power-factor indicator. In reply to his question, the stability test was made under normal running conditions, and nothing beyond ordinary load and power-factor changes took place. The load on the interconnector varied through very wide limits, as one would expect with a circuit rated at only about 15 per cent of the load on one of the stations, but there was no evidence of instability. In the protective scheme shown in Fig. 6 the usual precautions are taken to ensure stability. Biasing transformers are used on one equipment, and induction relays with a slight time-lag on others. The inclusion of the booster within the protective scheme does not

* *Journal I.E.E.*, 1931, vol. 69, p. 1208.

* This has since been done (see page 363).

appear to have introduced any appreciable risk of instability.

It is interesting to hear from Mr. Norris that a moving-coil regulator is now in use on a 10 000-kVA 6-kV interconnector. No doubt it is difficult and costly to provide a larger number of smaller steps on an on-load tap-change transformer, but I would again emphasize that coarse adjustment of the voltage at the station busbars will inevitably increase the cost of maintaining the voltage within the prescribed limits at the consumer's terminals.

The case of phase-swing of sufficient magnitude to trip out one line when another was switched out, mentioned by Mr. Ayres, is interesting and emphasizes the importance of consideration being given to these matters when alterations or additions to a system are contemplated. It should be noted that this effect is not confined to two power stations connected together, but can occur where suitable conditions exist between a generating station and any kind of rotary load.

Mr. Young considers that more attention should have been given to a critical examination of the competing apparatus for the control of voltage and power factor, but it was felt that an examination of the facilities necessary to control the voltage and power factor would be more useful. In response to the suggestion made by Mr. Young and by Mr. Ayres, an Appendix has been added* giving samples of the calculations relating to the graphs.

* See page 363.

SOME SUGGESTIONS ON THE EQUIPMENT AND ROUTINE OF THE METER DEPARTMENTS OF SUPPLY UNDERTAKINGS.

By J. L. FERNS, B.Sc., Graduate.

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SUMMARY.

The increasing use of alternating current for distribution purposes has considerably altered the character of the work done in many meter departments. A.C. meter testing now exceeds d.c. meter testing in regard to the quantity of work involved, and, furthermore, the imminent reductions in the cost of electricity will lead to still further demands for a.c. meters. In consequence of these changes the testing equipment of many undertakings will require extension or redesign, and Parts 1 and 2 of the paper contain suggestions to help the meter engineer in this task.

As the work of the meter engineer involves the accurate measurement of voltage, current, and power, it is essential that the errors of the testing instruments should be determined from time to time. It should be possible for all undertakings having more than 5 000 consumers to carry out this operation on their own premises, and Part 3 suggests the minimum of standardizing equipment with which they should be provided.

Although, owing to its almost ideal construction, it was found possible to dispense with the rotor clamp in the induction meter, this omission is not an unmixed blessing. If the light-weight induction meter is to be given the same treatment in transport as the robust, clamped-type, d.c. meter, then the removal of the clamp is to be regretted. Part 4 deals with this matter and with other points connected with the installation of meters.

After the meters have been installed on consumers' premises the meter engineer has by no means finished with them—even neglecting the question of meter reading. The meters will create various types of complaints, and naturally faults will occur from time to time. In Part 5 the equipment for dealing with this work is discussed, and the correct attitude towards meter maintenance is defined.

Part 1. SINGLE-PHASE TESTING EQUIPMENTS.

Although the adaptability of the transformer makes it easy to provide a suitable voltage or current supply separately, it is not quite so easy—with limited finances—to provide a voltage and current supply which permits the phase angle between the voltage and current to be adequately controlled. Before the suggested type of equipment is described, a number of existing methods of providing current supplies and phase-angle control are discussed. Without this review it might appear that the other possibilities of solving this problem had been overlooked.

CURRENT SUPPLIES.

Type A.

The load to be tested, usually composed of meter series-coils or current-transformer primary windings, is connected directly to the secondary terminals of a special step-down transformer, and the load current is controlled by a rheostat in the primary circuit (see Fig. 1, type A). Coarse adjustment may also be obtained by tap-changing on the secondary winding of the step-down transformer.

The drawback to this circuit is that the loading transformer is not working under the proper conditions if the

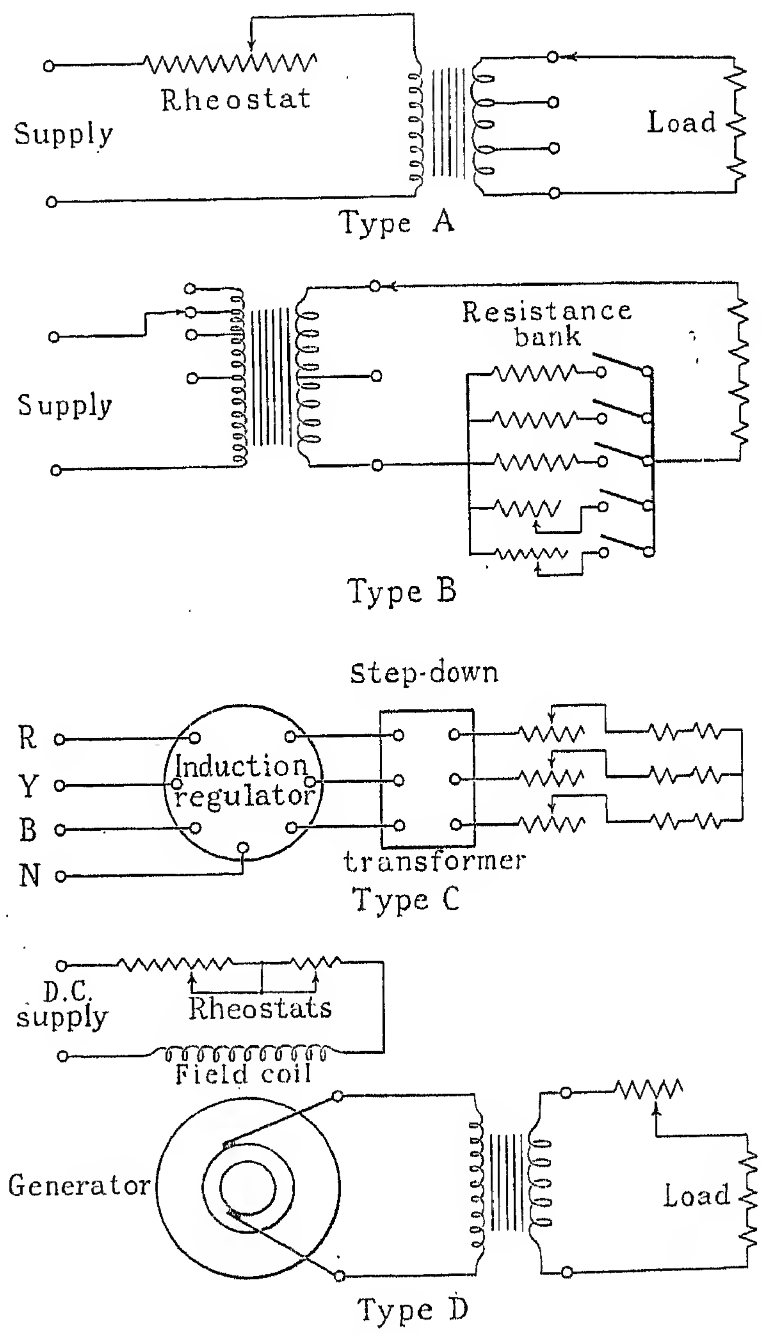

FIG. 1.

flux density in the core is not kept very low. Unless precautions are taken to prevent misuse (e.g. a voltmeter, with a red danger mark, connected across the primary of the loading transformer) this type of supply cannot be recommended. Its great advantage, of course, is the simplicity of the rheostatic control.*

* See Reference (1).

Type B.

This method employs a step-down voltage transformer, and the load current is controlled by a bank of resistances in the secondary circuit (see Fig. 1, type B). Further control may be obtained by tap-changing on the primary or secondary windings.

An objection to this method is that if the magnetizing component of the secondary current becomes appreciable compared with the energy component, the wave-shape may be distorted. The meaning of this statement will be realized if it is remembered that the wave-shape of a current which magnetizes iron, at a high flux density, so as to produce in the windings a sinusoidal voltage, is not itself a sine wave. This disadvantage is overcome by allowing a reasonably high output voltage and giving the resistance bank generous proportions. These precautions are particularly necessary if induction-type overload relays with inverse time characteristics have to be tested.

Type C.

An extension of the previous type of supply is to provide an induction regulator in the primary or secondary circuit of the loading transformer. Its great advantage is the ease with which the current can be regulated, but there is the disadvantage that load adjustment alters the testing power factor unless the Barbour or Ferranti type of regulator is employed.

Type D.

When the loading transformer is fed from an independent motor-alternator set, the coarse and fine regulation of the current can be obtained by rheostats in the field circuit of the alternator. It must be remembered, though, that it is not advisable to draw testing currents from an under-excited alternator, and consequently a resistance bank is still required.

Phase-Angle Control.

Method A.

If the test bench is supplied by a 3-phase 4-wire main, then unity, 0·5 lagging, 0·5 leading, zero lagging, and zero leading, power factors can be obtained—approximately—by merely changing the phase connections of the current or voltage supply leads. The limitation of the method is that the various power factors cannot be accurately set owing to the inherent angle of lag in the current supply circuit.

Method B.

This method is to employ a variable inductance in the secondary circuit of the step-down transformer supplying the load current. Normally it is not practicable to get lower power factors than 0·5 with this device, and to obtain zero lagging power factor one has to employ an arrangement patented by the Metropolitan-Vickers Electrical Co.* Owing to the difficulty of designing a variable inductance to cover a wide range of currents the method is only suited for equipments where the power-factor test is restricted to two or three loads. There is the further disadvantage that the meters are not tested on exactly unity power factor.

* See Reference (2).

Method C.

By means of a phase-splitting device a single-phase supply can be made to produce a rotating magnetic field in a machine similar to an induction motor. The rotor is normally stationary, but its position relative to the stator, and hence the phase of the rotor output, can be varied. If a 2- or 3-phase supply is available it is unnecessary to employ a phase-splitting device to obtain the rotating field in the phase-shifter, as it is termed. This method is the best one available for obtaining phase-angle control.

Method D.

If the testing equipment is supplied by a special motor-alternator set, a second alternator is sometimes put on the same shaft as the main alternator. The auxiliary machine is used to supply the voltage circuit only of the testing equipment, and its stator is so constructed that it can be rotated relative to the stator of the main alternator. Alteration of the stator position naturally alters the phase of the auxiliary alternator output relative to the main alternator output. The position of the movable stator is usually controlled by a small reversible motor operated by a switch on the test bench. This method is preferable to that where both voltage and current supplies are drawn from one alternator, as alterations of the load current do not then affect the testing voltage. It is essential, though, for the wave-shape of each alternator to be as near as possible to the sine wave.

Method E.

If a potentiometer rheostat or choke coil is connected between two phases of a 3-phase supply, it is possible to obtain a supply of variable phase. The output circuit is connected between the neutral or other phase, and some point on the rheostat or tapped coil. The usual form of this apparatus has various disadvantages. As the voltage varies with the tapping point it is necessary to use a variable-ratio auto-transformer between the phase controller and the output circuit. The rheostat form of this device is not desirable on account of the danger of wave-form distortion, whilst the tapped-coil form does not enable fine adjustment to be obtained.

Suggested Type of Single-Phase Testing Equipment.

Bearing in mind the financial and practical difficulties it is evident that the type of equipment supplied from a separate motor-alternator set is out of the question for most testing departments. Therefore the meter engineer must rely on a 3-phase 4-wire supply to the equipment to derive the phase-angle control. If the financial restrictions also prohibit the purchase of a phase-shifter, one is further restricted to methods A, B, and E, for this duty. Individually each of these methods has severe disadvantages, but if methods A and E are combined, as described later, then these disadvantages may be overcome. This solution of the phase-angle control problem automatically involves the use of a type B current supply.

The suggested circuit is shown in Fig. 2, and some idea of the appearance of such an equipment may be gathered from Fig. 3. It is designed with the object of being

built by the undertaking's own engineers. The numerical values given in Fig. 2 are based on the standardized voltages and currents for meters (400, 230, and 110 volts, and 5, 10, 20, and 50 amperes respectively). The voltage supply circuit is simply a variable-ratio transformer fed from the red phase and neutral of the 3-phase 4-wire supply. A transformer is used instead of an auto-transformer, in order to isolate the testing circuit from in Fig. 2. The main phase-angle control is obtained by applying method A to the primary of transformer A, whilst finer adjustment is obtained by means of the rheostat X connected across the secondary terminals of transformer B. The latter device is an adaptation of method E. A potentiometer type of rheostat is necessary because the leading voltage injected by transformer B cannot be made constant owing to variations in the

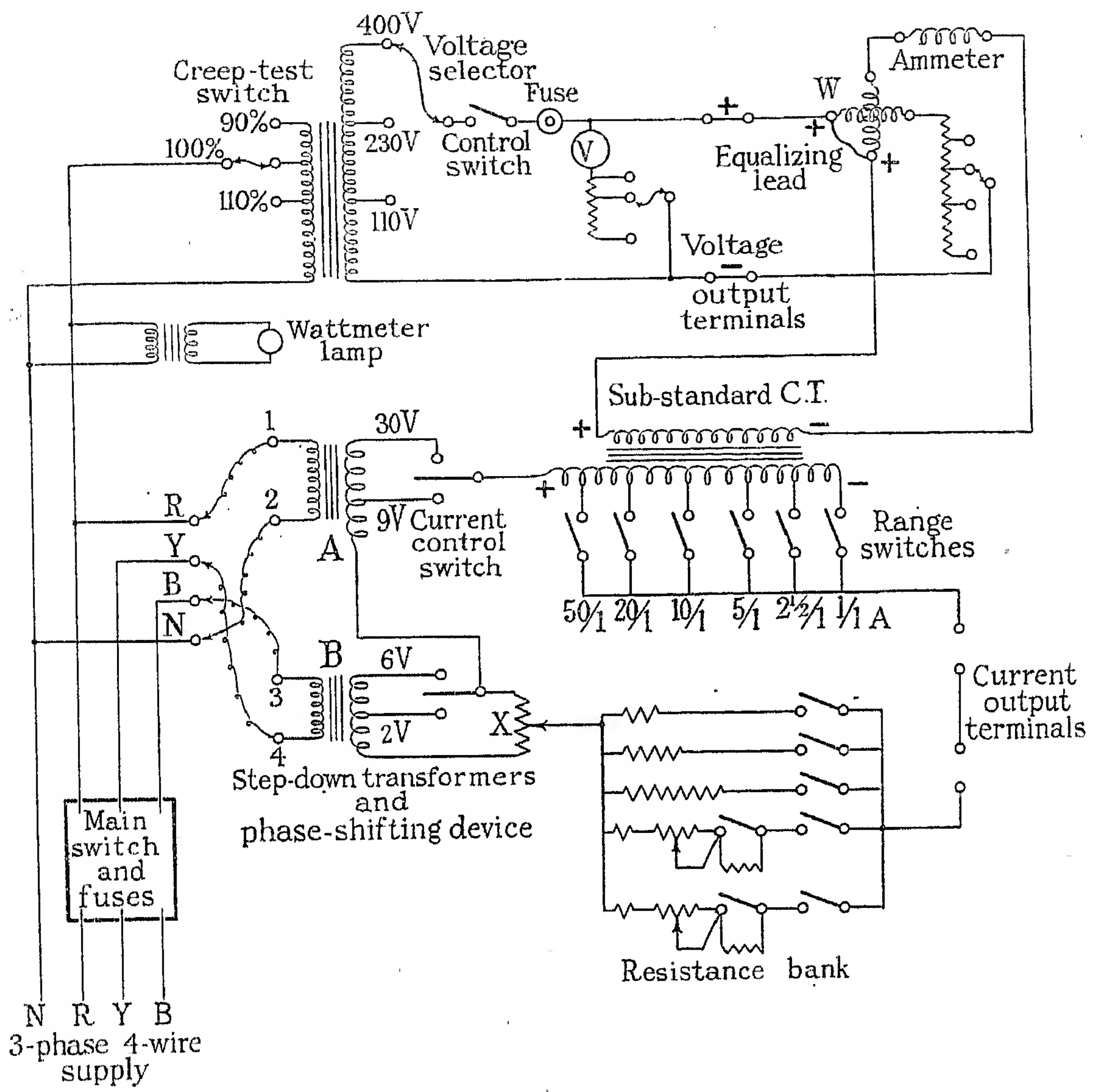

FIG. 2.—Testing circuit for single-phase meters.

Terminal	Power factor			
	Unity	0·5 lag	zero lag	0·5 lead
1	R	N	Y	N
2	N	B	B	Y
3	B	R	R	B
4	Y	Y	N	R

the mains. It will be noticed that provision is made on the primary winding, by means of 100 and 110 per cent taps, for providing the creep testing voltage. This enables the correct creep voltage to be secured irrespective of the particular secondary tap in use. The addition of a 90 per cent tap will also be found very useful, as sample meters usually require to be tested for the effect of voltage variation from normal.

The current supply circuit consists of two step-down transformers, A and B, which are connected as shown inherent angle of lag in the current circuit. The arrangement of plugs and sockets shown in Fig. 3 makes it impossible to cause a short-circuit by an incorrect connection, but an alternative and perhaps better method is to use a polarized 4-pin plug and four sockets. The current circulating in rheostat X is approximately 30 amperes on the maximum tapping, but as the output voltage of transformer B is only one-fifth that of transformer A the I^2R losses are of no importance. For convenience of load control the secondary windings of

transformers A and B are provided with 30 per cent taps. Fine regulation of the load current is obtained by having two graded rheostats in the resistance bank.

Having disposed of the supply problem we next encounter the problem of measuring the voltage, current, and watts (or watt-hours), supplied by the equipment. It is generally admitted that the most accurate way of testing a.c. meters (at the present state of the art) is by comparing them with a wattmeter and a stop-watch, if they are used in the proper manner. Therefore the chief measuring instrument for the suggested equipment is a shielded type of dynamometer wattmeter, having a current rating of 1 ampere, and voltage ratings of 60, 120, 240, and 480 volts. The scale would read from 0 to 120, so as to make the wattmeter-constant unity on the 120-volt range.

The disadvantage of the wattmeter under ordinary circumstances is the unreliability of low-scale readings, but this defect is avoided by feeding the wattmeter series coil from the secondary of a high-accuracy, multi-range, nickel-iron, ring-type current transformer. If the current-transformer ranges are 50/1, 20/1, 10/1, 5/1, 2·5/1, and 1/1, then several advantages are obtained:—

(*a*) The wattmeter current coils can be kept isolated from the main current circuit on all loads. This enables the ± terminals of the wattmeter coils to be commoned without any practical difficulties, such as a dropped voltage link on one of the meters under test, affecting the wattmeter. It also removes the difficulty of maintaining four coils at a common potential when standardizing the wattmeter against a dynamometer secondary standard.

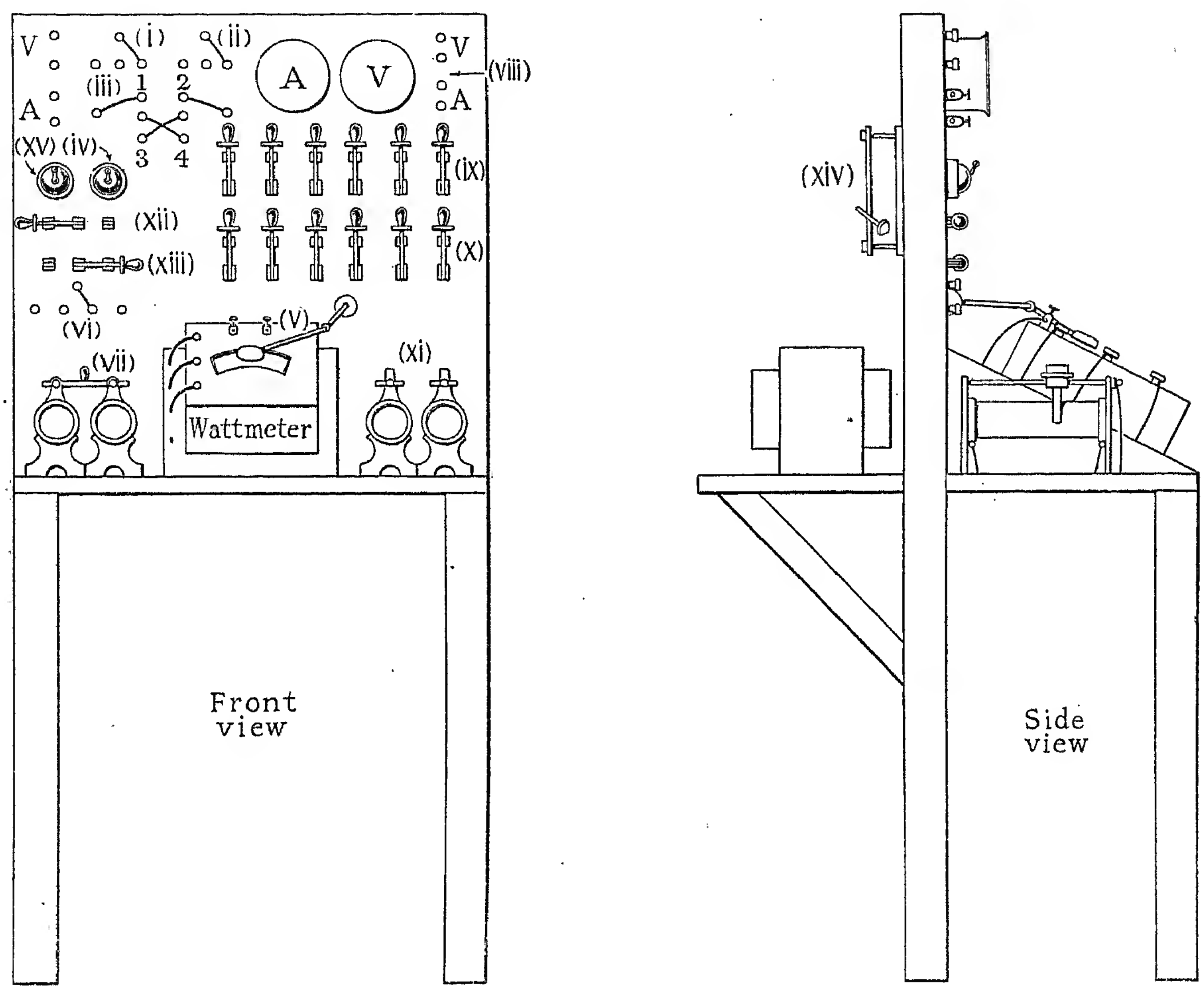

FIG. 3.—Single-phase testing panel.

(i) Voltage selector.
(ii) Voltmeter range selector.
(iii) Selector plugs for transformers A and B.
(iv) Creep-voltage switch.
(v) Wattmeter lamp and magnifying lens.
(vi) Wattmeter voltage-range selector.
(vii) Phase-angle control rheostat.
(viii) Output terminals.
(ix) Current-transformer range-selector switches.
(x) Load-current switches.
(xi) Load control rheostats.
(xii) Current control switch.
(xiii) Phase control switch.
(xiv) Main switch and fuses.
(xv) Voltage switch.

(*b*) There is no appreciable change of error with change of range (with a suitable current transformer), such as occurs when a wattmeter is used either direct or current-transformer operated. Incidentally, there is no danger of leaving the sub-standard current transformer open-circuited.

(*c*) A throw-over switch for changing the current coils of a wattmeter from direct connection to current-transformer secondary is a potential source of danger. Bad contacts in the switch are liable to alter the current-transformer burden, and hence the wattmeter errors.

(*d*) High scale readings can be achieved without resorting to the use of a low-power-factor type of wattmeter. In other words the current transformer enables a robust, high-torque wattmeter to be used. If it can be standardized in situ it may also be of a cheaper type, although it is absolutely essential to have a wattmeter whose self-heating errors are negligible.

If the meter engineer already has a wattmeter of 5-ampere capacity he can obtain the above effects by making the current-transformer ratios 50/5, 20/5, 10/5, 5/5, 2·5/5, and 1/5.

Turning now to the voltage circuit of the wattmeter, each range, except the 480-volt range, is designed so as to be capable of working continuously at twice its rated voltage. This enables full-scale readings (approximately) to be obtained on the usual 0·5 power-factor loads, and half-scale reading for $\frac{1}{20}$ load on 5-ampere meters.

No switches are provided in the voltage supply between the wattmeter and the testing bench. Experience shows that such switches (often cheap tumbler switches) are sometimes the cause of considerable inaccuracies due to contact resistances. The control switch on this equipment is therefore placed where it cannot create such an error.

The required voltage range is selected by fastening an easily-handled lug under the appropriate terminal of a group of four terminals mounted on the panel above the wattmeter. This relieves the wattmeter terminals of the wear and tear of screwing and unscrewing. The working voltage and the voltmeter range are chosen in a similar manner.

The wattmeter is mounted in a sloping position. This is done to ease the task of holding the load, but of course this practice is only permissible if the wattmeter can be standardized in its working position. The voltmeter and ammeter are of the moving-iron type, and of British Standard 1st grade accuracy. The panel should be made of asbestos board, but well-seasoned teak or oak may prove quite satisfactory. Plenty of space is provided under the wattmeter for the personal convenience of the load holder.

Part 2. TESTING EQUIPMENT FOR POLYPHASE METERS.

There has been a good deal of controversy in the past over the question whether polyphase meters should be tested on 3-phase or single-phase circuits, but much of the discussion has been misguided. Even if special precautions* are taken the single-phase test is only satisfactory for certain types of polyphase meters. If other types than these have to be tested, or rapidity of testing is required, then the polyphase method is essential. As there is no great difficulty in measuring the power in a low-tension 3-wire circuit the higher accuracy of the 3-phase test is indisputable.

There are several other reasons why the polyphase equipment—with two single-phase wattmeters—is preferable to the single-phase equipment, namely:—(*a*) The polyphase equipment is also eminently suited to the testing of single-phase meters. If desired, the components may be so arranged that two separate single-phase tests may be carried out, although only one will have fine control of the testing power factor. (*b*) Three-phase protective gear cannot be fully tested except on a polyphase circuit. (*c*) Current transformers can be tested for ratio and phase-angle errors without any extra equipment. (*d*) Voltage transformers could be tested if a standard voltage transformer and a source of high voltage were added to the equipment. (*e*) There is no difficulty in testing the phase rotation of the consumers' supply leads in order to ensure that the standard phase rotation is maintained throughout the system.

The lay-out of the suggested equipment is shown in Figs. 4 and 5. The current supply is of type B, and the phase-angle control by method C. The latter choice was inevitable for a modern, mains-operated, polyphase equipment. The type B current supply was chosen because it is superior in principle to type A.

The coarse control of the current is managed by a 3-phase bank of fixed resistances, and by having tapped primary windings on the step-down transformer. Fine control for load-holding purposes may be achieved by using an induction regulator in the supply to the step-down transformer, or by means of a magnetic shunt (see Fig. 6) if the current supply is drawn from a 3-phase transformer. No experience of the latter device has been obtained, but there is no apparent objection to it if the packet of laminations is made deep as shown in Fig. 6. If an induction regulator is used it is advisable to use the Barbour or Ferranti type, in order to avoid variations in the testing power factor.

At the end of the resistance banks (Fig. 5) will be noticed a set of three throw-over switches (Z). These switches (which are quite independent) are introduced to simplify the balancing of the elements when testing 3-phase 4-wire meters. They avoid the need for disturbing the current-transformer connections when such meters are under test. It should be emphasized that even though certain tests on current-transformer-operated meters necessitate having one or more current-transformer primaries idle, the secondary connections should not be disturbed.

To facilitate the connecting-up of current transformers the star-point connection is made by means of a copper bar, which is connected to the special switches (Z) by a heavy conductor. The current-transformer table is permanently connected to the meter bench by a suitable number of leads, arranged in pairs, and having large post-type terminals at each end.

The voltage supply is drawn from a bank of three single-phase transformers, which are provided with tappings for: 1 to 9 volts in steps of 1, 10 to 90 volts in steps of 10, 100 to 400 volts in steps of 100. The tappings are brought out to a terminal board, and the various sections are interconnected by means of leads supplied with easily handled lugs, as this method is considered preferable to those employing rotary switches or plugs and sockets. No switches are used between the wattmeters and the meters under test, for the reason given above. The phase-shifter is inserted, of course, in the constant-voltage side of the voltage supply transformers. Three moving-iron voltmeters are fitted to the panel, as simultaneous readings of the

* See Reference (3).

various voltages are essential for certain tests. Suitable links enable either line or phase voltages to be measured.

Each of the two wattmeters is similar to the one specified for the single-phase equipment, and the voltage-range selection is managed in the same way. The wattmeters are completely current-transformer operated, and the three high-accuracy, nickel-iron, ring-type current transformers have the ratios: 500/1, 250/1, 100/1, 50/1, 20/1, 5/1, 2·5/1, and 1/1.

The primary windings are arranged in the following manner, all the turns being in series on the 1-ampere range: 1 turn to carry 500 A; 1, 250 A; 3, 100 A; 5, 50 A; 15, 20 A; 25, 10 A; 50, 5 A; 100, 2·5 A; 300, 1 A.

The secondary windings have 504 turns with tappings at 1, 2, 498, and 501 turns. These tappings enable the mean scale error of each wattmeter (if any) to be counterbalanced, so as to make the overall errors practically zero. If desired, the wattmeters may be mounted in a sloping position as specified for the wattmeter of the single-phase equipment. The general arrangement of the equipment, which follows the usual principles, will be seen from Fig. 5.

Current-Transformer Testing.

The above equipment is adapted for testing current transformers by the comparison method, by bringing out the primary tappings for the blue sub-standard current-transformer ranges of 5/1, 2·5/1, and 1/1 A, to terminals on the current-transformer table (marked K in Figs. 4 and 5).

The test is carried out by connecting the primary of the current transformer under test between the red and yellow current output phases, and the secondary to the appropriate terminals of the blue current-transformer primary circuit (in series with any necessary added burden). The required current is passed through the primary of the current transformer under test, whilst the red and blue wattmeter voltage circuits are supplied with a common voltage of suitable magnitude so as to obtain high scale readings. The phase of this common voltage is adjusted so as to compare the primary watts with the secondary watts at unity power factor and 0·5 lagging power factor respectively. If the nominal ratio of the current transformer is known then this comparison enables the errors of the latter to be obtained at the two power factors. The unity power-factor error is equal to the ratio error of the current transformer, whilst the phase-angle error in minutes is obtained (provided the error does not exceed 60 minutes) by multiplying the algebraic difference between the unity and 0·5 lagging power-factor errors by 20. If the 0·5 lagging power-factor error is more positive than

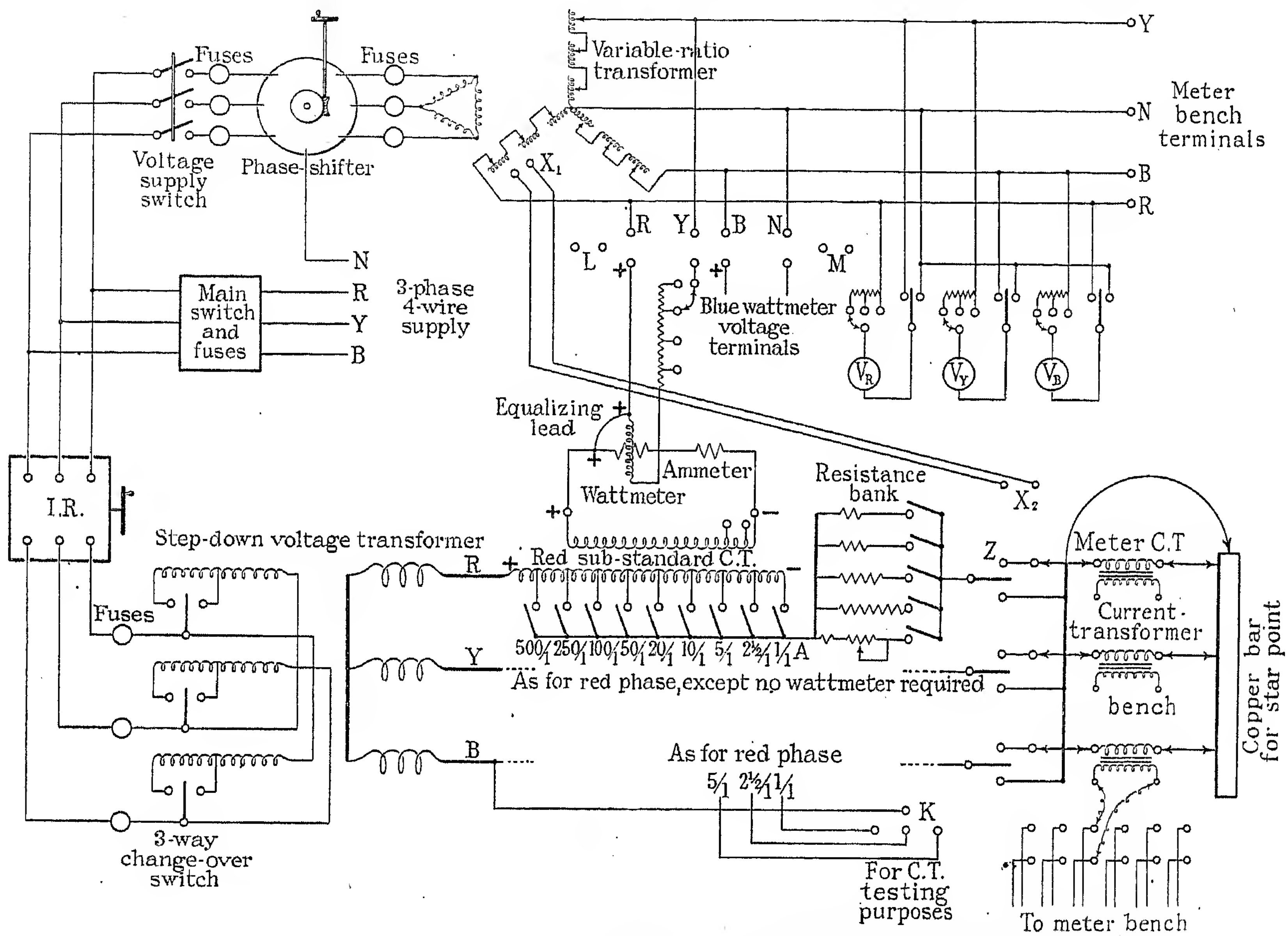

Fig. 4.—Testing circuit for polyphase meters.

I.R. = induction regulator. V_R, V_Y, V_B = voltmeters for red, yellow, and blue phases respectively.

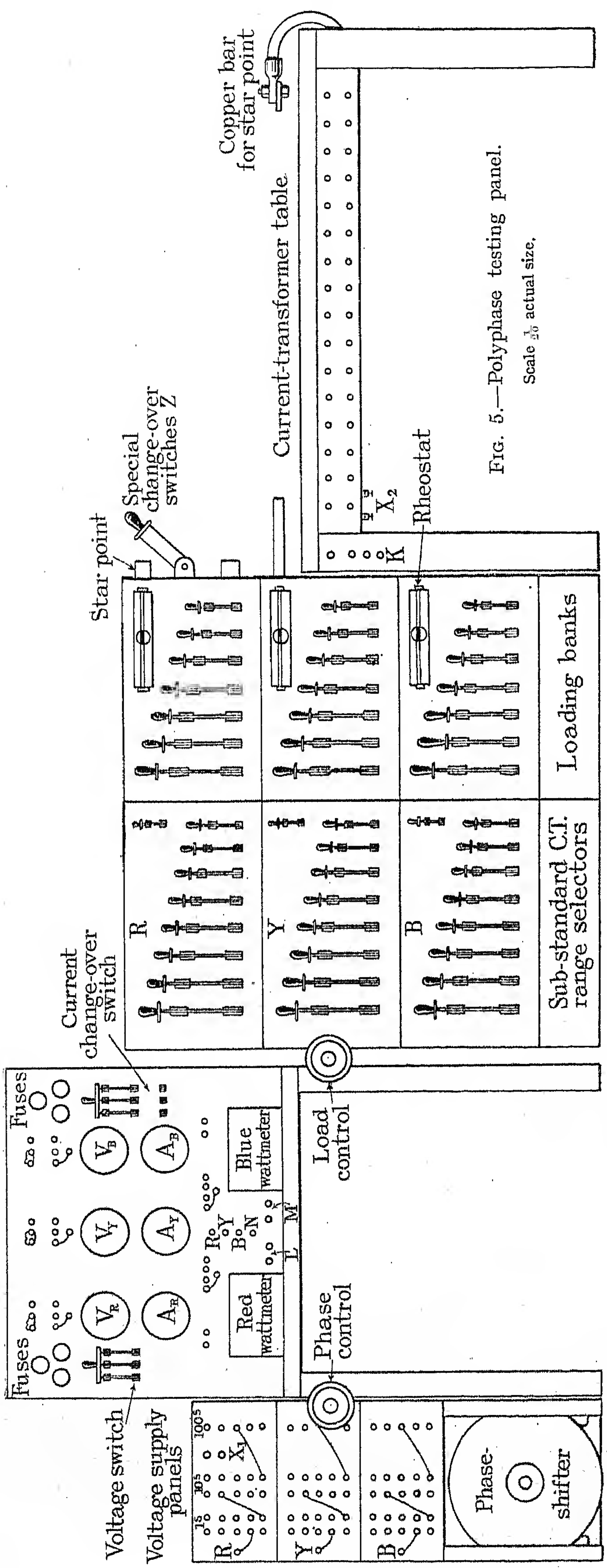

FIG. 5.—Polyphase testing panel.
Scale $\frac{1}{20}$ actual size.

the unity power-factor error, the phase-angle error is a leading one, and vice versa.

The accuracy of the comparison method depends on knowing the actual scale errors of the two wattmeters and the ratio and phase-angle errors of the sub-standard current transformers. Under straightforward working conditions the ratio error of a current transformer may be measured within $\pm$ 0·4 per cent, and the phase-angle error within $\pm$ 10 minutes. If special precautions (such as repeated readings, direct comparison of the two wattmeters on a common load, and tests at other power factors) are taken, however, the limits may be narrowed down to $\pm$ 0·2 per cent and $\pm$ 5 minutes. Whilst this degree of accuracy may not seem very high in comparison with that attained by the latest National

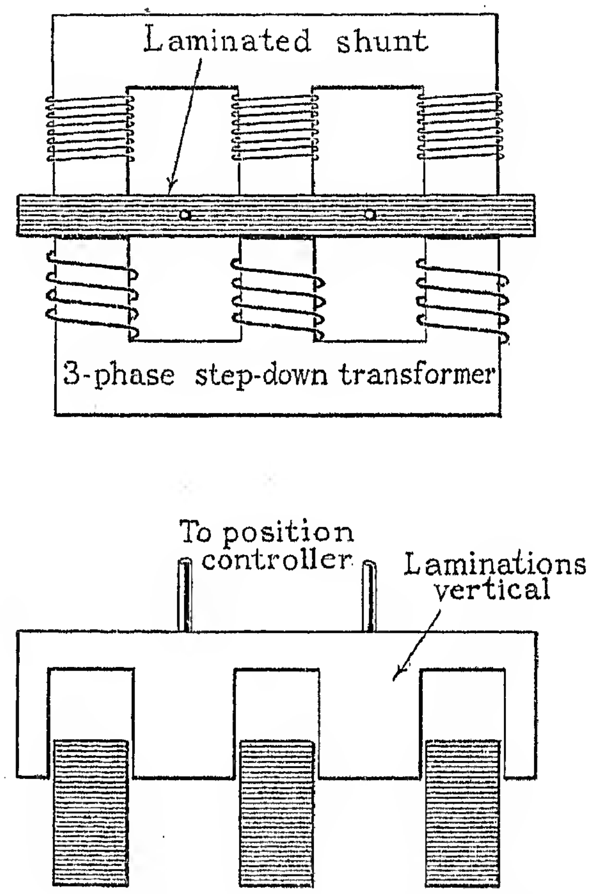

FIG. 6.—Three-phase magnetic shunt regulator.

Physical Laboratory equipment,* it is quite sufficient for routine testing of commercial current transformers. The great advantage of this method, of course, is the utilization of existing equipment for an additional purpose with a minimum of trouble to obtain the desired results.

It should be pointed out that even though meters may be calibrated in conjunction with their own current transformers it is still necessary to check the current transformers individually for accuracy. In one recent case it was found, as the result of individual testing, that a current transformer had its winding insulation completely burnt out even though the "as received" errors of the associated meter—an off-circuit, 4-wire, polyphase meter operated from three current transformers—were quite reasonable. The faulty current transformer showed no outward signs of distress.

* See Reference (4).

Voltage-Transformer Testing.

The comparison method is again employed; in addition to the apparatus shown in Fig. 4 a source of high voltage is required and a standard voltage transformer. This extra equipment is placed in a convenient position, as regards safety and the handling of heavy apparatus, and two pairs of leads are taken from this position to the testing panel (terminals L, M, in Figs. 4 and 5). The voltage transformer under test and the standard voltage transformer have their primaries connected in parallel across the high-voltage supply, whilst their secondaries are connected via the abovementioned leads to the blue and red wattmeter voltage circuits respectively. A common current of approximately 1 ampere is supplied to each wattmeter current coil by connecting the red and blue current output terminals to a low-voltage supply drawn from the voltage circuit via the terminals X_1 and X_2 (see Figs. 4 and 5), and using the 1/1-ampere ranges. Owing to the low value of the circulating current there is no danger of excessive voltages being set up in the primary of the current supply transformer.

The outputs of the two voltage transformers can now be compared at unity and 0·5 lagging power factors—due precautions being taken as regards polarity and power factor, on account of the altered connections of the equipment—and the errors of the voltage transformer under test are determined at the two power factors.* An added convenience for this work is a loading device which enables various values of secondary burden to be imposed on the voltage transformer under test, as it is usually necessary to test a voltage transformer at more than one burden.

Ratio errors may be determined within $\pm 0 \cdot 2$ per cent and phase-angle errors within ± 5 minutes, if the two wattmeters are first compared with a common voltage supply from the standard voltage transformer. Although this degree of accuracy is not very high from the modern point of view, it would nevertheless be a useful asset to a supply undertaking in view of the cost of providing more accurate methods of measuring voltage-transformer errors. Voltage transformers are similar to current transformers in that they can develop faults which may not exhibit outward signs of distress, and which can only be discovered by periodical tests.

Part 3. STANDARDIZATION OF TESTING INSTRUMENTS.

Electricity meters are usually labelled as registering in watt-hours or ampere-hours, yet they are tested, generally speaking, by instruments which are two or three degrees removed from the legal standards of power and current. The distance of this relationship and other factors make it desirable to consider whether a testing department should possess secondary standards as well as sub-standards. The following reasons clearly show that secondary standards are essential for all undertakings having a capacity of 5 000 kW or over.

* The ratio and phase-angle errors of the voltage transformer under test are calculated as for a current transformer, except that a more positive error on the 0·5 lagging power-factor test now denotes a lagging phase-angle error.

(a) The fact that the error of an indicating sub-standard may not exceed $\pm \frac{1}{4}$ per cent of full-scale reading, at any scale point, does not prevent the actual error from being quite appreciable; e.g. it may be $2\frac{1}{2}$ per cent at $\frac{1}{10}$ full scale. The possession of a secondary standard would enable the actual errors at all points of the scale to be determined.

(b) The continual use of a sub-standard under service conditions (and often by young, inexperienced testers) is always liable to alter the characteristics of the resistances and springs; particularly if overloading of the windings is carried on to any extent. If any damage does occur to a sub-standard it is usually by far the quickest way to repair and re-standardize it on the department's own premises.

(c) The majority of the revenue of a supply undertaking is derived from the readings of its electricity meters, and is therefore directly dependent on the accuracy of the latter. This accuracy is, in turn, dependent on the accuracy of the sub-standards. The effect of a mean error of 1 per cent in the sub-standards (assuming all the service meters to have been tested by them) is about £700 per year on the annual income of a 5 000-kW undertaking; which is an appreciable item. Many engineers expectantly hope that the meters which over-register will compensate for those which under-register; but it is better to be sure than sorry. The growing popularity of the 2-part tariff makes it essential that every unit consumed should be registered by the meters.

(d) If meters are issued which over-register the consumption and several consumers successfully dispute their accounts, the undertaking's reputation for fair dealing will be tarnished. Such an occurrence will lead to other disputed accounts, even if they are not justified by the facts. An efficient standardizing equipment may be regarded as an insurance against such trouble.

(e) It is far more convincing, and cheaper in the long run, to be able to standardize an instrument in one's own test room than to return it to the maker, or the nearest laboratory, to obtain its errors.

The nature of the secondary standards will depend on the supplies given by the undertaking. If the supplies are direct-current only then a d.c. potentiometer equipment is essential, but if the supplies are alternating-current only one cannot put forward the a.c. potentiometer. This is because very few testing departments possess a steady-voltage a.c. supply, and because the cost of the a.c. potentiometer is rather high. These disadvantages are to be regretted, as the a.c. potentiometer is an extremely versatile piece of apparatus.

Therefore, for a.c. work one is compelled to use the d.c. potentiometer or dynamometer-type secondary standards. The d.c. potentiometer, however, cannot detect the phase-angle errors in a.c. wattmeters, and thus the mean d.c.-error curve cannot altogether be relied upon if the power factor differs from unity. Again, certain a.c. instruments are operated from current transformers and in these cases the d.c. potentiometer is useless.

Before the introduction of the nickel-iron ring-type

current transformer the phase-angle error difficulty compelled testing departments to use low- and heavy-current dynamometer wattmeters, having numerous ranges, for a.c. standardizing purposes. Such instruments are very costly, and this probably accounts for the fact that few undertakings possess a.c. secondary standards. The perfection of the current transformer, however, has now altered the whole situation. The combination of a low-current deflecting-type laboratory standard wattmeter with a suitable multi-range current transformer enables a tremendous range of powers to be accurately measured at a reasonable cost. The multi-range nickel-iron ring-type current transformer is now a robust and reliable piece of apparatus, and it can be justifiably used as a secondary standard. Therefore the standardizing equipment which should be in the possession of all undertakings of over 5 000 kW capacity is :—

(1) *Undertakings supplying direct current only.*—D.C. potentiometer equipment.

(2) *Undertakings supplying alternating current only.*—Low-current (1-A or 0·5-A) dynamometer wattmeter; dynamometer ammeter of the same current rating as the wattmeter; dynamometer voltmeter; multi-range, nickel-iron, ring-type current transformer, the secondary having the same rating as the wattmeter, and the primary having sufficient ranges to cover the requirements of the department.

(3) *Undertakings supplying both alternating current and direct current.*—All the above-mentioned secondary standards.

The Wheatstone bridge has not been mentioned, so far, as it is assumed that this sub- or secondary standard is an essential instrument necessary to carry out the everyday work of any undertaking.

The standards should be so disposed and permanently wired as to enable all the usual tests to be carried out with a minimum loss of time and effort. Where possible, the wiring should be so arranged that the test-bench instruments may be checked in their normal working positions. The permanent housing and wiring of the secondary standards avoids disturbance, and thus minimizes the risk of mechanical damage.

Part 4. TRANSPORTATION AND INSTALLATION OF METERS.

Generally speaking, the a.c. meter is not provided with a rotor clamp and therefore the liberties that are allowed in storing and transporting d.c. ampere-hour meters should not be permitted with a.c. meters. The transporting of unclamped meters on the floors of motor vans or on bicycle carriers, for instance, is not conducive to a long life of accuracy.

If it is recognized that the most vulnerable points of a meter are its bearings, steps should be taken to avoid trouble in this direction. It is suggested that all a.c. motor meters which are not provided with clamps should have their rotors packed with a piece of soft cardboard, when not in use, so as to prevent the pivot making contact with the jewel. This packing should be inserted immediately the testing is completed, and not removed until the meter is ready for load. This precaution is particularly necessary in the case of meters for large consumers. The point which it is desired to emphasize is that a bad spot on the pivot or jewel caused by mishandling will set up rapid deterioration of the bearing under service conditions. Where meters are not in duplicate the loss in revenue due to such a fault cannot, in general, be detected from the periodic readings of the meter. If the suggested policy is adopted it is essential for the meter fixers to be employees of the meter department. They must be shown how to remove the packing, see that the meter gaps are clean, and check the running of the meter on a low load, before finally sealing-up the meter.

As regards the installation of a.c. meters, the following suggestions for improving this work may be put forward:

(1) The meter fixers must be given strict instructions to place the series windings of the meters in the live line. If this is not done it is possible for the consumer to obtain (on earthed systems) energy without paying for it. It is also possible for the meter rotor to run backward if the neutral portion of the consumer's wiring has an earth fault.

(2) As periodical visits have to be paid to every meter for reading purposes, it is extremely desirable to place the meter where such attentions will cause the consumer a minimum of trouble. The lack of cupboard space in modern houses is well known, and if a meter is placed at the rear of one of these cupboards it will probably be hidden by an assortment of household goods or appliances. To read the meter it is often necessary to remove a good deal of this paraphernalia. Difficulties of this kind annoy the consumer and reduce the number of visits per hour which the meter readers can make. Attempts to read meters over or through obstructions are also liable to cause errors. Co-operation with the builders would appear desirable, as in some cases the latter make special provision for the electricity supply.

(3) A few types of meters are liable to be noisy, and such meters should not be placed where they will give annoyance to the consumers, e.g. over a bedroom door when the premises are supplied by overhead mains.

(4) The installation of all large-capacity meters should be supervised by a meter engineer. This is particularly necessary in the case of polyphase meters, where correct phase-rotation and polarity are of supreme importance. It is also desirable to check the mountings of ring-type current transformers because, in spite of their simplicity, they can easily be fixed incorrectly by fitters who have not received a technical training. Supervision is also desirable to ensure that the metering connections suit the tariff under which the consumer receives a supply.

Part 5. MAINTENANCE OF METERS.

COMMON FAULTS.

Whilst a correct initial treatment of electricity meters will greatly assist in obtaining a long and trouble-free life, it is inevitable that "on circuit" attention will be eventually required. Obvious faults, such as (1) meter not registering; (2) differences in advances, where duplicate meters are in use; (3) noise from the meter; (4) maximum-demand indicator reading too high or too

low; (5) some physical defect in the meter; (6) meter working when the consumer's load is zero; (7) incorrect operation of prepayment mechanism; (8) meter working backwards; are notified by the consumers, or other departments of the undertaking, to the meter department, and they should be given attention immediately.

On-Site Testing and Repair Equipment.

The detection of the causes of these faults, and their rectification, require an adequate equipment of tools and spare parts. Where the testing assistant is not usually provided with a motor-car this equipment cannot be very extensive, but it should at least contain:—Range of suitable spanners; large, small, and pinion screwdrivers; cutting pliers, sealing pliers, seals, and binding wire; flashlamp and neon test lamp; special tools required for certain makes of meters; time-switch winding key; feathers, seccotine, vaseline, and emery paper; spare nuts, terminal screws, and pointers; spare jewels, pivots, and top pins; peg wood, and a piece of pith; all the necessary keys.

These components will fit into a leather attaché case measuring approximately 12 in. × 9 in. × 5 in. In addition, the testing assistant should carry a stop-watch in order to be able to check the accuracy of the meter (approximately) on site, or to determine the consumer's load, assuming the meter to be correct. The department should also possess the following equipment for use on certain types of "on circuit" work:—Motor-type phase-rotation indicator; insulation-resistance tester; buzzer; bottle of clean mercury and rubber overshoes, if d.c. mercury motor meters are in use.

Of the above components only a few require any comment. The neon test lamp is extremely useful as it enables one to detect the live line even if no earth connection is available near the meter position. Furthermore, it enables one to detect whether a supply is alternating-current or direct-current. The buzzer and phase-rotation indicator are required for checking the wiring of a newly installed polyphase metering equipment. Sterilized feathers are very valuable for cleaning the gaps of motor meters, and they are mentioned here because their use is not so well known as it might be.

The important point in connection with this work is the necessity for discovering and rectifying (where possible) the fault at the first visit. This is particularly important if the area of supply is very extensive.

Importance of Regular Maintenance Visits.

There are other types of faults, however, which pass undetected unless special efforts are made to discover them. The general custom at the present time is to issue meters and leave them in service until the consumer changes his loading, or some definite fault develops, or the accounting department suspects that the meter is faulty from a big variation in the periodic advances. Experience shows that this practice is to be deplored, especially as the undertaking is usually the loser by this neglect. There are comparatively few meters which are worked in the region of full load for the greater part of their effective life, and the average load is probably about one-tenth. At this loading any friction due to foreign matter or bearing deterioration has an extremely important effect on the meter accuracy and, unfortunately for the supply undertakings, these troubles reduce the registration of the meters. Although meters are, in this respect, delicate instruments, they are often placed in positions where the conditions are opposed to successful operation over long periods. Typical instances of adverse situations are:—pelt yards in tanneries; currying rooms, where leather is polished; rooms often full of steam; coal cellars; walls where the meter becomes covered with the debris of decaying plaster and other droppings.

Consequently if the best financial results are expected the meters should be given suitable attention before the drop in registration compels the accounting department to investigate the matter. Low incorrect bills can be just as much a nuisance as high incorrect bills. It is extremely difficult to convince a consumer who has been receiving low bills for some time, owing to a faulty meter, that his new, or repaired, meter is working correctly.

Other faults which a system of regular maintenance visits will reveal and rectify are:—(1) incorrect type of meter employed; (2) meter tampered with, or without seals; (3) meter too large or too small for the work which it has to do. The form which such a maintenance scheme should take is too dependent on local conditions for it to be possible to prescribe any set procedure. The remainder of the paper is therefore confined to one aspect of such a scheme.

On-Site Measurement of Meter Errors.

One of the difficulties in the path of the maintenance engineer is the practical one of measuring the meter errors on site with the minimum expenditure of time and trouble. The insertion of measuring instruments in circuit with the meter is not, in general, a very easy task, and if a reasonable number of meters are to be checked per day some simpler method of attack must be found. Such simplicity is desirable even at the expense of reduced accuracy of measurement, and as the chief function of the maintenance visit is to find errors of the order of 3 per cent it is clear that simplicity is quite in order. With this proviso it is suggested that the following method of testing low-tension meters might be applied to small consumers taking not more than 15 kW.

A box containing a number of calibrated resistances is arranged so that it can be supplied from the various types of outlets available on small consumers' premises. (For the sake of safety, each adaptor has a cover provided for it when not in use.) The measuring instruments, mounted on the top of the box, consist of a moving-iron voltmeter and a multi-range moving-coil ammeter, both of sub-standard grade. The ammeter is for use with d.c. ampere-hour meters, and it is cut out of circuit when watt-hour meters are being tested.

On the arrival of the testing assistant at the consumer's premises the natural load, if any, is taken off and the above device connected to the most convenient point. The assistant then switches on the appropriate resistances and watches the voltmeter, or the ammeter, to take the average reading for the period during which the other

tester times the meter revolutions. From these figures, and the known load of the resistance box, the error of the meter can be determined. This procedure is repeated for the other loads which require checking. If these errors are satisfactory the meter is then tested for creeping on voltage only, and for running on starting current.

If, in the case of watt-hour meters, the device is not connected to the meter output terminals (neglecting the series-coil losses), the consumer's wiring resistance will cause an error in the measurements, owing to the voltage drop in this resistance. Whilst this error will be appreciable at full load—of the order of 1 per cent—it is negligible on the more important loads such as $\frac{1}{10}$ and $\frac{1}{20}$. Furthermore, this discrepancy is in the favour of the consumer, as it makes the meter errors appear more positive than they really are. If, in a particular instance, the lowness of the voltmeter reading suggests that this discrepancy is serious, a test can always be made at the meter terminals.

Consumers taking more than 15 kW are considerably less in number than those taking under 15 kW, and consequently the meters for these consumers may be justifiably given a more orthodox test to determine their accuracy on site. Also, the money value of these consumers is relatively larger, and thus they are worthy of more attention on this account alone. The various methods of making such tests are well known.* It is not suggested, of course, that test-room work should be given up in favour of on-site testing of meters. Checking the accuracy of a meter at a few particular loads is a very different operation from properly calibrating and testing it.

The author would urge meter engineers to develop simpler but more widely useful instruments and accessories in order to make such testing work safer, and possible without interrupting the supply to the consumer. Highly accurate clip-on current transformers for high-tension circuits, and accurate clip-on ammeters for d.c. circuits, are two instances of the special requirements of meter engineers.

In conclusion, the author wishes to thank Mr. H. C Lamb, chief engineer and manager of the Manchester Corporation Electricity Department, for the facilities which he has provided in enabling this paper to be published.

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Discussion before the Meter and Instrument Section, 7th December, 1934.

Mr. O. Howarth: In Part 1 the author points out that distortion of the current wave may occur when the resistances are connected on the secondary side of the transformer which supplies the current. Some engineers object to resistances in the primary of that transformer on the ground that they distort the wave; but, if we consider the circuit as a whole, we have to realize that it does not matter very much where the resistances are put provided that the circuit is suitable. Resistances in the primary have been found to be quite satisfactory in practice, provided the loading current transformer works at a low flux density.

The methods described for providing the desired load (Fig. 2) seem to me to involve rather a lot of resistances. In the method adopted by the undertaking with which I am connected an auto-transformer is employed, one section having tappings every 10 volts up to 250 and another every volt up to 15. Two contacts are adjusted to apply a suitable voltage to a loading current transformer. A variable resistance, which enables fine regulation to be obtained, is connected between the transformers. On the secondary side, when the range of the wattmeter is changed, the number of secondary turns is also changed. The one resistance gives fine control for currents from 0·25 to 1 000 amperes. This method avoids the many resistances which have to be used to get a current of 1 000 amperes by the arrangement shown in Fig. 2.

Another point is the arrangement for providing the desired voltage. No provision for giving fine adjustment of voltage is shown at the top left-hand corner of Fig. 2, and it appears to me that if a few taps were brought out at the bottom end of the transformer on the primary side a fairly fine adjustment could be obtained. The arrangement shown for adjusting the phase (by means of the resistance X) involves a resistance to carry 60 amperes. I have been wondering why that phase-shifting device was not placed on the voltage transformer, where the currents are small and there would have been less difficulty due to the range of currents the resistance had to carry. Instead of the plug for throwing over from unity to 0·5 power factor, i.e. stepping the phase of the current supply by 60°, a 3-pole throw-over switch could be used. We have used 2-pole throw-over switches in our circuits for this purpose and we find that they save a lot of time. The range-change arrangement in the sub-standard transformer made possible by the use of mumetal is an excellent one. I agree with the author that the most accurate way of testing a.c. meters is by means of a wattmeter and stop-watch.

In Part 2 he discusses the testing of 3-phase meters, but does not say a great deal about the testing of 4-wire meters. When testing 4-wire 2-element meters it is necessary to arrange the voltage circuit so that the sum of the three voltages is equal to zero. This can readily be arranged by using three single-phase transformers of ample size and connecting them in delta/star.

* See Reference (5).

I should like to know whether, with the arrangement shown in Fig. 4, it is possible to obtain a symmetrical 3-phase loading. This is not essential, because the accuracy of the test is not involved. Nevertheless, one likes to have symmetrical loading as far as practicable on a 3-phase test. In Fig. 6 the author shows a magnetic shunt for adjusting the load. Has he had any experience of such an arrangement, and what sort of success has been obtained by the method?

Dealing with current-transformer testing, it seems to me that to use two wattmeters, as he suggests, is an obsolete method, in view of present-day facilities which can be quite cheaply provided. To read the full value of the quantities in order to obtain a difference is not a very accurate method, and is rather tedious if accuracy is to be obtained. The author refers to Dr. Arnold's paper* on methods of testing current transformers; we have used a modification of Dr. Arnold's method, employing an ordinary wattmeter to measure the difference current; it is quite a rapid and accurate way of testing current transformers, provided, of course, that one has a standard current transformer of the same ratio available.

Turning to page 376, to read two wattmeters seems a clumsy way of testing a voltage transformer. Methods can be devised for measuring a ratio where the accuracy of the instruments used does not enter very seriously into the observation, i.e. the wattmeter can have an error of 4 or 5 per cent without introducing an error of more than 0·1 per cent in the results.

I am glad to see that the author says that the growing popularity of the 2-part tariff makes it essential that every unit consumed should be registered by the meters. It is frequently stated that in view of the standing charge the accuracy of the meters does not matter much, because not much of the revenue depends upon them; but when a supply undertaking introduces a 2-part tariff it usually expects its total revenue to go up. We know that some of the revenue comes from the fixed charge, but with the increasing use of electricity there is an increasing revenue from the running charge, which does depend on the registration of the meters; and as the meters in general have to be larger, and still work quite a long time on small loads, it is essential that they should be at least as accurate to-day as they have been in the past.

When referring to standards the author mentions the a.c. potentiometer. I have never regarded the a.c. potentiometer as an instrument of precision. It is a very useful instrument for a lot of measurements, but I do not think one can compare it with a wattmeter for measuring power. I agree with the author's remarks about the transportation and installation of meters. I am strongly of the opinion that a.c. meters should have their rotors packed to prevent damage. He mentioned using a number of calibrated resistances in measuring loads for on-site testing. I do not know whether it is the custom to do much on-site testing of small meters; as a rule it is more economical to change the meter and then test it in the test room. I do wish that those manufacturers who make miniature instruments would produce a miniature wattmeter with an accuracy as high as that of their miniature voltmeters and ammeters, and if possible at a comparable price.

* *Journal I.E.E.*, 1934, vol. 74, p. 424.

Mr. G. F. Shotter: Referring to Fig. 1, it is obvious that various combinations of the methods the author suggests could be used, and we ourselves make use of that shown as type B, but with 12 tappings on the secondary of the transformer. This combination is very successful for use with rotating sub-standards. For standardizing with dynamometer wattmeters we use a similar arrangement to that shown in Fig. 2, with the exception that the phasing transformer is used on the potential side and with many more tappings than are shown by the author. The use of such a transformer has been found to be very convenient and flexible. In addition, particularly where the secondary load has a high power factor, the injection of a quadrature e.m.f. does not vary the total voltage to any appreciable extent.

I agree with the author in his contention that polyphase meters should be tested on a 3-phase supply, and also where possible with their own current transformers.

Fig. 4 shows a circuit very similar in principle to that used by ourselves, with the exception that, in place of the induction regulator on the primary, we use multi-connected carbon rheostats with mechanical pressure control. These have been in use for many years, and have proved to be extremely satisfactory.

With regard to the author's method of current-transformer testing, I do not think that this is accurate enough. A ratio error of $\pm$ 0·4 per cent and an angle error of $\pm$ 10 minutes can be greatly improved upon, and my knowledge of this method of testing suggests that the above figures will be greatly exceeded at low loads.

Turning to Part 3, I agree with the remarks (paragraphs *c* and *d*) regarding the accuracy of sub-standards, although I am rather of the opinion that the "pious hope" interpretation of the law of averages is not so prevalent as it used to be. Even then, this interpretation was not so much due to the meter engineer as to those responsible for the purchase of the instruments.

With regard to Part 4, our experience, which extends over many years, shows that—at least with oiled bearings—the only damage that is caused by transportation of the meter is the production of surface cracks, and for this reason alone I should like to see a.c. meters fitted with clamping arrangements. There are practical difficulties to be overcome, but there is no doubt that these can be effectively dealt with at a price. On the other hand, I do not think the author's method of packing with soft cardboard would be satisfactory. In the case of a big undertaking it is in my opinion essential to send meters out from the meter department sealed.

With regard to Part 5, I think the author is wrong in stating that the general custom at the present time is to leave meters in service until the consumer changes his load. As far as my own company is concerned this is certainly not the case, as a periodical routine change has been in operation for many years. Also, with regard to large equipments, these are periodically tested and, where necessary, overhauled; this also under a definite schedule which has been in operation for 12 years. I agree with the author that this is the only satisfactory method of dealing with the matter.

Finally, I should like to emphasize the author's statement regarding the low load on which the large majority of meters are working. In these days of overload meters there is no necessity for this to occur.

Mr. E. Fawssett: The author gives a list of types of current supplies, but he seems to have omitted the first and most simple one, which is suitable for the ordinary 5-ampere meter, forming the main stock-in-trade of the undertaking. These can be tested direct on the supply, and then there is no fear of wave-form trouble. A group of about 50 meters on the test bench has not sufficient reactive effect on the characteristics of a circuit to upset the power factor (using cross connection) by more than 1 or 2 per cent, which does not matter in the least if one is taking a characteristic at about 0·5 power factor as compared with unity.

Fig. 2 is very interesting; I should like to suggest one addition to it, arising out of a fatal accident which occurred within my knowledge. The idea is to couple with the main switch and fuses—shown in the left-hand bottom corner of Fig. 2—a fourth switch. Couple the four switches together with that switch reversed so that, if it is closed when the others are open, it closes on to an Osglim lamp connected between one phase and neutral. Then put up a large notice: " Treat this bench as alive, and touch nothing unless this lamp is alight." This is the most reliable safety device one can have.

I presume that the current transformer indicated in Fig. 2 represents only a diagrammatic sketch of the arrangement. If not, it would be better to have a series-parallel primary instead of idle copper, as indicated in the figure, because probably that idle copper will mean uneven distribution of turns and not the same characteristics on all ranges. The description of the use of the apparatus (page 372) suggests that the chief measuring instrument for the author's equipment is a shielded type of dynamometer wattmeter. I join issue with him and with the previous speakers as to that being the regular day-to-day method. I think it should be used for standardizing check-meters, and that all commercial tests, particularly on the single-phase bench, should be dial tests. The dial reading is the basis on which the consumer pays, and we have all too little chance in the test room of finding any faults in the dial. The polyphase equipment (Fig. 4) probably looks more elaborate than it really is. The chief advantage seems to be the possibility of the meter department making that gear up *sub rosa*, without applying to the chief engineer for very much money. In the " bad old days " that was a very real advantage. The motor-alternator set is, however, by far the best, and is not to be grudged by the enlightened chief engineer of any undertaking of reasonable size. In general, the equipment described in the paper is rather elaborate for the very small station, and other methods suit the big undertakings. The author's testing arrangements are mainly useful for the medium-size station; but there are plenty of these, and therefore the paper is fully justified. I agree with Mr. Shotter that the accuracy of 0·4 per cent mentioned on page 375 is rather doubtful of attainment. As other speakers have said, the method in question is not a very good one. I think it is perhaps not quite as accurate as the author suggests.

Turning to the remark on page 376, " Many engineers expectantly hope that the meters which over-register will compensate for those which under-register," I take it that there the author means station engineers and not testing engineers. Nowadays, with the increasing use of cobalt steel for magnets, I do not think there is the faintest chance of that happening. It is almost certain that the balance of the meters, if neglected, will be on the slow side. I agree with previous speakers as to the fallacy that the 2-part tariff makes it unnecessary to bother about the accuracy of the meters. In point of fact the range over which the meters have to work has been increased by the use of the 2-part tariff, and their accuracy over a wide range is more needed than ever.

I quite agree with all the author says in Part 4 as to the transportation of meters. Single-phase meters now have no clamps, and no provision for checking chattering due to transportation. While this caused no trouble when meters were transported in pairs by the meter fixer, whose arm acted as an excellent shock absorber, they are now despatched in hundreds and must be sent by some form of mechanical transport, in the larger undertakings at any rate. I have in mind an area where there are no less than 670 separate networks, and relatively unskilled workmen have to fix the meters. It would not be practicable to let these men open the meters and remove the packing. It is also hardly practicable to have a meter representative capable of doing that in every area, fixing every meter that is sent out—perhaps a distance of 50–70 miles—from the test room. I think Mr. Shotter is right in saying that we must point out to the manufacturers that the balance of advantage is now in favour of having the meters clamped.

I agree with the author that a few types of meters are liable to be noisy. The effect of the noise is dependent not only on the meter but also on the kind of fixing and the kind of wall to which the meter is fixed. I have made measurements of the noise that one may get from a meter in a quiet situation; it can be a serious nuisance to the consumer, rising to 30 decibels or more. The important point in connection with the work of maintaining meters is the necessity of discovering and rectifying the fault on the first visit, a point that can hardly be too strongly stressed in these times when long distances have to be travelled. The second, and sometimes even the first, visit may wipe out the whole profit on that consumer for the year. This method of on-site measurement of meter errors is very interesting to me because something of the sort seems to be necessary in connection with a scheme which I shall mention in a moment, which we have just lately adopted. The figure of 15 kW, however, seems rather high to provide for at the ordinary voltage. It means a current of 60 amperes, which the author seems to contemplate taking at one outlet. Only the cooker switch could deal with so much current. The method is in general a good one for a rough test. The power is measured in terms of V^2/R, and I suggest that the voltmeter might be scaled in terms of the square of the voltage. It would then merely be necessary to divide by the resistance value in use, and the result obtained would be the power consumed. The development to which I have just referred has been put into operation on the North-East Coast. We have a fleet of

vans on specially-sprung passenger chassis (" ambulance springing ") equipped as travelling workshops and meter stores, carrying half a dozen meters of sorts likely to give trouble, and driven by expert repairers. These vans visit outlying districts. We have three test-rooms, 40 miles apart each way, with one van for each; and in addition there is a van for each of the several outlying districts, which may be 30 or 40 miles from the nearest test-room. Each of those repairers has 20 000 meters in his charge, and the number of faults which occur is just about sufficient to keep him busy, the majority of the meters being of the slot type.

The author ends with a very good piece of advice: he urges meter engineers to develop simpler instruments and instruments of wider use, so as to make all the testing work safer. I think that can hardly be too strongly advocated.

Mr. M. Whitehead: On page 370 the author points out that an objection to the Type B current supply is that, if the magnetizing component of a secondary current becomes appreciable compared with the energy component, the wave-shape may be distorted. This statement is rather ambiguous, and even the statement which follows it does not make it quite clear. I should like the author to amplify that point. Also on page 370 he mentions that method B of phase-angle control has the disadvantage that the meters are not tested on exactly unity power factor. Here, I think, the author might have stated the magnitude of the error involved. I feel that it is very small.

Mr. E. W. Hill: The test arrangements described in the paper are quite suitable for manufacturers' test departments, or are adaptable to them. It should not, however, necessarily be assumed that those same arrangements will meet all the requirements that a manufacturer has to satisfy. A manufacturer has difficulties to overcome which do not confront the supply undertaking. For instance, owing to economic pressure the intrusion of the time factor affects manufacturers very keenly; they therefore have to try to secure the utmost rapidity of manipulation of the test apparatus by their testers. The fundamental difference between the procedure which a manufacturer has to adopt and that which a supply undertaking has to adopt is that between calibration and testing. I do not think this is a mere verbal distinction. It is a fair generalization, though it should not be taken too literally, that the supply undertaking has to receive meters which have already been calibrated by the manufacturers, and has only to subject them to acceptance tests. On the other hand, the manufacturer has to deal with meters that, so to speak, have only just been born. They are raw pieces of apparatus; they have to be calibrated into correct measuring instruments. Therefore, all the manufacturer's test apparatus has to be manipulated many more times than that of the supply undertaking. It is, of course, true that the manufacturer has one advantage: he deals, generally speaking, with very much larger quantities of a uniform product than do most supply undertakings. On the other hand, however, he has to do under pressure of time a lot of adjustment work on all his meters: hence the testers must be provided with means of exceedingly rapidly switching and changing the load. To my mind the most efficient piece of apparatus for that kind of work is one that is not mentioned by the author, although it is admirably suited also to supply undertakings' test departments. That is the well-known testing set where one sets up the load—full, half, quarter, and so on—merely by the touch of a switch. In pieces of apparatus of the testing-set type one finds that, without any detriment to accuracy, resistance control in the primary circuit of the generating transformer is used.

A test procedure which has not been mentioned so far is the stroboscopic method, which is extremely good for ascertaining the rate and the speed of the meter at any moment. It enables instantaneous adjustment of the meter to be made inch by inch, as it were, until it attains to its proper state of correctness. There are, however, one or two difficulties in its use, the principal one being that it is only suitable for testing meters when they are running at high loads. With the stroboscopic method the aspect of the test marks becomes hazy at 1/10 load or less, so that the test becomes almost impracticable. Another difficulty in connection with this method is that it generally involves testing one meter at a time; the meters thus do not become properly acclimatized by their own self-heating. One has to make allowances for this effect if the meters are subject to alteration in consequence of their self-heating, as sometimes happens, especially with regard to the quadrature adjustment.

I fully agree with the author about the use of standard wattmeters in conjunction with the modern highly accurate nickel-iron current transformer. This avoids the embarrassment of having multi-range high-current-capacity wattmeters. The use of small-capacity wattmeters in conjunction with current transformers provides the utmost flexibility, combined with far greater accuracy and convenience than one can get with heavy-current wattmeters.

In conclusion, I am very pleased that nearly every speaker hitherto has commented on the author's remark about the 2-part tariff not in the least diminishing the necessity for accuracy in meters. I know that many people do consider it a debatable point; they say that if the revenue has two constituents, one being fixed independently of the meters and the other depending upon the registration of the meters, it does not matter quite so much whether the meters are accurate or not. That view in relation to modern conditions is fundamentally fallacious. Since the meters now have to work over a larger range and have to measure more and more energy, they are still required to be extremely accurate.

Mr. J. W. Carter: I have had extensive experience with meter testing circuits which are in principle the same as those recommended by the author, and have found them to be quite excellent. The absence of waveform distortion is ensured by placing the current-regulating resistances in the secondary circuit, and there is no doubt that this is the best practice, but for 3-phase benches for heavy currents the method is cumbersome. The author's 3-phase bench for 500 amperes probably represents the limit to which we would be prepared to go in this direction. It is not safe to assume that 500 amperes is likely to be the highest current that will be required for testing purposes, as the heavy-current transformer with built-in primary is quite common. I

have found that primary regulation by induction regulators is quite satisfactory under these circumstances. If harmonics are introduced they will be at the low loads; a test on a 4 000-ampere bench showed a 2 per cent 3rd harmonic on all currents corresponding to the full load of the meters under test, and the amplitude increased till it reached a maximum of 10 per cent at 1/20th load. These figures were repeated for all sizes of meters from 4 000 amperes to 5 amperes. There is of course, no distortion of the wave in the voltage circuit, and in consequence the error introduced in the testing of watt-hour meters is negligible even at 1/20th load.

The author's recommendation of stop-watch testing for single-phase meters, on account of its accuracy, applies only so far as one is prepared to follow the lines of laboratory testing rather than commercial testing. Testing by dial registrations is the nearest approach that can be made to service conditions, and therefore the errors found in this way are the more likely to hold good under the usual methods of testing meters in quantities.

Mr. A. G. Kemsley: I should like to deal with the measurement of current-transformer ratios. We have made more progress in this direction in the last 12 months than we had made during the previous 12 years, and this is entirely on account of the current-transformer testing set recently described by Dr. Arnold.* We decided to try to make a set after his design, and, with the exception of the vibration galvanometer, the whole of the equipment was constructed in our own workshop. The results obtained from it during the last 6 months have been so remarkably good that I strongly recommend any meter engineer who has a lot of current-transformer testing to do to install such a set. Only one observer is required to make both the ratio and the phase-angle test, and the method of using it is so simple that we use it now for testing all transformers that come into the works; it was originally intended for testing only instrument transformers. It is much easier to obtain accurate results with the Arnold testing set than to obtain inaccurate results by other methods. The method of using it is very similar to that of using the d.c. potentiometer, and the results are of the same order of accuracy. Further, it has completely done away with the necessity of testing integrating meters with their current transformers.

Mr. W. A. Wales: Should not the type B current supply (Fig. 1) be connected up as a potential divider? This arrangement would be more economical than the one given in the paper.

I should like to refer to a type of current transformer which is now available, built on the same lines as the tubular type of resistance. The voltage winding is wound over the core, which is made of iron stampings, a tube is placed over the top, and then the current turns are wound on. A slider makes contact with these current turns just as in the potential rheostat. This transformer is at present only available in the small sizes.

I take it that there is not much difficulty about keeping the frequency at 50. Frequency control comes into great prominence when very precise measuring devices such as a.c. potentiometers are used. I am afraid that a.c. potentiometers have several disadvantages, one of these being that they take rather too long to set up.

* *Journal I.E.E.*, 1934, vol. 74, p. 424.

For testing current transformers I am a little uncertain whether the a.c. potentiometer gives a correction curve which does not take into account the wave-form. I think it simply takes into account the first harmonic and neglects the others, and therefore the error curve as obtained by an a.c. potentiometer only gives corrections for the fundamental and not for the other harmonics.

Mr. A. H. Gray: The author refers on pages 370 and 373 to the use of the induction regulator as a method of current control, and advocates the Barbour or Ferranti type to obviate the possibility of a power-factor change. It seems to me that the double regulator with distributed windings will produce less phase displacement than either of the above types. The former are in effect variable chokes, hence their internal reactance will be greater than that of the distributed-winding type, which is in effect a transformer.

Referring to method B for phase-angle control, the author states that this method is only applicable to two or three loads; this is not strictly accurate, since the use of multi-chokes makes the method practically universal. Actually, this method has recently been modified and now uses a form of potentiometer control whereby all power factors and all currents from unity down to zero lagging can rapidly be obtained. The apparatus consists essentially of two potentiometers connected across the supply voltage, one for coarse and the other for fine regulation. These two potentiometers, connected in series, feed a small power transformer with two primary windings and one tapped secondary winding. Unity power-factor loads are obtained by feeding only one of the primary windings, whilst other power factors around 0·5 lagging are obtained by introducing a choke coil into the same circuit. Still lower power factors are obtained by introducing the second primary winding, the voltages impressed on the two windings being varied by means of the two potentiometers.

I am very surprised to see that the author advocates the use of a stop-watch and wattmeter for single-phase testing; apart from the errors of the two standards, there is the error of fluctuating load to be considered. I should have thought that the rotating sub-standard would have been more easy and accurate to use, particularly for on-site testing and single-phase meter testing generally.

On page 373 the author states that the polyphase method of testing is more accurate than the single-phase method. My experience differs from this; I find the single-phase method very much quicker, and, providing the meter under test is of modern design (hence its interaction errors are small), the method gives quite good "commercial" accuracy.

With regard to the question of current-transformer testing, the comparison method is, I believe, rapidly falling into disuse where high accuracy is essential. As manufacturing firms spend large sums of money—possibly as much as £1 000—on testing equipment to obtain accuracies of the order of 3 minutes, it would appear that extreme care is required to operate the author's method and obtain the accuracies he specifies. Perhaps he would give us more details of his experiences in obtaining these accuracies.

Turning to the question of clamps on meters, I strongly deprecate the use of paper clamps. The use of these necessitates the meter being opened by persons other than the authorized tester, and hence provides a possible source of trouble.

I am pleased to see that the author favours site testing, contrary to many engineers' opinions to-day. The method he suggests of using graduated resistances is interesting, but I would draw his attention, particularly where polyphase testing is concerned, to the method commonly used, which involves phantom loads. By using a 3-phase delta-connected transformer with tapped secondaries it is possible to obtain unity, 0·5, and zero power factors, both leading and lagging, together with the usual rheostatic current control.

Finally, I should like to show two slides illustrating commercial designs of testing equipments. (Mr. Gray here exhibited a lantern slide.) This illustrates a polyphase testing set employing tapped auto-transformers similar to those already described by Mr. Howarth. The set is comparatively small, yet it gives fine regulation from 1 000 amperes down to 0·25 ampere. The voltage is arranged in steps of 5 volts up to 600 volts. (Mr. Gray here exhibited another lantern slide.) This shows a portable polyphase testing equipment utilizing phantom loads. In this arrangement the supply is taken from the meter voltage supply and is fed into the delta-connected transformer, the secondary of which is used for supplying the necessary currents. Various power factors are obtained by means of the plugs and sockets connected to this transformer, whilst the current is regulated by means of rheostats.

Mr. F. E. J. Ockenden: There are some aspects of the author's testing methods with which I do not quite agree. Turning to Fig. 1, he suggests that the wave distortion in circuit B is likely to be less than that in circuit A. The fundamental law for the avoidance of secondary wave distortion is very simple; there must be no impedance in the primary circuit of the supply transformer. Consider a transformer with a primary winding, the resistance and leakage reactance of which are lumped together, externally to the winding itself, and a secondary winding connected to the load in the usual manner. So long as the core flux is sinusoidal the secondary voltage wave is also sinusoidal. In order to produce a sinusoidal flux it is necessary to draw a non-sinusoidal current from the mains. This current must flow through the "lumped" impedance and cause a non-sinusoidal voltage across it. This is deducted from the voltage applied across the primary winding itself, and therefore appears as distortion of the secondary voltage wave. Even in the best of transformers there must be some resistance and leakage reactance in the primary winding and therefore some slight wave distortion in the secondary; the moment, however, that resistance is added to the primary circuit (as in A) the wave must be seriously deformed. The only effect of adding resistance to the secondary circuit (as in B) is to decrease the current in the circuit without in any way impeding the flow of magnetizing current in the primary circuit. This arrangement is therefore to be preferred.

The question of induction regulators has been raised; the value of this type of apparatus for testing purposes depends very largely on its design. The usual form consists of two parts, a primary voltage being applied to one part and an induced voltage being derived from the other. The applied voltage is fairly certain to be sinusoidal. If, however, as is likely, the derived voltage is not so, then, when the net voltage is reduced to zero by opposing the two quantities, a sinusoidal wave is opposed by a non-sinusoidal wave and the only residue is the harmonic content. The result is that the harmonic content of the wave derived from the regulator is a maximum at the lower settings.

Dealing with the question of current- and voltage-transformer tests, I do not think we should criticize the author for using wattmeters for the purpose. It would appear that one of the characteristics of his design is that it enables the wattmeters which are already in use for other purposes to be employed for transformer testing. It would, however, be better to use only one of these, connected differentially between the standard current transformer and the one under test. With this arrangement only the difference current flows in the wattmeter, and it is read directly by a single reading of the needle. It may be said that the author's 5-ampere winding is unsuitable for the tiny difference current applied in this way; but it is really just as suitable as the combination of two 5-ampere meters, and the single reading is easier to obtain than the small difference between two comparatively large readings. Turning to voltage-transformer tests, I think that here again it would be better if he took a single wattmeter and used it differentially, exciting the current coils from another supply of variable phase position. Apart from the fact that he uses two wattmeters, I do not disagree with the author's method.

With regard to the testing of voltage transformers in general, a supply undertaking which buys voltage transformers may safely presume them to be correct when supplied, so that the question of correctness of turns ratio does not then arise. If during service any fault develops it will show itself in one or two ways; firstly, by alteration of the magnetizing current, and, secondly, by reduction of the short-circuit impedance. I think that in most cases supply companies could keep adequate check on the condition of their voltage transformers by periodic measurement of these two quantities.

The author deals at length with the use of nickel-iron-cored current transformers, and with this aspect of his work I am wholly in agreement. He makes, however, the rather curious statement that it is not necessary to buy low-power-factor wattmeters, which, he suggests, are at best but poor articles, because by altering the current-transformer range he can increase the reading as required. He suggests, for example, taking a 1-ampere wattmeter and passing through it a current of 2 amperes, thus doubling the reading at 0·5 power factor. This, however, merely means overloading the current coils, in which there cannot be any special virtue. The manufacturer, on the other hand, by effecting a compromise between the conflicting requirements of sensitivity and working forces, can produce a meter which is both accurate and robust and which will give full-scale deflection at power factors as low as 0·2.

Lastly, the author discusses the question of testing wattmeters for absolute accuracy. I support him when

he refrains from advocating the use of the a.c. potentiometer for this purpose owing to the difficulty of obtaining a voltage which is entirely free from fluctuation, but he also says that he does not consider that d.c. tests on a wattmeter are adequate since they do not take into account the possible phase-angle error of the wattmeter, and this will affect the readings at low power factors. Actually, the phase-angle error of a well-constructed wattmeter is extremely small. There are two possible sources of such errors: firstly, the eddy-current losses taking place within the windings themselves or within the framework on which the wattmeter is built; and secondly, the slight angle of lag in the voltage coil, or angle of lead in the series resistance due to self-capacitance. These effects tend to cancel out, and the net phase-angle error rarely exceeds 0·5 minute. An error as small as this cannot be regarded as serious, and it should therefore be possible to test wattmeters by means of a d.c. potentiometer only, with all its advantages of accuracy and steadiness of reading.

Mr. G. H. Fowler: The author appears to have devoted a considerable amount of time to the question of wave-form in selecting his apparatus for testing meters. To what degree he has been successful in obtaining a pure sine wave can best be judged by the remarks of Mr. Ockenden on the production of harmonics. What I am more concerned with is the extent to which these harmonics influence the accuracy of an induction-type meter. If these harmonics are introduced only into the current wave, and not into the voltage wave, the dynamometer wattmeter will still read the true power, i.e. 50-cycle volts with 50-cycle amperes. Therefore, harmonics cannot influence the dynamometer-type wattmeter. To what extent do they affect the induction wattmeter? I should like the author's views on this point.

In regard to on-site testing, the power—15 kW—which has to be drawn from service terminals has to be dissipated in the form of heat. I can imagine a good many places—in the City of London, for instance—where meters are situated in warehouses containing paper and similar inflammable material, where it would be rather awkward to dissipate 15 kW in the form of heat. I see no objection to an ordinary transformer, with a ratio of, say, 200/3 volts, drawing the necessary current through an arrangement which could be easily transported and would not be dangerous from the point of view of setting fire to its surroundings.

Mr. W. G. Daker: With the author's testing set one has first of all to adjust a resistance; this causes an alteration in the power factor, making it necessary to adjust the power-factor regulator; this again alters the load, making necessary one more adjustment. How much simpler it is to use the testing set described by Mr. Hill! With this, one has only to turn three knobs and the switch, and can then read off the load.

The author's method of on-site testing necessitates taking several assistants and a number of resistances to the sites; a far simpler arrangement seems to be to take a rotating sub-standard.

Mr. F. Byrne: I am particularly interested in Mr. Fowler's remarks on the question of wave-form. I submit that it is no better practice to take precautions to calibrate a meter under sinusoidal wave-form conditions, and then to introduce it on to a circuit the wave-form of which is unknown or badly distorted, than it is to calibrate a meter under uncertain conditions and to put it on a circuit where the wave-form is sinusoidal. The whole case seems to be summed up in this: how near is the induction meter to the perfect measurer of electrical energy? The undertaking with which I am connected has attacked the problem in the following way. A meter which had been connected on a circuit having a highly distorted current wave-form (15 to 20 per cent 5th harmonic) was tested by means of two dynamometer wattmeters, this type of instrument being presumably independent of the type of error to which the induction meter is subject. First, a number of tests were taken under the circuit conditions, and the results showed the induction-type meter to be 1·1 per cent fast as compared with the dynamometer-type wattmeters. Then, under the same conditions, with the exception that the current was of sinusoidal wave-form, the meter was tested again. The second series of results only differed from the previous results by $\frac{1}{10}$ of 1 per cent, the error measured in the second case being 1·2 per cent. It seems to me that the point about producing correct wave-form in a test room may be laboured a little too much. One must admit that it is desirable to produce that kind of wave-form when other instruments than induction-type meters are tested. To put the question in another way, the British Standard Specification for meters allows for a variation of 1 per cent with a 10 per cent variation in voltage applied to the meter. Comparatively, therefore, a 0·1 per cent variation in error due to the introduction of a 15 per cent 5th harmonic does not seem to be worthy of very much consideration.

Mr. J. L. Ferns (*in reply*): The current loading-circuit described by Mr. Howarth has many desirable features, not the least of which is the reduction (under proper operation) of the primary resistance to a comparatively small value, since the majority of the regulation is obtained on the auto-transformer. Precautions must be taken, however, in designing the resistance and transformer, or the circuit will still possess the possible disadvantage of the type A circuit. It is unfortunate that the limiting current for the current circuit shown in Fig. 2 was not more severely stressed in the paper. Actually the circuit was designed chiefly for low-capacity (5- and 10-ampere) meters, of which sizes most undertakings have large numbers. Owing to the limitation of slide-wire rheostats to about 25 amperes per tube it follows that 50 amperes is the limiting capacity of the suggested circuit.

If the injected voltage phase-shifting device is applied to the voltage circuit—as suggested by several speakers—it is essential to do away with the potentiometer rheostat and obtain the phase regulation by finely adjustable tappings on the secondary of the auxiliary transformer. This arrangement would avoid any danger of distortion due to resistance in series with the voltage coils of the meters, and it would also enable any suitable type of current loading circuit to be employed. Mr. Howarth mentions several devices for improving the control of the change-over circuits, and it would be interesting to give here a simplification of the circuit

suggested in Fig. 2. Instead of using 4-pin plugs (which are not readily obtainable) it is possible to manage with 3-pin plugs by permanently connecting terminal 4, Fig. 2, to the yellow phase, especially if 0·5 leading power factor is not desired. Three-pin plugs are obtainable in any district, if not actually stocked by the undertaking. Experience has shown this method of phase control to be even more rapid than the induction phase-shifter method.

I am pleased to see that the suggested wattmeter and current-transformer arrangement meets with the approval of Mr. Howarth and several other speakers.

Mr. Howarth then raises the problem of maintaining the condition $V_r + V_y + V_b = 0$ on the polyphase testing (voltage) circuit, and mentions that the accuracy of test is not affected by a departure from symmetrical loading. This is quite true as regards 3-wire 2-element meters but it is incorrect as regards 4-wire 2-element meters. The latter type of meter gives a true measurement only if the above condition is fulfilled, whereas the measuring instruments always give the correct measurement since they are connected on the 3-wire system and the condition $I_r + I_y + I_b = 0$ is always satisfied when polyphase tests are being carried out. To avoid trouble in this respect it is desirable to use liberally rated transformers and phase-shifters, and if a large number of 4-wire meters are to be tested together it is an easy matter to make the loading symmetrical by connecting a suitable number of meter coils between yellow and neutral. In order to ensure that the main supply satisfies the above voltage condition I should also take steps to measure the negative phase-sequence voltage and, if possible, leave the device always in commission so as to know when to take steps to balance the voltage load. As regards Fig. 6, I cannot claim any practical experience with the magnetic-shunt device beyond verifying (for the single-phase case) that the principle of operation is quite in order.

In common with several other speakers, Mr. Howarth emphasizes the defects of the methods of instrument-transformer testing described in the paper. These defects are indicated in the paper itself, and it is pointed out that the field of usefulness of the methods is limited. As a matter of fact, if the ratio of the transformer under test is nominally equal to that of the sub-standard transformer it is easy to connect the single-phase wattmeters so that one measures the testing load and the other the difference current or voltage. It was assumed that this would be understood, and consequently the general case where the ratios differ was described in the paper. I cannot understand why some service engineers favour instrument transformers of peculiar ratios, but this condition has to be faced in some undertakings. The main point in the paper is that the suggested polyphase equipment can combine this work, with a minimum of trouble, with single-phase and polyphase meter testing. The undertaking with which I am connected utilizes the bridge methods of testing for transformers requiring complete curve tests or very accurate results for the errors, but for certain routine tests on current transformers the wattmeter comparison method is quite satisfactory and very quick.

Mr. Howarth is in agreement with me as regards the care required in transporting meters, but questions the value of the suggested method of on-site testing. If suitable precautions have always been taken to transport and install meters in the proper manner such as he employs, then the method of replacing the meters at regular intervals is probably the best. The on-site testing scheme mentioned in the paper was intended, however, for those undertakings where meter maintenance has not been given the attention it deserves and where it is intended to make a start on the problem. In making a new start it would seem more desirable to pay the meter sites a visit and determine whether they also are suitable, besides checking the accuracy of the existing meter. After this preliminary attention it would be easier to set up, say, a definite scheme of replacement for future maintenance work. Several speakers prefer the rotating sub-standard method on account of its higher accuracy, but apparently they have not considered the time element, on account of which the method in the paper was suggested. It cannot be too strongly emphasized that the insertion of a rotating sub-standard on site is not a very easy matter; moreover, if the meter can be checked without touching the seals the consumer will be deprived of any excuse for suspecting that the tester has tampered with the meter.

It is pleasing to hear from Mr. Shotter that the injected-voltage type of phase control has actually been used and found very successful. With the proviso mentioned above (of doing away with the resistance) there is no doubt that placing the phase control on the voltage circuit, as carried out by Mr. Shotter, avoids any limitations on the current loading circuit and is a better method from the practical point of view. The disadvantages of the method of current-transformer testing described in the paper have already been dealt with; I can assure Mr. Shotter that the method is only intended for fairly high current-transformer load currents, and as a convenience for those testing departments who cannot afford more accurate equipment. His experience that transportation of meters results in surface cracks, even with oiled bearings, justifies the case made out in the paper for some form of clamp. I quite agree that cardboard packing is a little obnoxious when hundreds of house service meters have to be handled, but until manufacturers provide mechanical clamps what else can one do to protect the meter bearings? A big difficulty in a number of undertakings is the fact that the meter fixers are not controlled by the meter department, and hence cannot be trusted to carry out the requirements of the meter engineer. Even, however, if the cardboard-clamp method is too troublesome to apply to domestic meters, it should certainly be applied to power consumers' meters (presumably large revenue-producing meters). My remarks regarding the practice of leaving meters in service until a fault or a change of load occurs are based on personal observation and conversations with other engineers, but they should be looked upon as referring more to the number of undertakings rather than the number of meters in use. There is no doubt that this state of affairs is being improved upon, and the object of the paper was to assist the meter engineers concerned. I am pleased that Mr. Shotter confirms that the majority of meters work on a very low average load.

The type of current loading-circuit mentioned by Mr. Fawssett is a very special case which would only be suitable for a large undertaking, and it was therefore excluded from Fig. 1. Then again it is not good practice to waste electricity (fifty 5-ampere meters would only require about 10 volts to circulate full-load current) even if the loss of energy is negligible compared with other system losses. His suggestion for a warning device is very useful, but for safety in the test room I should prefer (1) to arrange for the testers' surroundings to be of non-conducting materials, e.g. heating-pipes and radiators to be cased in wood; (2) to place bare, live switches and terminals in semi-remote situations on the testing panels if they cannot be avoided; and (3) to provide insulating couplings between control handles and induction regulators or phase-shifters, which are presumably earthed.

Whilst the series-parallel method of varying a current-transformer ratio may perhaps give a slightly closer agreement between the various ranges, it is not a convenient method to use in practice. With the modern nickel-iron ring current transformer the type of winding described in the paper gives very tiny differences between the various ranges, and consequently its practical advantages make this method of range-changing essential. I note that Mr. Fawssett favours the dial-test method of testing meters, presumably in the belief that it is a quicker method. My own opinion, however, is that the wattmeter plus stop-watch method is better and as quick, if the testers are supplied with sufficient watches. I do not agree with Mr. Fawssett when he says the suggested equipments are too elaborate for the small undertaking. After all, we expect the scales in a small toffee shop to be just as accurate as those in a big multiple shop. He prefers the motor-alternator set for supplying testing equipments, but now that the supply frequency is being standardized at 50 cycles per sec. I do not think the motor-alternator set has any special advantages for the modern meter test-room unless the local voltage regulation, or wave-shape, is very bad.

The meter transportation and installation problem in Mr. Fawssett's widespread area is certainly difficult, because the cardboard-clamp idea is out of the question with unskilled labour for meter fixing. Until the manufacturers begin to fit the mechanical clamp there appears to be no solution to his transport problem. As regards on-site testing, the figure of 15 kW was only intended to indicate the limiting capacity of the type of consumer under consideration, and as a matter of fact the testing gear would not require to be rated higher than 3 or 4 kW to obtain the desired data. His suggestion for scaling the voltmeter in terms of V^2 is certainly a good one and would much simplify the calculations, particularly if the resistance values were conveniently adjusted. The meter maintenance scheme which he has just developed is an extremely bold attack on a difficult problem, and its leading features deserve the attention of all meter engineers.

Mr. Whitehead's query regarding the type B current circuit is very natural if one has not actually met this cause of distortion. If one of the components (not necessarily belonging to the apparatus under test) of the secondary circuit is an iron-cored inductance in which the testing current causes the iron to become saturated, then harmonics will be introduced into the testing current. The amount of distortion depends on the degree of saturation and the impedance of the iron-cored inductance compared with the total circuit impedance. In reply to his question regarding the departure from unity power factor when using methods A or B of phase-angle control it should be stated that the errors liable to be introduced are chiefly on account of the shape of the characteristic error curve, and not on account of the departure of the power factor from unity. To obtain the nominal testing watts the load current has to be increased by as much as 10 per cent if the so-called unity power factor is simply controlled by the circuit characteristics. There are quite enough sources of discrepancy without allowing this one to occur.

The first portion of Mr. Hill's useful contribution answers itself; as regards the testing-set equipment he mentions, I do not think this can be easily designed for the purposes indicated in the paper. The stroboscopic testing method has certain practical disadvantages, but there is no doubt that the time is not far distant when its use will become common in meter test-rooms. By employing 400 marks on the rotor, the American Westinghouse Co. have been able to introduce the stroboscopic testing method as a complete means of testing meters. The English manufacturers, in general, use only 100 marks on the rotor, and hence the limitations mentioned by Mr. Hill. In connection with testing more than one meter at a time by this method he may be interested to know that such an equipment has been constructed—in the Paris testing department, I believe.

Mr. Carter raises the question of the limiting current for the suggested type of polyphase equipment, and I agree that 500 amperes is about the maximum capacity if it is desired to use 1-ampere wattmeters. By having two sub-standard current transformers and more secondary tappings on the loading transformer, however, it is possible to extend the maximum current capacity. This problem indicates the necessity for other departmental engineers co-operating with the testing engineer before deciding to buy any measuring or protective equipment. The ring-type current transformer is certainly the most convenient for the testing department.

If watt-hour meters are being tested, then harmonics of the order mentioned by Mr. Carter will certainly be negligible on most meters, but in designing the testing equipments suggested in the paper it had to be kept in mind that they must be suitable for kVAh meters and relays, as well as kWh meters. This is the reason for taking such precautions to avoid wave-form distortion.

I agree with Mr. Kemsley as to the recent progress in current-transformer testing methods, and that most testing departments should have something more accurate than the ordinary polyphase testing equipment for this duty. One must crawl before walking, however, and there are still too many undertakings without a polyphase equipment, let alone a current-transformer testing equipment. I do not agree, though, that current-transformer testing does away with the need for testing meters with their current transformers.

Mr. Wales suggests that type B supply (Fig. 1) should be connected as a potential divider. This is already

envisaged in the diagram, since secondary tappings can be provided *ad lib.* The latest form of regulator which he describes has the limitation of being only suitable for low currents. Mr. Wales emphasizes some of the defects on account of which the a.c. potentiometer was ruled out of consideration for standardizing work.

Mr. Gray points out the advantages of the twin induction regulator as compared with the other types. This type should have been mentioned in the paper, but in making a decision between the three types the relative cost and reliability will be the deciding factors in this case rather than the electrical characteristics. The testing circuits described by Mr. Gray are extremely ingenious, and, provided the chokes and transformers are correctly designed, they should prove very satisfactory. The single-phase circuit is particularly good since it only requires a single-phase supply for its operation, but it still has the disadvantage of only approximating to unity power factor. For ordinary single-phase meter testing the rotating sub-standard has the big disadvantage that only one meter can be tested at once, besides certain other practical disadvantages.

With polyphase meters of modern design there is no doubt that single-phase tests will produce reasonable results, but Mr. Gray has overlooked the fact that supply undertakings have perhaps as many as 20 different types of older polyphase meters, in addition to modern meters. The method of current-transformer testing described in the paper is not intended to compete with specialized current-transformer testing equipments.

Mr. Gray deprecates the use of paper clamps, but he does not state whether he advocates fitting proper mechanical clamps to modern a.c. meters. The on-site testing equipment he describes is very useful for particular testing requirements, but it necessitates making tests on all the components of a metering equipment whilst it is in the test room.

Mr. Ockenden discusses the fundamental characteristics of transformers; I disagree with his statement that " the moment resistance is added to the primary circuit (as in A) the wave must be *seriously* deformed." Resistance can be safely inserted in the primary provided the magnetizing current does not contain any appreciable harmonics, i.e. when the flux density in the iron is kept low. The danger with the type A circuit is in overstepping the limiting value of the flux density for correct operation. I am glad to see that he appreciates the conditions under which the suggested polyphase equipment has to work, and that he realizes that dual-purpose apparatus cannot usually give the performance of specialized apparatus. I feel, however, that Mr. Ockenden has misunderstood the reference to the 1-ampere wattmeters. It is not intended to pass 2 amperes through the current coils to obtain high readings on 0·5 lagging power factor, but to use the property of the voltage circuits for carrying twice the rated voltage. The wattmeters have to operate at full-scale deflection for hours on end, and it is my experience that the high-power-factor type of wattmeter is more suitable for this duty than the low-power-factor type. As regards the small phase-angle error of dynamometer wattmeters, the customer cannot verify this fact without an a.c. test.

Mr. Fowler's query regarding the effect of harmonics has already been discussed, but I should add that the effect of the current harmonics alone, on induction meters, is to increase the braking effect of the current flux and to cause the current-coil laminations to work at a different point on the magnetic characteristic. The meaning of the figure of 15 kW in respect of on-site testing has been explained in the reply to Mr. Fawssett.

I am afraid Mr. Daker hardly does justice to the simplicity of the methods suggested in the paper, and I am sure that a little practical experience of them would alter his opinion, bearing in mind the testing requirements of supply undertakings.

Mr. Byrne's initial comment partakes of the nature of a conundrum and the answer in the same form is that " two wrongs do not make a right." The reasons for taking precautions against harmonics have already been given. If Mr. Byrne is only concerned with testing kWh meters then the current harmonics he mentions will have a very small effect provided his supply voltage contains no corresponding harmonics. What steps does he propose taking, however, to make sure that his voltage wave-shape will be satisfactory, say, a year hence? In these days of gaseous discharge lamps, rectifiers, and neon signs, the system wave-shape may quite easily become affected in certain places.

In conclusion, the thanks of meter engineers are due to those speakers whose alternative suggestions or confirmatory remarks regarding the various testing equipments will help forward the development which the paper was written to assist.

THE DESIGN, CONSTRUCTION, AND USE OF RESISTORS OF CALCULABLE REACTANCE.*

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SUMMARY.

The design, construction, and use of two-terminal resistors of calculable reactance, covering the range 100–10 000 ohms are described. The resistors form a series of primary standards suitable for the measurement of residual inductance and are of two forms; those in the range 100–1 000 ohms consist of a single straight wire concentrically mounted within a return conductor, while those in the range 1 000–10 000 ohms consist of single-layer solenoids similarly mounted. The former are suitable for measurements up to a frequency of 10 000 cycles per sec. and the latter for measurements up to 3 000 or 4 000 cycles per sec.

The inductances and capacitances of the standards are calculated as far as possible from their dimensions, although certain of the capacitances involved were measured directly. The variation of reactance with frequency is discussed and experimental confirmation of theoretical predictions is given.

Comparison measurements between the standards have been carried out using simple bridge methods, and the general consistency of the results shows that, using these standards and methods, the phase angle of any resistor in the range under discussion can be determined with an accuracy of 6×10^{-6} radian at 1 000 cycles per sec. even when bridge ratios as high as 10 : 1 are involved. As an additional check, the values of reactance were determined in terms of the constants of high-frequency self-inductance standards, and the calculated and observed results were found to show the same consistency.

CONTENTS.

(1) INTRODUCTION.

The measurement of the "residual inductance" of resistors is one which is in frequent demand, and it is the purpose of the present paper to describe the development of the methods used for such measurements at the National Physical Laboratory. The need has been primarily for simple and reproducible methods for measurements at frequencies up to about 5 000 cycles per sec. on resistors ranging from 100 to 10 000 ohms, and a set of primary standards of residual reactance has been constructed to cover this range.

The design and application of such standards has been discussed by several writers, of whom one of the earliest was S. L. Brown.† Later workers in this field include Nukiyama and Shoji,‡ Wilmotte,§ Davis,|| and Berberich;¶ and the use of such standards has been discussed in detail by Hartshorn and Wilmotte,** and by Bartlett.††

The proposed method of use of a standard of calculable reactance is an important factor governing the design, but in order not to limit the usefulness of the standard it is desirable that its properties should be independent of its surroundings. This can be achieved by completely enclosing the resistance in a metal screen, which must then be brought to a definite potential, usually earth potential. Such a standard is then most satisfactorily employed in a simple four-arm bridge with a Wagner earthing device, and the screen may be connected either directly to earth or to a detector point on the bridge (and incidentally to one end of the resistance), which is maintained at earth potential by the Wagner earthing arm. In the former case, the currents which flow to the screen in virtue of the capacitance between it and the resistor are eliminated from the measurements, and in the second case such currents are included in the measurements. The effective reactance is, of course, different in the two cases.

It was decided to make use of the above principles in designing the present standards; their general pattern is that of a uniform resistor concentrically mounted within a brass cylinder of negligible resistance and connected to it at one end. The ideal form is that in which the uniform resistor is simply a single straight wire, but a limitation is imposed by the necessity for restricting the size of the instrument. The most highly resistive wire which is mechanically suitable for the purpose is of nichrome, 2·8 mils diameter. This alloy has approximately double the resistivity of the ordinary resistance alloys—manganin and eureka—and its tensile strength is about 50 per cent greater.‡‡ A resistance of 100 ohms is given by a length of 43 cm of this wire. A 1 000-ohm standard would thus be 430 cm long, which in the ordinary way is entirely out of the question in a laboratory standard. The use of a concentric return conductor, however, renders the system astatic, so that it can be bent into any convenient form and still retain the properties of a straight standard. It becomes impracticable, nevertheless, to pursue this principle in constructing resistances greater than 1 000 ohms, and for larger values than this it was decided to make the resistor in the form of a uniformly wound solenoid. A

* The Papers Committee invite written communications, for consideration with a view to publication, on papers published in the *Journal* without being read at a meeting. Communications (except those from abroad) should reach the Secretary of the Institution not later than one month after publication of the paper to which they relate.

† See Reference (1).
‡ *Ibid.*, (2). § *Ibid.*, (3). || *Ibid.*, (4).
¶ *Ibid.*, (5). ** *Ibid.*, (6). †† *Ibid.*, (7).
‡‡ Finer wire could be obtained, but its mechanical strength and stability of resistance are not considered sufficient for use in a standard instrument.

solenoid 1 cm in diameter and 1·9 cm long, wound with the wire described above, has a resistance of 1 000 ohms, so that it is easy to construct a manageable standard of resistance as high as 10 000 ohms by this means.

Simplicity of construction, metrology, and use, being eminently desirable in the present instance, the following details were finally decided upon. In the decade 100–1 000 ohms, ten straight brass tubes, each carrying a single nichrome wire of 100 ohms resistance concentrically within it, were made and so mounted that they could be connected together with concentric connectors to give resistances of 100, 200, 300, . . . 1 000 ohms, the necessary condition for the connection of the screen being realized in each case. For the 1 000–10 000 ohm decade, it was decided to build ten units having resistances of 1 000, 2 000, 3 000, . . . 10 000 ohms, each consisting of a uniformly-wound solenoid of 1 cm diameter mounted within a return conductor of brass tubing. These units range from 2 to 20 cm in length and have an overall external diameter of 1·5 cm.

(2) Theoretical Considerations.

The simple theory of a shielded resistor of the type under consideration is well known. It assumes that the resistance, inductance, and capacitance to screen, of the resistor are uniformly distributed along its length. In any practical case, however, it is necessary to consider in addition the effect of mutual capacitances between the various parts of the resistor itself, the effect of "lumped" capacitances (i.e. capacitances of finite magnitude which may be regarded as acting between two points of the system), and the effect of eddy-current losses in the material of the resistor and its screen.

It must be observed that the present concern is with the effect of these various factors on reactance only, and, further, the limitation of the analysis to frequencies not greater than 10 000 cycles per sec. makes it permissible to consider every effect separately and to obtain the total effect by simple summation, as all second-order terms are negligible and are disregarded in what follows.

The effect of mutual capacitances in standards of the form proposed was first investigated with the aid of models. The models took the general form of a straight rod or uniform solenoid, divided into two equal lengths, and mounted concentrically within a cylindrical screen but not connected to it. The capacitance between the two parts of the rod or solenoid in the presence of the screen was measured directly, and its variation with total length was investigated. In the case of the rod models the effective length was varied using adjustable screens. A similar device was adopted with the solenoid models, on which some results were checked by actual removal of turns from the winding.

From the measurements on the rod models in screens of various diameters, it was deduced that the total distributed capacitance between the two halves of a straight-wire standard of the type proposed for the 100-ohm units could not exceed 0·01 $\mu\mu$F at most, and was probably considerably less. It is impossible, therefore, that the reactance of such a standard can be appreciably affected by mutual capacitances between various points on the resistor.

From measurements on the solenoid models (which were of precisely the same dimensions as the solenoid standards) it was evident that the mutual capacitance between any two points on the solenoid was sensibly zero when such points were separated by a distance greater than about 0·05 cm, since the total mutual capacitance between the two parts of the model appeared to be independent of total length when this exceeded 0·1 cm. The significance of this result is that all elements on a resistor of this form, except those lying very near the ends, can neither gain nor lose (displacement) current in virtue of mutual capacitances, and the effect on reactance of those elements near the ends is easily shown to be negligible. On the basis of this experimental evidence it therefore seemed reasonable to disregard all question of "turn-to-turn" and other mutual capacitances in the solenoidal resistors.

Turning now to the question of lumped capacitances, it is of course well known that if a resistance R be shunted by a capacitance C, then the impedance is altered by an amount $-j\omega CR^2$, where j is the operator rotating through a right angle and ω is the angular frequency of the current. In any resistance standard such a shunt capacitance is provided by that existing between the terminals of the instrument, and in order to allow for this an actual measurement of the terminal capacitance must be made. In addition, in the case of the 100–1 000 ohm decade there exist, owing to the particular method of connecting the units, lumped capacitances between resistor and screen at equal intervals along the total length. The equivalent circuit of this decade is shown in Fig. 1, in which the condensers marked C_1 represent these lumped capacitances. If r_1 denotes the resistance of each section of the resistor, the total effect on the impedance may be determined in the following way.

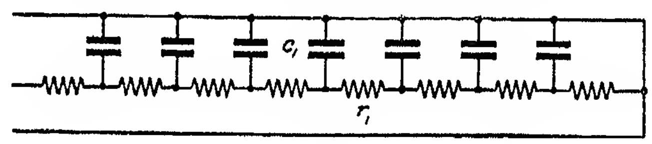

Fig. 1.

For one unit only there is no connector, and hence no effect.

For 2 units, the contribution to impedance is $-\alpha C_1 r_1^2$,

For 3 units, the contribution to impedance is $-(1+2^2)\alpha C_1 r_1^2$,

For 4 units, the contribution to impedance is $-(1+2^2+3^2)\alpha C_1 r_1^2$,

For n units, the contribution to impedance is

$$-\sum_1^{n-1} p^2 \alpha C_1 r_1^2,$$

where p is any integer between 1 and $(n-1)$, and $\alpha = j\omega$.

$$\text{Now} \qquad \sum_1^{n-1} p^2 = \frac{n(n-1)(2n-1)}{6}$$

so that the total contribution to impedance from this source will be

$$-\frac{\alpha n(n-1)(2n-1)}{6} C_1 r_1^2 = -\alpha \frac{2n-1}{6n} KR^2 \qquad (1)$$

where $K = (n - 1)C_1$ = total capacitance,
$R = nr_1$ = total resistance.

This result can be used to calculate the effect of both lumped and uniformly distributed capacitances to screen. For, if n be made infinite, the problem becomes that of a resistor with a uniformly distributed capacitance to screen, and it is easy to see that in this case the right-hand term in equation (1) reduces to $-\frac{1}{3}\alpha KR^2$.

If, then, a resistor has total resistance R, total inductance L, and total uniformly distributed capacitance to screen C, its impedance will be given by

$$Z = R + \alpha(L - \tfrac{1}{3}CR^2) \quad . \quad . \quad . \quad . \quad (2)$$

The effect of lumped capacitances can be calculated independently if necessary, and an appropriate term added to (2). The time-constant, τ, of the resistance will be given by

$$\tau = \frac{L}{R} - \frac{1}{3}CR \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Passing on to the question of the effect of eddy-current losses in the resistor and its screen, it need only be said with reference to the straight-wire standards that Russell's* comprehensive analysis of the problem shows that there can be no effect on the reactance of such a standard at frequencies less than 10 000 cycles per sec. and that the effect is negligible even up to frequencies as high as 1 000 000 cycles per sec. In the case of a solenoid enclosed within a coaxial cylindrical screen, however, the paths of the eddy currents in the screen are circles the centres of which lie on the common axis of the solenoid and screen, and effects of definite magnitude are to be expected even at low frequencies. In addition, the eddy-current losses in the material of the winding itself must also be considered.

A general expression for the effect of a screen on an inductance inside it has recently been given by Kaden,† but his equations are not readily adaptable for use at audio frequencies. The following treatment, while limited, is, however, sufficiently accurate for the purpose in hand.

The solenoid and screen may be regarded as an air-core transformer and we may apply the well-known results for augmentation of resistance and diminution of inductance of the primary windings—in this case the solenoid itself. If R_1, L_1, a, l, n = resistance, self-inductance, radius, length, and number of turns respectively of the solenoid; and R_2, L_2, b, l = resistance, effective self-inductance, radius, and length respectively of the screen; and if M = mutual inductance between solenoid and screen, then the effective resistance, R', and effective inductance, L', of the solenoid will be given by

$$R' = R_1 + \frac{\omega^2 M^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad . \quad . \quad . \quad . \quad (4)$$

$$L' = L_1 - \frac{\omega^2 M^2 L_2}{R_2^2 + \omega^2 L_2^2} \quad . \quad . \quad . \quad . \quad (5)$$

The problem now is to interpret the quantities M, R_2, and L_2. To a first approximation, it is clear that we may write

$$M = \frac{L_1}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$L_2 = \frac{4\pi^2 b^2}{l} k_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where k_2 is Nagaoka's constant* for the screen. If ρ is the resistivity of the screen, which has a thickness d, then

$$R_2 = \frac{2\pi b\rho}{ld} = \frac{4\pi^2 b^2}{l} r \quad . \quad . \quad . \quad . \quad (8)$$

where r is the resistance per cm of the screen when the lines of current flow are parallel to its axis. It will be observed that r is a quantity easy to determine experimentally, and that its use obviates the need for determining ρ and d.

We are now in a position to put equation (5) into a suitable form. Equation (4) need not be considered in detail, as the augmentation in resistance is in general insignificant. Rewriting equation (5), we have

$$L' = L_1 - \frac{M^2}{L_2\left[1 + (R_2^2/\omega^2 L_2^2)\right]}$$

$$= L_1 - \frac{L_1^2}{n^2 L_2\left[1 + (r^2/\omega^2 k_2^2)\right]}$$

or $$L' = L_1\left[1 - \frac{a^2}{b^2}\cdot\frac{k_1}{k_2}\cdot\frac{1}{\left[1 + (r^2/\omega^2 k_2^2)\right]}\right] \quad . \quad . \quad (9)$$

where k_1 is Nagaoka's constant* for the solenoid. This value of L' is used in equation (3) for computing the time-constants of the solenoidal standards. The experimental evidence in support of equation (9) will be discussed later. Meanwhile it is sufficient to state that on the 10 000-ohm standard at a frequency of 1 000 cycles per sec. the above frequency effect is less than 2×10^{-9} henry per ohm, expressing the result as a "time-constant," and this value is certain to at least 25 per cent. The possibility of reducing this effect by making a longitudinal slit in the screen has not been overlooked. This procedure would possess the disadvantage that it would not completely eliminate the effect, while at the same time it would render more uncertain the determination of distributed capacitance.

Heaviside† has shown that the field in the winding itself, when the current distribution is uniform throughout its section, contributes to the total self-inductance of a solenoid an amount L_3 such that

$$L_3 = \frac{1}{6}\frac{4\pi^2 n^2 h}{l}(4a + h)\mu \quad . \quad . \quad . \quad . \quad (10)$$

where h is the thickness of the winding and μ the permeability of the wire. The expression is accurate only for wire of square section, but it is sufficiently near the truth for the present purpose. It is this quantity L_3 which will vary with frequency, since the effect of increasing the frequency will be to render the current distribution non-uniform. L_3 will therefore represent the maximum possible change of inductance with frequency, since it will become zero at infinite frequency.

* See Reference (8). † *Ibid.*, (9).

* See Reference (10). † *Ibid.*, (11).

Cohen* has given a detailed analysis on these lines, but it is sufficient for the present purpose to show that if in any case L_3 is negligible in comparison with L, the inductance calculated in the ordinary way for the field in the core, then the inductance of the solenoid may be regarded as independent of frequency.

Neglecting h in comparison with a, the expression for L_3 can be put into the form $L_3 = \frac{2}{3} L\mu\ (h/a)$ so that the field in the winding itself contributes a fraction $\frac{2}{3}\ (h/a)\mu$ of the total inductance. For the solenoids actually constructed, $h/a = 2 \times 10^{-3}$ and $\mu = 1 \cdot 3$, so that the total contribution from this source is of the order of 2 parts in 1 000 of the total inductance, which is quite negligible for the present purpose, so that it is justifiable to regard the inductance of the solenoids as invariable with frequency on this account.

(3) The 100–1 000 ohm Decade.

The essential features of both decades have already been outlined in the Introduction, and the main con-

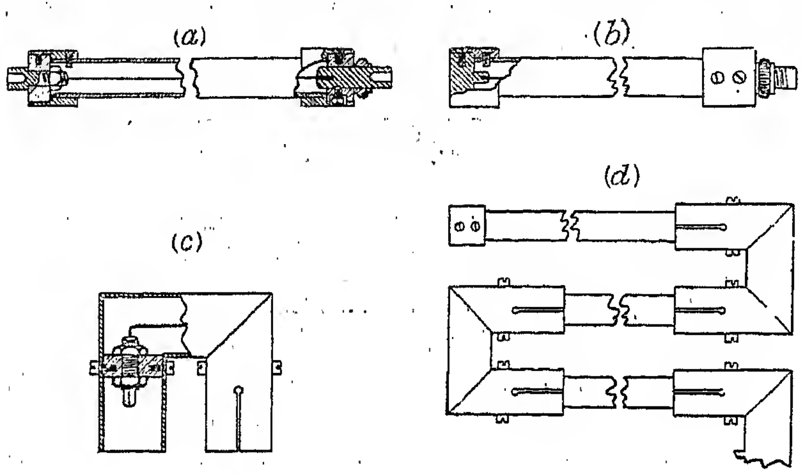

Fig. 2.

structional details of the 100–1 000 ohm decade are shown in Fig. 2. In nine of the ten tubes the resistance wire is supported between brass sockets held in keramot bushes (Fig. 2*a*), and in the tenth tube the wire is connected at one end to the screen (Fig. 2*b*). The connectors, the inner conductor of which is of heavy-gauge copper wire, are shown in Fig. 2(*c*), and the method of assembly is apparent from Fig. 2(*d*). The adjustment of resistance and the determination of dimensions were carried out, using a special jig, before the wires were mounted in the tubes. For convenience in use, the ten tubes are mounted in a " squirrel-cage " formation.

For the calculation of the reactances of this decade it was necessary to determine (*a*) the self-inductances of the wires, screens, and connectors, (*b*) the distributed capacitances, and (*c*) the lumped capacitances.

The self-inductance, L, of a go and return circuit consisting of a wire of radius a_1 and length l, concentrically situated within a thin tube of radius a_2 and length l, is given by*

$$L = 2l \left[\log_e \frac{a_2}{a_1} + \frac{\mu}{4} \right] \quad . \quad . \quad . \quad (11)$$

where μ is the permeability of the wire. In the present case it is sufficient to use a_2 in this formula and not the more precise quantity, the geometrical mean distance of the wire from the tube. The value of μ, determined from observations on a Curie balance, was $1 \cdot 3$ for the nichrome wire used. The calculation of the main inductances was thus straightforward, and small corrections were applied for the inductances of the sockets and connectors. The distributed capacitances between wire and screen were calculated from the dimensions of the system; and the lumped capacitances due to the socket ends, terminals, and connectors, were deduced from Schering-bridge measurements. These capacitance values (in $\mu\mu$F) are set out below: their application in formulæ (1), (2), and (3), will be obvious.

Total capacitance between resistor and screen	$= 11 \cdot 2_3$
Calculated capacitance between wire and screen	$= 4 \cdot 8_5$
Contribution from two sockets	$= 6 \cdot 3_8$
Total capacitance added by one connector	$= 6 \cdot 7_2$
Total lumped capacitance at connections	$= 13 \cdot 1$
Terminal capacitance	$= 0 \cdot 2$
Total end capacitance	$\doteq \frac{1}{2}(6 \cdot 38) + 0 \cdot 2 = 3 \cdot 4$

The mean inductance of one tube and screen was $0 \cdot 4_5$ μH. The time-constants of the decade, given in Table 1, are independent of frequency up to 100 000 cycles per sec. It will be seen that all the values are comparable with those given by commercial coils. It would of course be a simple matter to make coils in which the net reactance would be smaller than that of these resistors, but for standards of this kind the actual value is of secondary importance, its constancy and the limits within which it can be computed being the chief factors.

Table 1.

Time-constants of the 100–1 000 *ohm Decade, in Microhenrys per Ohm.*

Resistance, ohms ..	100	200	300	400	500	600	700	800	900	1 000
Time-constant ..	0·004	0·003	0·000	−0·004	−0·009	−0·015	−0·022	−0·030	−0·040	−0·051

(4) The 1 000–10 000 ohm Decade.

The possibilities in the design of a set of standards of the type just described are very limited, but immediately the construction of solenoids is considered many possible variations arise.

As a preliminary measure, curves were constructed showing the inductive time-constants of solenoids of various dimensions and resistances as a function of their diameters. These curves are reproduced in Fig. 3, from which it will be seen at once that the resistivity of the wire is of great importance in determining the time-constant and, further, that beyond a certain point very

* See Reference (12).

* See Reference (13).

little is gained by altering the shape of the solenoid. The simplicity of construction had to be taken as a guiding factor, however, and it was decided that solenoids 1 cm in diameter were probably the most suitable for the purpose in hand. This, it will be seen, gives an inductive time-constant of the order of 8×10^{-8} henry per ohm for any coil between 1 000 ohms and 10 000 ohms resistance, wound with nichrome wire of 2·8 mils

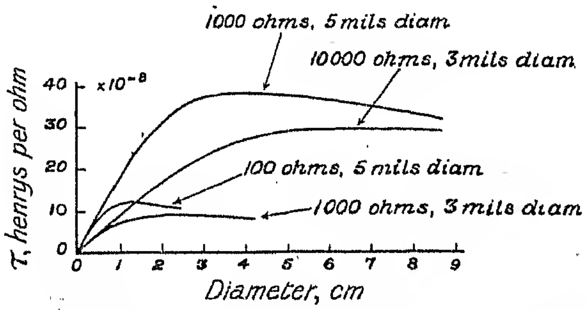

FIG. 3.—Time-constant (τ) of solenoids, as a function of diameter (nichrome wire).

diameter. While this value is large, it must be remembered that the contribution of the capacitances will oppose this inductive effect (see equation 2). It is easy to show that, if b is the radius of the screen and a the radius of the solenoid, l its length, and r the resistance per cm of the wire used, then a standard of the form proposed will be almost non-reactive if

$$\log_e \frac{b}{a} = \frac{l^2 r^2 \times 10^{-2}}{54k} \quad . \quad . \quad . \quad . \quad (12)$$

where k (which in general is nearly unity) is Nagaoka's function of $2l/a$.* A few preliminary calculations on this basis showed that while it would be easy to make the 10 000-ohm unit non-reactive, it would be impracticable to do so for the 1 000-ohm unit. Consequently, a compromise was effected by using screening tubes of approximately 1·5 cm diameter for all the standards, making the 1 000-ohm standard inductive and the 10 000-ohm standard capacitive.

The solenoids were wound with double silk-covered nichrome wire, 2·8 mils diameter, on turned rods of

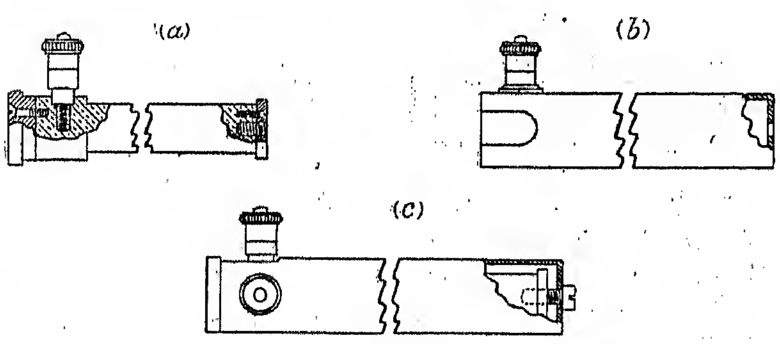

FIG. 4.

keramot 1 cm diameter having a shoulder carrying a terminal at one end, and fitted with a small brass plate on each end-face (Fig. 4a). The winding was secured to the terminal and to the brass plate remote from the terminal. The screening tube is of brass (Fig. 4b), the method of assembly being shown in Fig. 4(c). Ten such units, having resistances 1, 2, 3, . . . 10 × 1 000 ohms, form the higher decade.

For the calculation of the reactances it was necessary

* See Reference (10).

to determine (a) the inductances of solenoids and screens, (b) the distributed capacitances, (c) the lumped capacitances, (d) the resistance per cm of the screen (see equation 9).

The inductances of the solenoids were calculated using Nagaoka's formulæ and tables,* and ranged from 79 to 952 μH. The corrections for the field inside the wire,† the pitch of the winding,‡ the permeability of the keramot former, and the inductance of the screen, were individually and collectively negligible.

The distributed capacitances were determined by the following method. A simple bridge of the type shown in Fig. 5 was set up, one arm, X, being a solenoidal resistor complete with screen. Balance was effected by adjusting the ratio arms and the condensers C_1 and C_2. R_2 was a ballast resistance, usually $\frac{1}{10}$ the resistance of X. The solenoidal resistor was then removed from the bridge and completely filled with dry benzene, replaced, and balance restored by adjusting C_2, the condenser

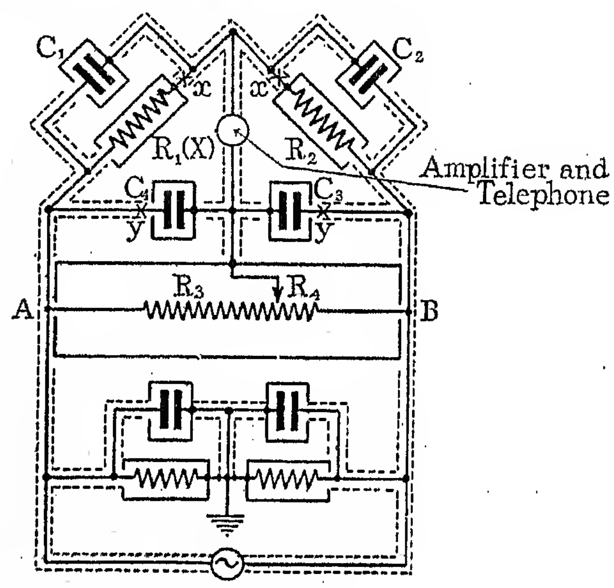

FIG. 5.

Disconnections at yy for main-bridge balance. Disconnections at xx for Schering-bridge balance. Condensers C_3 and C_4 not used in substitution measurements. Lead screens (shown dotted) are all earth-connected.

shunting the smaller resistor. If ΔC_2 is the difference in capacitance of this condenser for the two settings, then

$$\frac{1}{3}C = \frac{R_2}{X} \cdot \frac{\Delta C_2}{\kappa - 1} \quad . \quad . \quad . \quad . \quad (13)$$

where C is the total distributed capacitance to screen of the resistor under examination (see equation 2), and κ is the dielectric constant of the benzene.§

This method of determining the distributed capacitance can be applied to any screened resistor. Even in the present instance, with resistors of definite geometrical form, it appears to be more satisfactory than the calculation of C from the dimensions. In the case of the straight-wire standards, the dimensions are so readily determined and C is so small that no difficulty arises, but in the case of a solenoidal resistor with its irregular surface the measurement of the effective diameter presents some difficulty, and no estimation can readily be made of any end effects. It was therefore considered more satisfactory to accept the experi-

* See Reference (10). † *Ibid.*, (11). ‡ *Ibid.*, (14).

§ In order to avoid contamination of the benzene, the solenoids were dried over fused calcium chloride for several days and were thoroughly washed with benzene before use. The value of κ for the benzene actually used was determined immediately after the measurements.

mentally-determined values of C for the solenoidal standards.

It will be realized that the measurements made with the benzene will include only part of the "turn-to-turn" capacitance of the solenoid, the field in the interior of which will not be affected by the presence of the benzene, but, as the work on the models has shown (see Section 2), any effects due to this are so small as to be entirely negligible.

The total distributed capacitances, measured as described above, ranged from $4 \cdot 6$ $\mu\mu$F for the 1 000-ohm standard to $34 \cdot 2$ $\mu\mu$F for the 10 000-ohm standard, while $0 \cdot 2_5$ $\mu\mu$F was added by the terminals.

Table 2 gives the calculated values, in microhenrys per ohm, of the time-constants of these standards at zero frequency and at a frequency of 1 000 cycles per sec.

In addition to the ten resistors in the above decade, three others of the same type were made, two of nichrome (1 000 and 10 000 ohms) and one of eureka (10 000 ohms). These standards differ slightly in geometry and number of turns from the corresponding units in the decade, and with them it has been possible to obtain additional experimental checks on the calculated results.

where C_1 and C_1' are the readings of the capacitor C_1. If C_2 is varied, then

$$\tau_1 - \tau_1' = R_2(C_2' - C_2) + C_1(R_1 - R_1') \quad . \quad (16)$$

The difference of the time-constants of two resistors is thus determined directly in terms of the difference of two readings of a condenser, and in this way the three 1 000-ohm and the three 10 000-ohm standards were compared.

Comparisons were also made between corresponding units in each decade (i.e. the 100-ohm and 1 000-ohm, 200-ohm and 2 000-ohm, etc.) by the following modification of the above standard method. In the bridge described, arm 1 was provided by a unit of the lower decade and arm 2 by the corresponding member of the higher decade. At balance, we have

$$\tau_1 - \tau_2 = (C_1R_1 - C_2R_2) + \tau_4 - \tau_3$$

in which the capacitances C_1 and C_2 include all the stray shunt capacitances in the arms with which they are associated. The value of the quantity $(C_1R_1 - C_2R_2)$ was determined directly as follows. When balance had been effected, the high-potential lead was disconnected from each resistor and its bare end shielded,

TABLE 2.

Time-constants of the 1 000–10 000 *ohm Decade, in Microhenrys per Ohm.*

Resistance, ohms	1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000	10 000
Time-constant:—										
Zero frequency	0·075	0·075	0·070	0·056	0·044	0·032	0·016	−0·004	−0·022	−0·044
1 000 cycles per sec.	0·074	0·074	0·069	0·055	0·042	0·030	0·014	−0·006	−0·024	−0·046

(5) Measurements with the Calculated Standards.

Comparison measurements between the various calculated standards were carried out on a bridge of the type shown in Fig. 5. In this bridge a Kelvin–Varley potential divider, AB, provides an adjustable pair of ratio arms, and phase-angle adjustments are made by variable air condensers shunting the other arms of the bridge. It is well known that if τ_1, τ_2, τ_3, and τ_4, are the (small) time-constants of the arms numbered 1, 2, 3, and 4 respectively, then

$$\tau_1 - \tau_2 = \tau_4 - \tau_3 \quad . \quad . \quad . \quad . \quad (14)$$

The right-hand side of this equation is a quantity determined by the setting of the potential divider, and can be assumed to remain constant over small variations in setting.

The difference between the time-constants of two resistors of the same nominal value can readily be determined. Each in turn is placed in the arm 1, and balance effected by adjusting the potential divider and either capacitor C_1 or capacitor C_2; but, of course, not both. If τ_1 and τ_1' be the time-constants of the resistors, R_1 and R_1' their resistances (assumed nearly equal), then it is easy to show that

$$\tau_1 - \tau_1' = R_1(C_1 - C_1') + C_1'(R_1 - R_1')^* \quad . \quad (15)$$

* See Section (2), page 391, for the limitations of an equation of this form.

leaving what is actually a Schering bridge, which was then balanced by adjusting either C_1 or C_2 (not both), and by adding condensers in shunt across the arms of the potential divider, the setting of which remains unaltered at a ratio n, say. Then, if C_1 and C_1' are the two values of C_1, we have

$$C_1'/C_2 = 1/n = R_2/R_1$$

so that

$$C_1R_1 - C_2R_2 = R_1(C_1 - C_1') \quad . \quad . \quad . \quad (17)$$

giving the quantity required directly in terms of the difference of two settings on one condenser.

In order to determine separately, when necessary, the magnitudes of C_1R_1 and C_2R_2, the procedure is to balance the Schering bridge by adjustment of the ratio arms. One of the condensers C_1 or C_2 is then altered by a definite amount and the bridge re-balanced. If n_1 and n_2 are the two ratios given by the divider and C is the known change in C_2, say, then

$$C_1 = \frac{C}{n_2 - n_1} \quad \text{and} \quad C_2 = \frac{n_1}{n_2 - n_1}C$$

whence

$$C_1R_1 - C_2R_2 = \frac{CR_1}{n_2 - n_1}\left[1 - \frac{R_2}{R_1}n_1\right] \quad . \quad (18)$$

In order to carry out these measurements as accurately as possible, a completely screened junction-box

was made for the bridge centre. The leads from this box to the arms of the bridge were screened from each other and were closely twisted. The screens of the leads and of the junction-box were connected directly to the Wagner earth point, so that effectively the capacitance between the leads was nearly zero. The twisting of the leads reduces the inductance of the arms, which does not affect substitution measurements, but which is included in comparison measurements between two arms. The method outlined above deals adequately with the stray capacitances in the arms, but the stray inductances must be measured independently. It was found that they were such as to contribute an effective time-constant of $3 \cdot 4 \times 10^{-9}$ henry per ohm, when associated with a 100-ohm resistor.

A difficulty which arises in comparing the time-constants of two arms in the way described above is the simultaneous balancing of the main and Wagner arms in the Schering-bridge arrangement. The two sets of adjustments required are not independent, and therefore the values giving the simultaneous balance point can only be approached very slowly by a large number of successive small adjustments. This difficulty can be removed to some extent by the use of larger condensers, but this tends to reduce the accuracy of the measurements as the result is then made to depend on a small change in a large quantity. A suitable compromise can, however, be effected in most cases.

In this way ten values of the quantity $\tau_4 - \tau_3$ were determined for a particular setting (giving a ratio 1/10) of the potential divider. This quantity has been determined independently in terms of the constants of the standard high-frequency inductance coils of the Laboratory. The results of these measurements are given in Table 5. The calibration of the potential divider having been obtained in a similar way for a wide range of settings, the above methods become applicable to measurements on resistances of any value. The unknown resistance would form one arm of the bridge, and a calculated standard, of value chosen to give a convenient setting of the potential divider, would provide the opposite arm. The method is thus not limited to resistors the values of which closely approximate to an integral multiple of 100 ohms.

The variation with frequency of the 1 000-ohm, 5 000-ohm, and 10 000-ohm units of the higher decade was measured by comparison with the corresponding units of the lower decade, which on theoretical grounds must be considered invariable up to frequencies of at least 100 000 cycles per sec.

Tables 3 to 6 summarize the measurements made on the standards. The calculated time-constant values given in the tables include the frequency correction and are given in microhenrys per ohm.

The accuracy of the substitution measurements given in Tables 3 and 4 is probably higher than that of the direct comparison measurements given in Tables 5 and 6, which are much more difficult to carry out.

TABLE 3.

Measurements on Three 1 000-ohm Units, at 1 000 cycles per sec.

τ_1 = time-constant of 1 000-ohm unit in higher decade.
τ_2 = time-constant of separate 1 000-ohm unit.
τ_3 = time-constant of 1 000-ohm unit in first decade.

Quantity	Observed value	Calculated value
	μH per ohm	μH per ohm
$\tau_1 - \tau_2$	0·004	0·006
$\tau_2 - \tau_3$	0·118	0·119
$\tau_3 - \tau_1$	− 0·123	− 0·125

The discrepancies in Table 3 are probably due to the errors in calculating the inductance of a short solenoid, but the agreement between the calculated reactances of two standards of very different forms is very satisfactory. The agreement amongst the 10 000-ohm standards is excellent. It must be remembered that the experimental limitations in these measurements are of the order of $\pm 0 \cdot 001$ microhenry per ohm, which corresponds to a phase angle of 6×10^{-6} radian at a frequency of 1 000 cycles per sec.

TABLE 4.

Measurements on Three 10 000-ohm Units, at 1 000 cycles per sec.

τ_1 = time-constant of 10 000-ohm unit in higher decade.
τ_2 = time-constant of separate 10 000-ohm unit (nichrome).
τ_3 = time-constant of separate 10 000-ohm unit (eureka).

Quantity	Observed value	Calculated value
	μH per ohm	μH per ohm
$\tau_1 - \tau_2$	− 0·048	− 0·047
$\tau_2 - \tau_3$	− 0·031	− 0·031
$\tau_3 - \tau_1$	− 0·016	− 0·016

TABLE 5.

Values of $\tau_4 - \tau_3$, in Microhenrys per Ohm, for the Kelvin–Varley Potential Divider at the Ratio 1/10, at 1 000 cycles per sec., in Terms of the Calculated Standards.

Standard	100/1 000	200/2 000	300/3 000	400/4 000	500/5 000	600/6 000	700/7 000	800/8 000	900/9 000	1 000/10 000
$\tau_4 - \tau_3$	−0·003	−0·004	−0·004	−0·006	−0·006	−0·005	−0·004	−0·005	−0·004	−0·004

Mean value = $0 \cdot 004_5$: this compares with the value 0·004 obtained from other measurements.

The values given in Table 5 are the mean values of two sets taken first by varying C_1 (Fig. 5) and then by varying C_2. The mean difference between the values in each set was about 0·001 microhenry per ohm, and the mean variation from the mean of the values given in the table is less than this. The mean of the values in Table 5 is $0\cdot004_5$ microhenry per ohm, which is in striking agreement with the value—0·004 microhenry per ohm—obtained from measurements on standard high-frequency self-inductance coils.

TABLE 6.

Variation with Frequency of the Solenoidal Resistors. Values of $\Delta\tau$, the Amount by which the Time-constant at Zero Frequency exceeds that at the Given Frequency.

Resistor	Frequency	$\Delta\tau$ (microhenrys per ohm)	
		Observed	Calculated
	cycles per sec.		
1 000-ohm	1 000	0·001	0·001
	1 500	0·003	0·002
	2 000	0·005	0·004
	3 000	0·009	0·008
5 000-ohm	1 000	0·003	0·002
	1 500	0·005	0·005
	2 000	0·006	0·006
	3 000	0·011	0·012
10 000-ohm	1 000	0·002	0·002
	1 500	0·004	0·004
	2 000	0·007	0·007
	3 000	0·013	0·013

The measurements on frequency variation given in Table 6 show that, while equation (9) is only approximate, it is sufficiently accurate to permit the time-constant of any standard to be calculated with an uncertainty not greater than 0·001 μH per ohm up to at least 3 000 cycles per sec., and above that frequency experimental difficulties encountered with large resistors can lead to uncertainties greater than any from this source.

It is very doubtful whether greater accuracy in measurements of time-constant on high resistances can be obtained without extreme refinement. A stray capacitance of 0·1 $\mu\mu$F shunted across a 10 000-ohm resistor, or a stray inductance of 0·1 μH in series with a 100-ohm resistor, will produce an effective contribution to time-constant of 0·001 microhenry per ohm, and it is a matter of experience that "strays" of this order of magnitude are very common and generally difficult either to determine or to eliminate.

(6) ACKNOWLEDGMENTS.

The author is greatly indebted to Dr. L. Hartshorn for advice and many helpful suggestions. Thanks are also due to Mr. C. E. Webb, who measured the permeability of the wire and keramot used in the standards, and to Mr. F. Cheeseman, who assisted with some of the measurements and computations.

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THE GRID-CONTROLLED RECTIFIER WITH ZERO-POINT ANODE.*

By George I. Babat.

(*Paper first received* 10*th May, and in final form* 31*st August*, 1934.)

Summary.

This paper describes new grid-controlled rectifier circuits, called "zero-anode" or "neutral-point anode" circuits, in which the rectifiers have an auxiliary anode connected to the mid-point of the main power transformer. Formulæ and curves are given for the computation of the d.c. component of the rectified current, the main power factor, the displacement factor, the overload capacity, and the amplitudes of the harmonics, of these circuits. They prove that such circuits possess many valuable qualities which are lacking in the circuits usually employed.

(1) Introduction.

In 1902 there was suggested by Cooper Hewitt a method of controlling the rectified voltage of a mercury-arc rectifier by means of an alteration in the starting angle of the rectifier anodes. The grid-controlled rectifier, which originated from this suggestion, has now reached a high degree of perfection and is finding wide application in various branches of industry.

In the present paper a description is given of the new grid-controlled rectifier circuits, to which the author has given the name "zero-valve" or "zero-anode" circuits. These circuits have many advantages over the usual grid-controlled rectifier circuits.

(2) General Principles of Action of Zero-Anode Circuits.

Let us consider the principle of action of the "zero-anode circuits" for the general case of the m-phase rectifier circuit dealt with in Fig. 1. The current waveforms for this circuit are shown in Fig. 2.

The peculiarity of "zero-anode circuits" which distinguishes them from the usual "parallel circuits" of grid-controlled rectifiers is that the former have a valve connected between the mid-point of the main transformer and the common cathode of the controlled valves. This valve is called the "zero valve."

In the steel-tank grid-controlled mercury-arc rectifiers such a valve may take the form of an additional, auxiliary anode (see Fig. 1) connected to the mid-point of the main transformer, and called the "zero anode."

Suppose that the starting angle of the grid-controlled anodes (see Fig. 1) has been set in such a way that at the period of the cycle marked t_1 (see Fig. 2) current begins to pass through the anode (1). Then at the moment t_2 the potential of the transformer phase which is connected to the anode (1) reverses and becomes negative. In the usual parallel circuit the current flows through the anode (1) even after the moment t_2. In the zero-anode circuit, however, at the moment t_2 the load current will begin to pass through the zero anode, because after the instant t_2 the potential of the circuit of anode (1) will be directed against the current, and it will therefore be easier for the cathode inductance to "push" the current through the zero anode than through the controlled anode.

After the instant t_2 the load current will flow exclusively at the expense of electromagnetic energy stored in the cathode inductance. To make the load current, after the instant t_2, pass through the grid-controlled anode (1), the cathode inductance would have not only to supply the energy to feed the load but also to give a part of the energy stored by itself back to the a.c. main. After the moment t_2 the anode current I is directed against the potential of the transformer phase feeding this anode, whereas at the passing of the load current

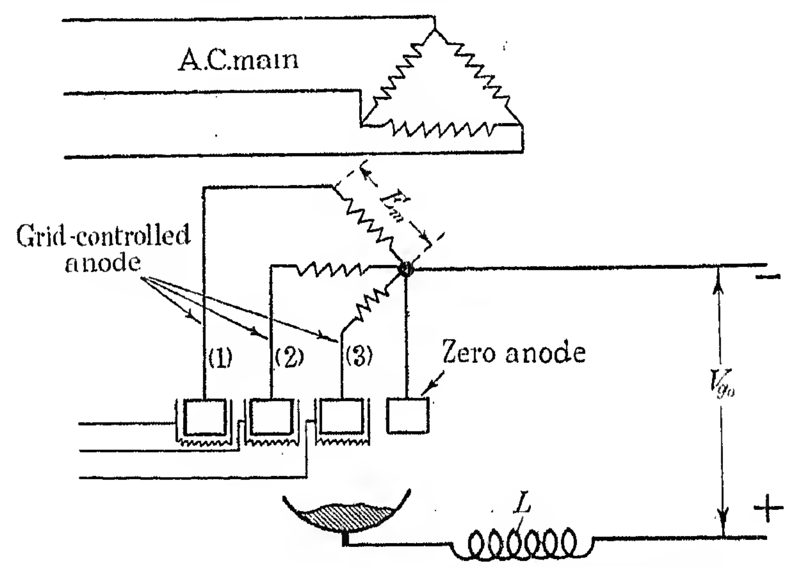

Fig. 1.—The grid-controlled rectifier with zeroanode.

through the zero anode the cathode inductance only gives energy for feeding the load.

The load current will flow through the zero anode till the instant t_3 when the next grid-controlled anode (No. 2, Fig. 1) comes into operation. Then, when the voltage of the phase connected with the anode (2) becomes equal to zero, at the moment t_4, the load current will return to the zero anode, through which it will flow till the instant when the anode (3) comes into operation.

Thus, in the zero-anode circuits, the current stops flowing through any of the grid-controlled anodes when the potential of the main transformer phase connected with the given anode reverses, irrespective of the starting angle of the grid-controlled anodes. Consequently an alteration of the starting angle of the grid-controlled anodes leads to a change in the duration of the plate-current impulse. Thus the time of current flow through each of the grid-controlled anodes will not be $2\pi/m$, as for simple parallel circuits, but $\pi - \alpha$. The continuity of load current is accomplished by ensuring that the intervals between the currents of the individual grid-

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controlled anodes are covered with the current flowing through the zero anode. Thus, during a cycle of rectified voltage, m impulses of current pass through the zero anode.

It is necessary to point out, however, that the zero anode begins to work only at the moment when the voltage of any one of the grid-controlled anodes is zero before the moment of ignition of the next grid-controlled anode would be reached. If the starting angle of the grid-controlled anode is shifted so that the potential at any one of the grid-controlled anodes is still positive at the moment of ignition of the next grid-controlled anode, then the zero anode will not function.* It would clearly be useless to employ the zero anode in the non-controlled rectifying circuits, as in the latter the rectified-voltage curve never falls below zero potential.

The minimum starting angle at which the zero anode begins to work is obviously $\pi - \frac{2\pi}{m}$. Throughout the paper this will be referred to as the "critical angle" of ignition, and will be denoted by $\alpha_{crit.}$. For the 3-phase rectifying circuits $\alpha_{crit.} = 60°$, which corresponds to

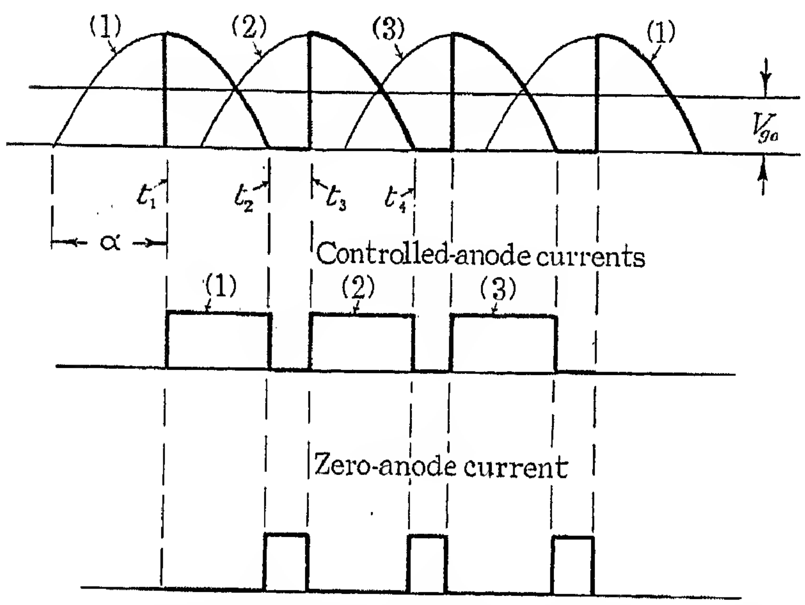


FIG. 2.—Voltage and current wave-forms of circuit shown in Fig. 1, for the case of large cathode inductance.

$V_{g_0} = 0{\cdot}87 V_{g_{max.}}$. For the 6-phase circuits $\alpha_{crit.} = 120°$, which corresponds to $V_g = 0{\cdot}5 V_{g_{max.}}$. For single-phase (full-wave and half-wave) circuits $\alpha_{crit.} = 0$. This means that in the full-wave rectifying circuit the zero valve starts to work at the very beginning of the rectified-voltage control. With multi-phase circuits, on the other hand, all the advantages of the zero valve are realized only in the cases where one has to control the rectified voltage within a comparatively wide range. We will now pass to the deduction of the principal formulæ for the zero-valve circuits.

(3) THE DIRECT-CURRENT COMPONENT OF RECTIFIED CURRENT.

To calculate the d.c. component of the rectified current, one must determine the average value of the rectified-current curve.

* For the circuit of Fig. 1 this will mean that the zero anode will work only in the case when at the moment of ignition of anode (2) the voltage of anode (1) has already become negative.

Each of the rectifier phases, as we have already found, is working in the interval of time from α to π, while the frequency of the rectified-current curve is $2\pi/m$. Therefore, to obtain V_{g_0}, we have to integrate $E_m \sin x$* from α to π and divide the value obtained by $2\pi/m$. Thus

$$V_{g_0} = \frac{E_m m}{2\pi}\int_\alpha^\pi \sin x dx = \frac{E_m m}{2\pi}(1 + \cos \alpha) \quad . \quad (1)$$

By means of a simple trigonometrical transformation we can write formula (1) in the following way:—

$$V_{g_0} = \frac{E_m m}{\pi}\cos^2\frac{\alpha}{2} \quad . \quad . \quad . \quad . \quad (2)$$

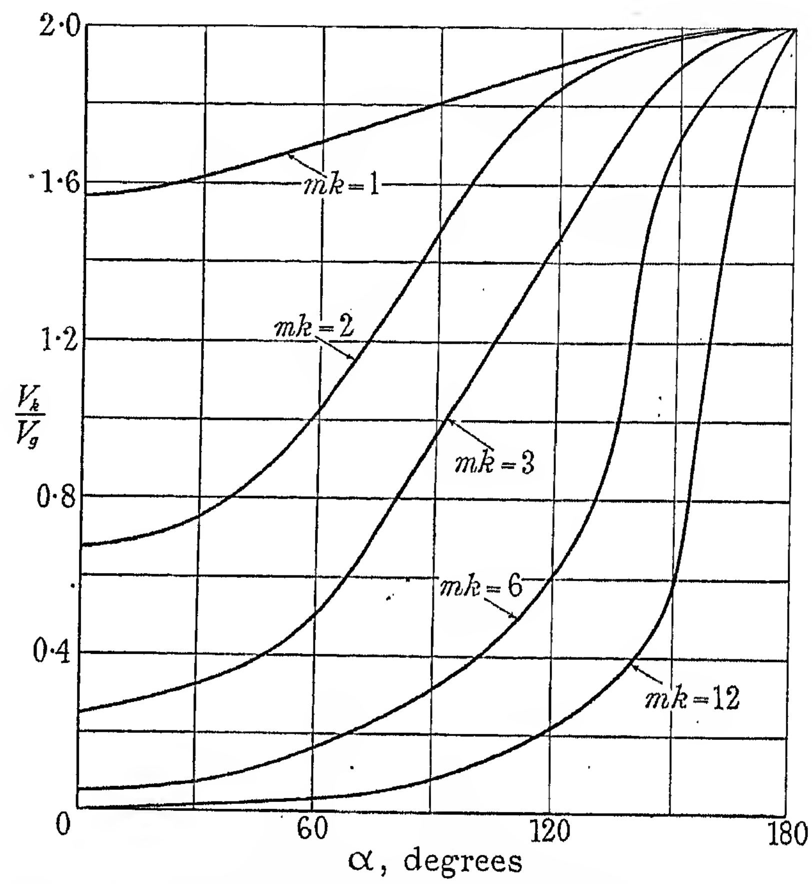


FIG. 3.—Harmonic-amplitude/starting-angle curves for full-wave circuit.

We must note, however, that this formula applies only for cases where $\alpha > \alpha_{crit.}$. If $\alpha < \alpha_{crit.}$, to determine V_{g_0} one must use the formula for simple parallel circuits. Thus

$$V_{g_0} = V_{g_{max.}} \sin\left(\alpha + \frac{\pi}{m}\right) = V_{g_{max.}} \cos\phi \quad . \quad (1a)$$

(4) THE ALTERNATING-CURRENT COMPONENTS OF THE RECTIFIED CURRENT.

To obtain the amplitudes of the harmonics contained in the rectified voltage we have to determine the sine and cosine terms, on account of the rectified-current curve being unsymmetrical. Thus, for the coefficients of the sine terms, we obtain

* E_m is the amplitude of the alternating-current voltage of the main transformer secondary. V_{g_0} is the calculated d.c. component of rectified current, ignoring the overlapping of anode currents. For a calculation of the d.c. component V_g, taking into account the voltage drop due to the anode currents overlapping (i.e. $V_g = V_{g_0} - \Delta V$), see Section (10).

$$a_k = \frac{E_m m}{\pi}\int_\alpha^\pi \sin x \sin (kmx)dx = -\frac{E_m m}{2\pi}\left\{\frac{(mk+1)\sin\left[(mk-1)\alpha\right] - (mk-1)\sin\left[(mk+1)\alpha\right]}{m^2k^2-1}\right\} \quad . \quad . \quad (3)$$

while for the coefficients of the cosine terms we have

$$C_k = \frac{E_m m}{\pi}\int_\alpha^\pi \sin x \cos (kmx)dx = \frac{E_m m}{2\pi}\left\{\frac{2-(mk+1)\cos\left[(mk-1)\alpha\right] + (mk-1)\cos\left[(mk+1)\alpha\right]}{m^2k^2-1}\right\} \quad . \quad . \quad (4)$$

Thus we can determine the amplitudes of the harmonics by means of the formula:—

$$V_k = \sqrt{(a_k^2 + c_k^2)} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In designing a rectifier set, particularly the smoothing arrangements (cathode inductance, filter, resonant shunts, etc.), it is not so important to know the absolute value of V_k as to know the ratio V_k/V_{g_0}. The expression obtained for this ratio by dividing (5) by (1) and substituting the values of a_k and b_k obtained from (3) and (4), is very cumbrous. The ratio V_k/V_{g_0} has therefore been expressed graphically (see Fig. 3).

In zero-valve rectifying circuits, as in the rectifying circuits usually employed, the value of the ratio V_k/V_{g_0} does not depend upon the number of phases in the rectifier set (this only determines the ordinal number of the principal harmonic). The curves given in Fig. 3 may therefore be used for the determination of the ratio V_k/V_{g_0} for circuits with any number of phases.*

The most important point in the calculation is the determination of the maximum value of the ratio V_k/V_{g_0}. This ratio increases with increase of the starting angle α. Let us consider the variation of V_k/V_{g_0} when $\alpha \to \pi$, i.e. when $V_g \to 0$. Writing the expression for V_k/V_{g_0} in the most general form, we have

Hence $$\lim \left[\frac{V_k}{V_{g_0}}\right]_{\alpha\to\pi} = 2$$

Thus the maximum possible value for the ratio of the amplitude of the harmonic to the d.c. component of rectified current is equal to 2, and depends neither upon the number of rectifier phases nor upon the ordinal number of the harmonic.

Fig. 4 gives comparative curves showing the variation of amplitude of the first harmonic with the d.c. component of rectified current, for parallel and zero-anode circuits.

The tendency of (*a*) the ratio of harmonic amplitudes in the zero-anode circuits and (*b*) the d.c. component, to some finite limit, and not to infinity (as in simple parallel circuits), is to be expected, because in the rectified-current curve V_g and V_k both tend to zero with increase of α.

Further, owing to the presence of the zero valve, the pulsating curve of rectified current is never negative (as all its sections below the axis of abscissæ are cut away by the zero anode); and it is known that in the pulsating current the ratios of the amplitudes of the harmonics to the d.c. component, with any wave-form of current, will never exceed the value 2, unless the current is negative.*

$$\frac{V_k}{V_{g_0}} = 2\left[\sqrt{\left(\frac{\int_\alpha^\pi \sin x \sin (kmx)dx}{\int_\alpha^\pi \sin x dx}\right)^2 + \left(\frac{\int_\alpha^\pi \sin x \cos (kmx)dx}{\int_\alpha^\pi \sin x dx}\right)^2}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

When the starting angle α approaches the value π, i.e. when $\alpha + \Delta\alpha = \pi$, then for the first term under the radical in (6) we obtain†

$$\frac{\int_{(\pi-\Delta\alpha)}^\pi \sin x \sin (kmx)dx}{\int_{(\pi-\Delta\alpha)}^\pi \sin x dx} = \frac{\sin\pi \sin (mk\pi)\Delta\alpha}{\sin\pi \,.\, \Delta\alpha} = \sin mk\pi = 0$$

For the second term under the radical we have

$$\frac{\int_{(\pi-\Delta\alpha)}^\pi \sin x \cos (kmx)dx}{\int_{(\pi-\Delta\alpha)}^\pi \sin x dx} = \frac{\sin\pi \,.\, \cos (km\pi) \,.\, \Delta\alpha}{\sin\pi \,.\, \Delta\alpha} = \cos km\pi = \pm 1$$

* With the multi-phase rectifier set, until the zero valve begins to work, the following formula is used for the calculation of the amplitudes of the harmonics:—

$$\frac{V_k}{V_{g_0}} = \frac{2\sqrt{[1 + (mk\tan\phi)^2]}}{m^2k^2 - 1}$$

† This follows from the well-known relation $\int_a^{a+\Delta a} f(x)dx = f(a)\,\Delta a.$

(5) Power Factor.

Rectifier Set.

To determine the power factor of the rectifier set with the zero valve, we use the same method as has been applied to simple non-controlled rectifiers. The power factor, generally speaking, is the ratio of watts to volt-amperes. The voltage supplied to the rectifier set is sinusoidal, but the currents are not; they may be resolved into a series of sinusoidal components, namely, the principal wave and the higher-harmonic waves. Only the fundamental component of the current carries power; the higher-harmonic components cannot carry power since there is no voltage of the corresponding frequency. If the effective value of the principal component is I_1, that of the voltage is $E_{eff.}$, and ϕ is the phase-displacement angle between them, then the power (in watts) consumed by the rectifier is $E_{eff.}I_1 \cos\phi$, while the

* See Berg: Gosudarstvenoie Energetitsheskoie Izdatelstvo (State Publishing Department), 1932, "Thermionic Valve Generators," pp. 107–121.

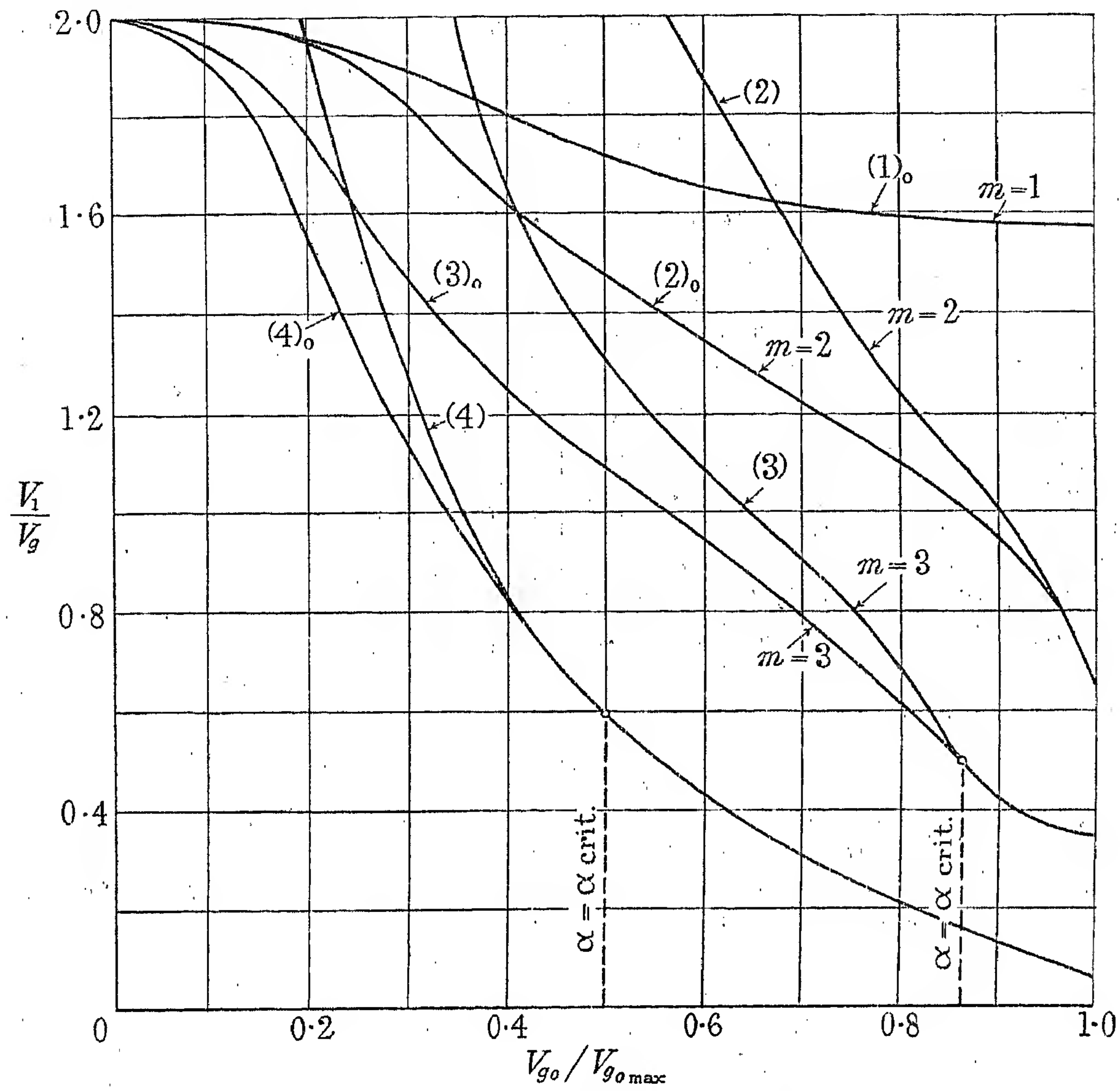


FIG. 4.—Comparative curves showing variation of amplitude of first harmonic with d.c. component of rectified current, for parallel and zero-anode circuits.

Curves (2), (3), (4): Simple parallel circuits.
Curves $(1)_0$, $(2)_0$, $(3)_0$, $(4)_0$: Zero-anode circuits.

volt-ampere product is $E_{eff.}I_{0eff.}$. Hence the power factor is given by

$$K = \frac{E_{eff.}I_1 \cos \phi}{E_{eff.}I_{0eff.}} = \frac{I_1}{I_{0eff.}} \cos \phi$$

The ratio $I_1/I_{0eff.}$ is called the distortion factor (μ), while $\cos \phi$ is the displacement factor.

Ignoring the losses in a rectifier set (which are, generally speaking, very small) it may be considered, approximately, that $P_2 = P_g = I_g V_g$. Then the main power factor, which we denote by K_0, may be determined from the relation

$$K_0 = \frac{I_g V_g}{I_{0eff.}E_{eff.}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$I_{eff.}$ has different values in each of the transformer windings; we have therefore to determine separately the power factors of the transformer primary (which we will denote by K_1) and of the transformer secondary (K_2).

Main Transformer Secondary.

Current is flowing through each of the grid-controlled anodes in the interval of time from α to π, which is

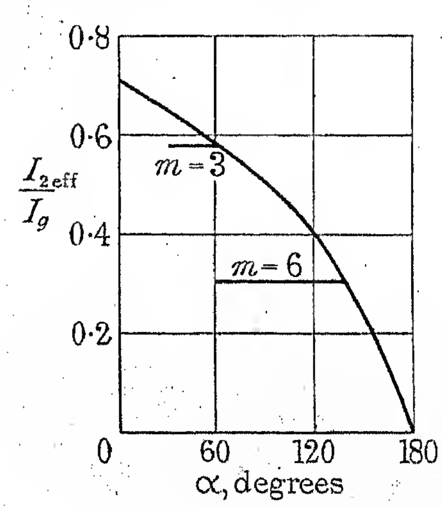


FIG. 5.—Effective value of current in main transformer, as a function of the starting angle (α).

consequently also the duration of the current flow through each of the phases of the main transformer secondary. The effective value of the current in the

transformer secondary may therefore be determined according to the formula

$$I_{2eff.} = \sqrt{\left(\frac{1}{2\pi}\int_{\alpha}^{\pi} I_g^2 d\alpha\right)} = \sqrt{\left(\frac{\pi - \alpha}{2\pi}\right)} I_g \quad . \quad . \quad (8)$$

Fig. 5 shows the variation of $I_{2eff.}/I_g$ with α.

Since K_2 is equal to the ratio of the watts to the volt-amperes,

$$K_2 = \frac{V_g I_g}{m I_{2eff.}(E_m/\sqrt{2})}$$

$$= \frac{[E_m m/(2\pi)](1 + \cos\alpha)I_g}{m\sqrt{[(\pi - \alpha)/(2\pi)]}I_g(E_m/\sqrt{2})} = \frac{1 + \cos\alpha}{\sqrt{(\pi^2 - \pi\alpha)}} \quad (9)$$

Formula (9) gives the variation of K_2 with α. It is far better, for the sake of clearness, to express K_2 as a function of $V_{g_0}/V_{g_{max.}}$. This is done in the curves of Fig. 6.

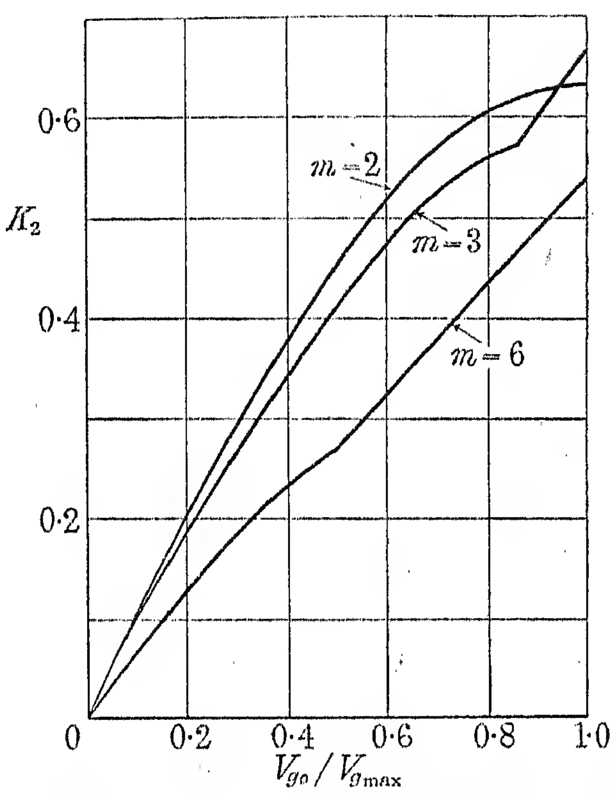


FIG. 6.—Power factor of main transformer secondary, as a function of $V_{g0}/V_{g max.}$

(6) OVERLOAD CAPACITY OF ZERO-ANODE CIRCUITS.

It is clear from formula (8) that with increase of the starting angle α (i.e. with decrease of V_g), $I_{eff.}$ decreases if I_g remains constant. This peculiarity of zero-anode circuits may be of the utmost importance in many cases.

Suppose that in our rectifier installation the main transformer allows some definite value of effective current. Then, according to formula (8), we shall be able to obtain from the set a rectified current given by

$$I_g = \sqrt{\left(\frac{2\pi}{\pi - \alpha}\right)} I_{2eff.} = \sqrt{\left(\frac{\pi - \alpha_{crit.}}{\pi - \alpha}\right)} I_{g_0} \quad . \quad (10)$$

I_{g_0} is the current that the set can supply before the zero valve begins to work, i.e. at $\alpha < \alpha_{crit.}$. It follows from formula (10) that with increase of α (i.e. with decrease of V_g) one may obtain from the set rectified currents of greater and greater magnitude.

If, for example, we take a full-wave circuit and denote by I_{g_0} the rectified current which it can supply at $\alpha = 0$ (i.e. at $V_g = V_{g_{max.}}$), and by $I_{g\alpha}$ the current that it can supply at values of α other than zero, we obtain

$$\frac{I_{g\alpha}}{I_{g_0}} = \sqrt{\left(\frac{\pi}{\pi - \alpha}\right)} \quad . \quad . \quad . \quad . \quad (11)$$

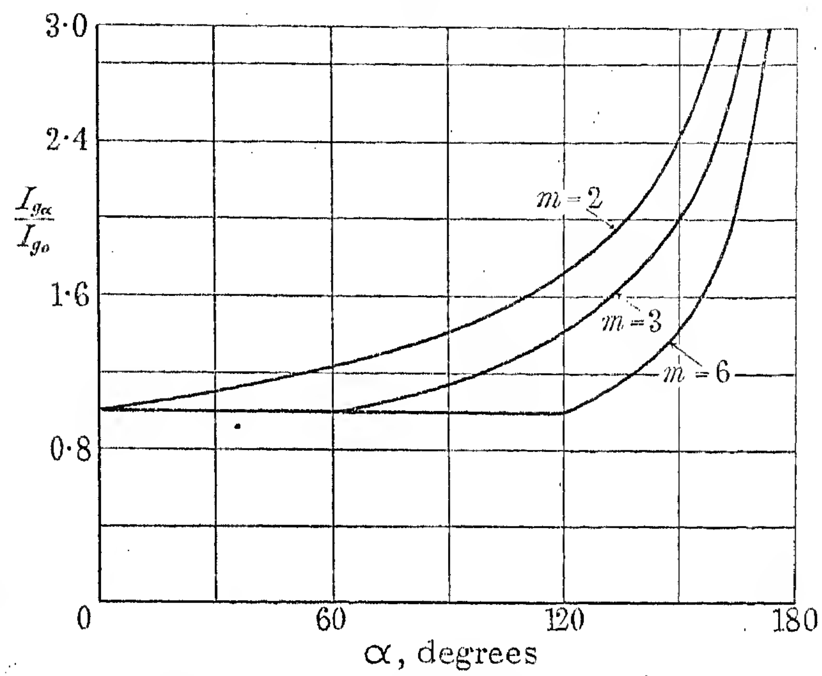


FIG. 7.—Variation of permissible overload with starting angle; for 2-, 3-, and 6-phase zero-valve circuits.

It follows from this formula that by halving V_g the current is increased 1·5 times compared with its value at $V_g = V_{g_{max.}}$; while by reducing V_g to one-quarter of its original value the rectified current may be made twice as great, the copper losses in the main transformer remaining constant.

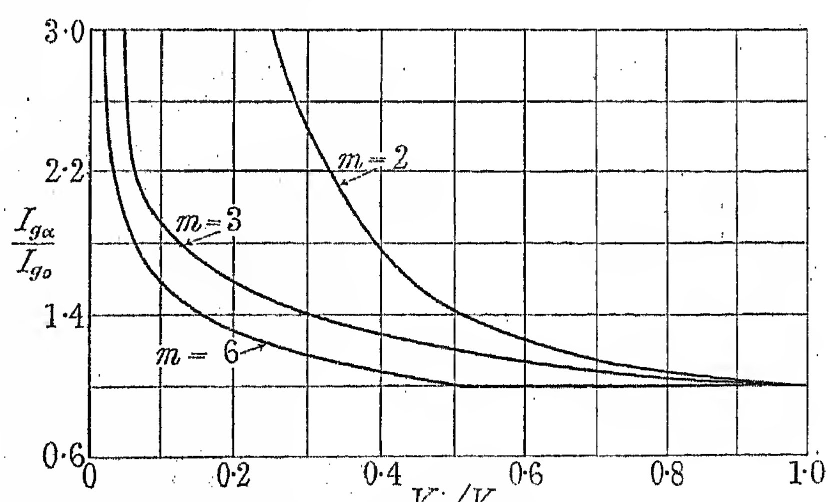


FIG. 8.—Variation of permissible overload with $V_{g0}/V_{g max.}$, for 2-, 3-, and 6-phase zero-valve circuits.

This property of zero-anode circuits may be of particular use in rectifiers designed for feeding electro-chemical installations. In such installations one has frequently to work at low voltage and high current. Figs. 7 and 8 show the relation of the ratio $I_{g\alpha}/I_{g_0}$ to the starting angle α and to the ratio $V_{g_0}/V_{g_{max.}}$ respectively. These curves give the permissible increase of the rectified current with decrease of V_g.

For the sake of completeness, let us deduce the relation between the average values of current flowing at the zero- anode and through the grid-controlled anodes.* The current at the zero anode (see the current waveform, Fig. 2) flows during the period of time from π to $\frac{2\pi}{m}+\alpha$, and m times during each period its instantaneous value is equal to that of the load current (I_g). Therefore, for the average value of the current at the zero valve, which we denote by I_{gz}, we have

$$I_{gz}=\frac{m(2\pi/m)+\alpha-\pi}{2\pi}I_g \quad . \quad . \quad . \quad (12)$$

Remembering that

$$\pi-\frac{2\pi}{m}=\alpha_{crit.}$$

we may rewrite formula (12) thus:—

$$I_{gz}=\frac{m(\alpha-\alpha)_{crit.}}{2\pi}I_g \quad . \quad . \quad . \quad . \quad (13)$$

It is quite obvious that the equation is effective only so long as $\alpha > \alpha_{crit.}$. If $\alpha < \alpha_{crit.}$, current cannot flow

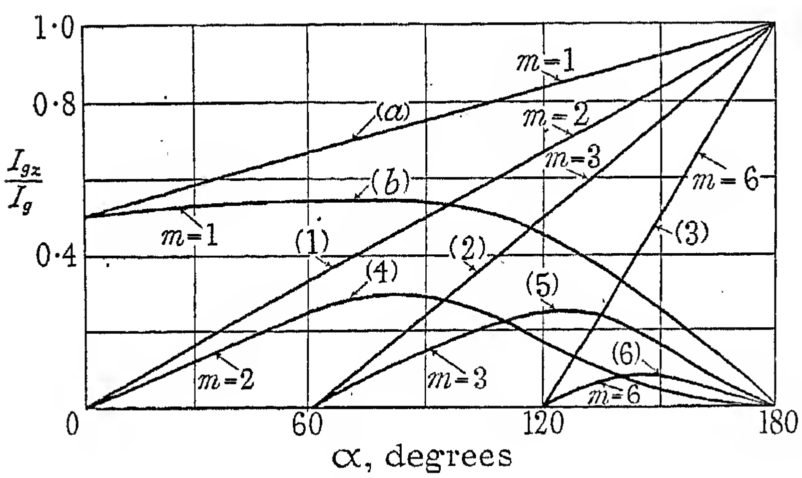


FIG. 9.—Variation of I_{gz}/I_g with α.
Curves (1), (2), (3): I_g = constant.
Curves (4), (5), (6): Load resistance = constant.
Curves (a), (b): Values for half-wave circuit.

through the zero valve. Fig. 9 shows the ratio I_{gz}/I_g as a function of α. It indicates that at $\alpha=\pi$, i.e. when $V_g=0$, $I_{gz}/I=1$. In other words, when V_g is nearly equal to zero all the rectified current flows through the zero anode.

We will denote the average value of the current through each of the grid-controlled anodes by I_{gy}; it is given by the equation

$$I_{gy}=\frac{\pi-\alpha}{2\pi}I_g \quad . \quad . \quad . \quad . \quad . \quad (14)$$

So long as $\alpha < \alpha_{crit.}$,

$$I_{gy}=\frac{2\pi}{m}I_g$$

Fig. 10 shows the variation of the ratio I_{gy}/I_g with the angle α.

Let us now consider the relation between the current through the zero valve and the angle α, assuming that the ohmic resistance of the load remains constant with change of V_g. In this case the current flowing through the load is expressed by the formula

$$I_g=I_{g_0}\cos^2\frac{\alpha}{2}$$

Then
$$I_{gz}=\frac{m(\alpha-\alpha_{crit.})}{2\pi}I_{g_0}\cos^2\frac{\alpha}{2} \quad . \quad . \quad . \quad (15)$$

For the full-wave circuit this expression may be written in the form

$$I_{gz}=I_{g_0}\frac{\alpha}{\pi}\cos^2\frac{\alpha}{2} \quad . \quad . \quad . \quad . \quad (16)$$

This formula shows that when the starting angle α is increased, I_{gz} first increases and then decreases.

In Fig. 9 the relation between the ratio I_{gz}/I_{g_0} and α is given for the full-wave and half-wave circuits, and also for the 3-phase and 6-phase circuits, the value of the load resistance being constant. It should be noted that in the case of the half-wave circuits (which will be dealt with in detail later) the value $-\pi$ has been substituted for $\alpha_{crit.}$ in equations (13) and (15). This effect is caused in half-wave circuits—even with the

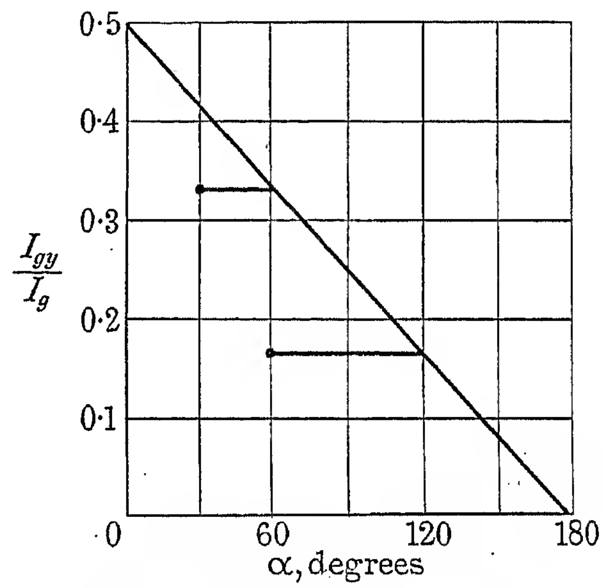


FIG. 10.—Variation of I_{gy}/I_g with starting angle (α).

starting angle of the grid-controlled anode equal to zero, i.e. with $V_b=V_{g_{max.}}$—by the current flowing through the zero valve during the half-cycle.

(7) DISTORTION FACTOR IN MAIN TRANSFORMER SECONDARY.

Since $K=(I_1/I_{0eff.})\cos\phi$, and denoting the distortion factor by $\mu=I_1/I_{0eff.}$, we find that $\mu_2=K_2/\cos\phi$, whence, taking into consideration formula (9), we have

$$\mu_2=\frac{2\cos(\alpha/2)}{\sqrt{(\pi^2-\pi\alpha)}} \quad . \quad . \quad . \quad . \quad (17)$$

(8) DISPLACEMENT FACTOR.

In the zero-anode circuits the current impulse through each of the grid-controlled anodes is of rectangular form. The phase-displacement angle ϕ, between the principal wave contained in this impulse and the main potential, is therefore equal to the angle between the impulse symmetry axis and the axis of the positive half-wave of

* A knowledge of the mean value of the current through the anodes is important in the determination of the power deliberated on the anodes.

the alternating voltage. From the wave-forms of currents and voltages for the zero-anode circuits (Fig. 2) it is clearly seen that this angle is equal to 0·5 of the starting angle. In other words, the following expression holds for the zero-anode circuits:—

$$\cos\phi = \cos\frac{\alpha}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Remembering, that according to (2),

$$V_{g_0} = \frac{E_m m}{\pi}\cos^2\frac{\alpha}{2}$$

we have

$$\cos\phi = \cos\frac{\alpha}{2} = \sqrt{\left(\frac{\pi V_{g_0}}{mE_m}\right)} \quad . \quad . \quad . \quad . \quad (19)$$

For the full-wave circuits, where $V_{g_{max.}} = 2E_m/\pi$, equation (19) may be written in the form

$$\cos\phi = \sqrt{\left(\frac{V_{g_0}}{V_{g_{max.}}}\right)} \quad . \quad . \quad . \quad . \quad (20)$$

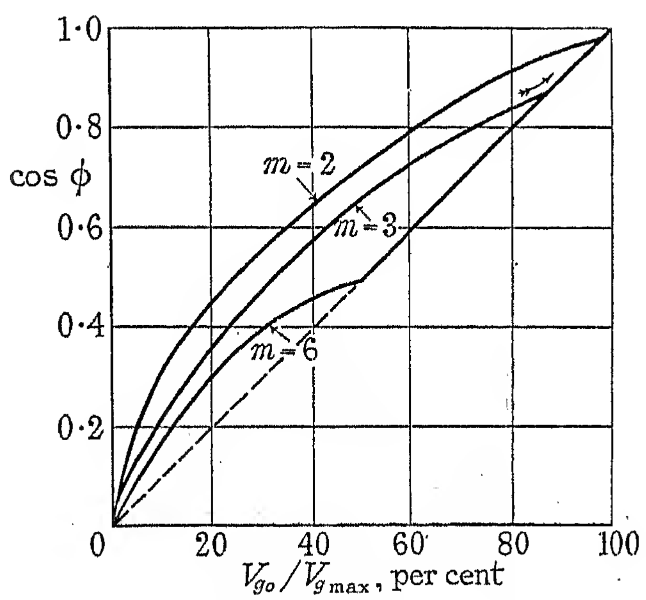


FIG. 11.—Variation of $\cos\phi$ with $V_{g_0}/V_{g_{max.}}$

The curves given in Fig. 11 show how $\cos\phi$ varies with the ratio $V_{g_0}/V_{g_{max.}}$. These curves show that in the zero-anode circuits the displacement factor decreases with V_g more slowly than in the usual grid-controlled rectifier circuits. The relation between $\cos\phi$ and $V_{g_0}/V_{g_{max.}}$ is linear for the usual circuits, and parabolic for zero-anode circuits.

(9) Power Factor of Main Transformer Primary.

The power factor K_1 of the main transformer primary is equal to the ratio of the primary watts to the primary volt-amperes. The latter depend upon the mode of connection of the transformer windings. We shall consider only a full-wave circuit.

In such a circuit the duration of the current flowing through the transformer primary is equal to $2(\pi - \alpha)$. Therefore the effective value of the current in the primary (with the transformer ratio equal to unity) will be

$$I_{1eff.} = \sqrt{\left(\frac{\pi - \alpha}{\pi}\right)} I_g \quad . \quad . \quad . \quad . \quad (21)$$

Hence for the power factor of the primary we obtain

$$K_1 = \frac{(1 + \cos\alpha)\sqrt{2}}{\sqrt{(\pi^2 - \pi\alpha)}} \quad . \quad . \quad . \quad . \quad (22)$$

Since $\cos\phi = \cos(\alpha/2)$, for the distortion factor in the transformer primary we obtain the expression

$$\mu_1 = \frac{K_1}{\cos\phi} = \frac{4\cos(\alpha/2)}{\sqrt{[2(\pi^2 - \pi\alpha)]}} \quad . \quad . \quad . \quad (23)$$

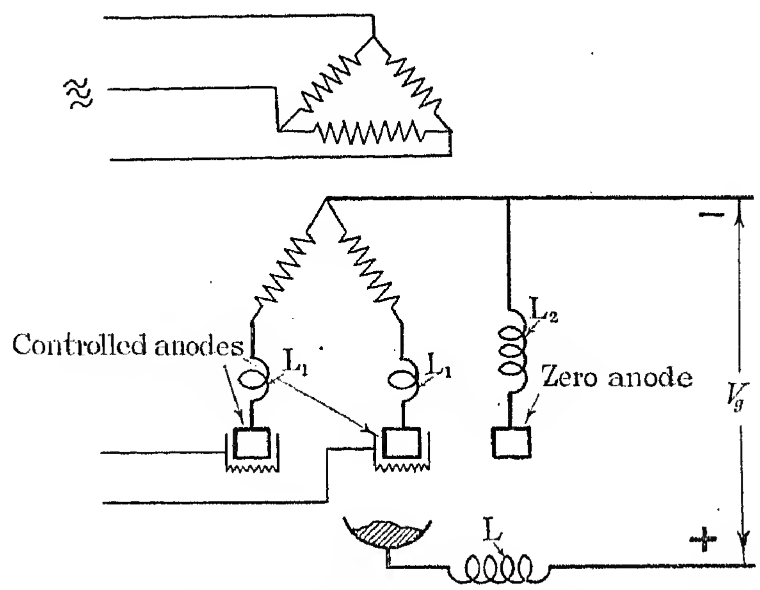


FIG. 12.—Grid-controlled rectifier with zero anode, having inductance in transformer secondary.

(10) Effect of Reactance of Transformer Secondaries in Zero-Anode Circuits.

All the above formulæ for zero-anode circuits* have been derived on the assumption that there are no impedances in the transformer secondaries. In reality, however, power transformers used in rectifier installations always have some amount of leakage, which is

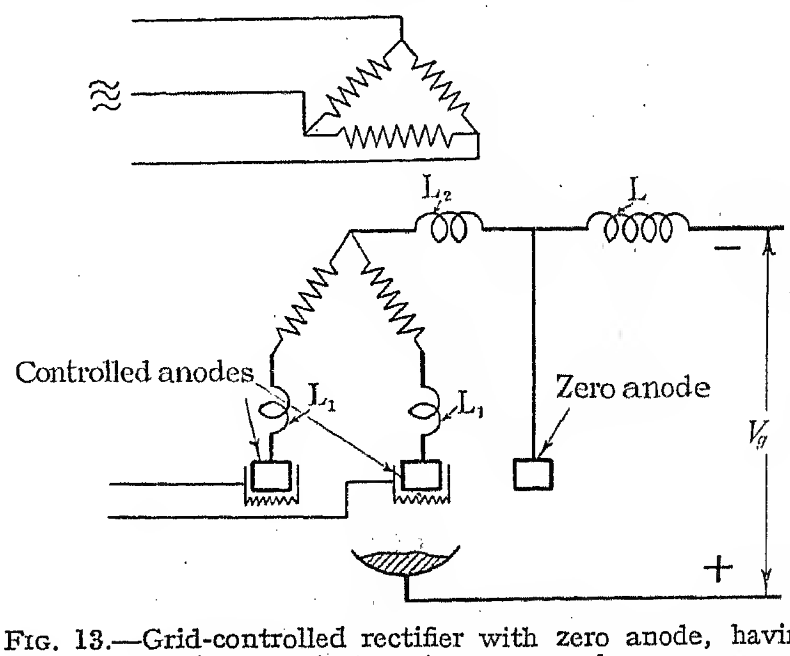


FIG. 13.—Grid-controlled rectifier with zero anode, having inductance in transformer secondary.

equivalent to the presence of reactance in the transformer secondaries. The latter also have ohmic resis-

* The circuits shown in Fig. 12 differ from those of Fig. 13 by reason of the fact that in the case of Fig. 12 only zero-anode current passes through the inductance L_1, while in the case of Fig. 13 the full load current passes through the inductance L_1. The two circuits are entirely equivalent as regards angle of overlap.

tance, but its effect is negligible. We shall therefore consider only the effect on rectifier operation of the reactance of the transformer secondaries.

Let us suppose that each transformer secondary grid-controlled anode has inductance L_1, and that the corresponding inductance for the zero anode (it consists only of the self-inductance of the connecting wires) is L_2.

Inductance effects in rectifier transformer secondaries, just as in simple parallel circuits, are expressed in the overlap of the anode currents. It is necessary to distinguish two angles of overlap in zero-anode circuits. The first corresponds to the period when the load current is being transferred from the controlled valve to the zero-anode valve: call this angle of overlap β_1. The second corresponds to the transition period of the value represented by β_1. Taking into consideration well-known statements of alternating-current theory, we may write down the following equation:—*

$$-I_g = \frac{E_m}{\omega(L_1 + L_2)} \sin\left(\beta_1 + \frac{\pi}{2}\right) - \frac{E_m}{\omega(L_1 + L_2)} \quad (24)$$

where $\omega = 2\pi \times$ frequency.

On solving this equation we obtain

$$\cos \beta_1 = 1 - \frac{I_g \omega(L_1 + L_2)}{E_m} \quad . \quad . \quad . \quad (25)$$

Let us now determine the value of the angle β_2. While the current is being transferred from the zero-anode valve to the controlled valve the corresponding phase of the power transformer can be regarded as short-

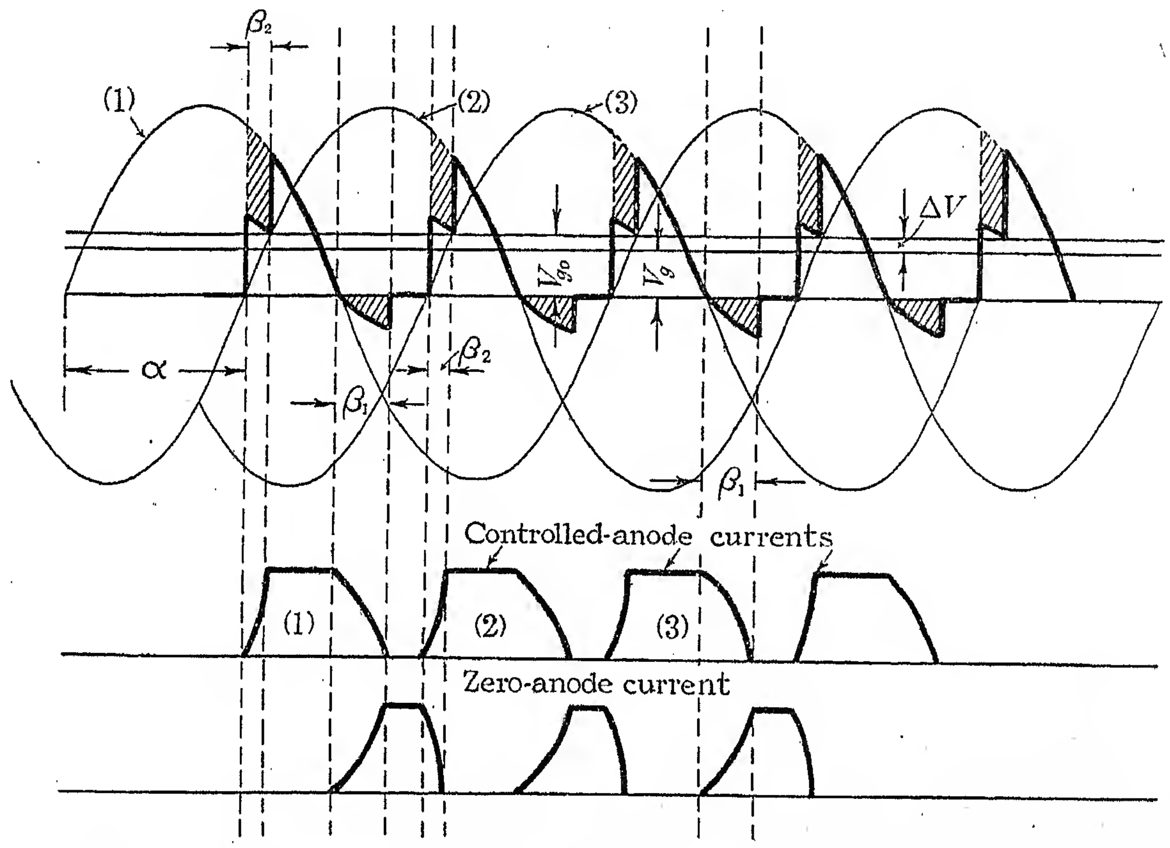


FIG. 14.—Voltage and current waves of zero-anode circuit with inductance in transformer secondary.

reversed current from the zero-anode valve to the controlled valve included in the next phase: call this angle of overlap β_2.

The current and voltage curves in Fig. 14 indicate that the period of transit of the current from the controlled valve to the zero-anode valve always begins at the instant when the voltage is passing through zero; this effect is independent of the number of rectifier phases and of the starting angle of the controlled valve. The value of β_1 will consequently depend only on the rectified alternating voltage and on the load current, for given values of L_1 and L_2. β_1 does not vary with the rectified voltage. From the above considerations the value of β can be derived.

When the controlled anode and the zero anode pass current simultaneously, the corresponding phase of the transformer can be regarded as short-circuited. Consequently it is necessary to determine the part of the cycle, measured in electrical degrees, during which the short-circuit current reaches the value I_g. This is the circuited; but the instant when the short-circuit begins is not fixed once and for all, as it is for the angle β_1; it is determined by the starting angle (α) of the controlled valve. We therefore have the following equation for the determination of β_2:—

$$-I_g = \frac{E_m}{\omega(L_1 + L_2)} \sin\left(\alpha + \beta_2 + \frac{\pi}{2}\right) - \frac{E_m \cos \alpha}{\omega(L_1 + L_2)} \quad (26)$$

Hence we see that

$$\cos(\alpha + \beta_2) = \cos \alpha - \frac{I_g \omega(L_1 + L_2)}{E_m} \quad . \quad (27)$$

Formula (4) indicates that the angle β_2 depends not only on I_g, E_m, L_1, L_2, and ω, but also on α, the starting angle of the controlled valve.

Knowing the angles of overlap β_1 and β_2, we can compute the voltage drop ΔV, of the rectified supply, due to the overlapping of the anode currents. The value of ΔV is given by the arithmetic mean of the

* K. A. KRUG: *Osnovy Electrotechniky*, 1932, vol. 11, p. 612.

shaded areas in Fig. 14, and it can also be computed from the formula

$$\Delta V = \frac{1}{2\pi/m}\left(\int_0^{\beta_1}\frac{E_m}{2}\sin x dx + \int_{\alpha}^{\alpha+\beta_2}\frac{E_m}{2}\sin x dx\right)$$
$$= \frac{E_m m}{4\pi}\left\{\Big[\cos x\Big]_{\beta_1}^{0} + \Big[\cos x\Big]_{\alpha+\beta_2}\right\}$$

Substituting values for $\cos\beta_1$ and $\cos(\alpha+\beta_2)$ from (25) and (27), we obtain

$$\Delta V = \frac{E_m m}{4\pi}\left[\frac{I_g\omega(L_1+L_2)}{E_m} + \frac{I_g\omega(L_1+L_2)}{E_m}\right]$$
$$= \frac{mI_g}{2\pi}\omega(L_1+L_2) \quad . \quad (28)$$

Where L_2 is negligibly small compared with L_1, as is the case in most rectifier installations, the expression for ΔV reduces to

$$\Delta V = \frac{m}{2\pi}I_g\omega L_1 \quad . \quad . \quad . \quad . \quad (29)$$

We therefore have the very interesting conclusion that the voltage drop in the controlled valve with zero anode has the same value as in the simple parallel-circuit grid-controlled rectifier, provided, of course, that the inductances in the transformer secondaries associated with the controlled valves are equal. Moreover, for a given value of V_g, the voltage drop is independent of the starting angle α; the d.c. voltage-regulation curves of the grid-controlled rectifier with zero anode at different starting angles α will therefore be parallel straight lines, as shown in Fig. 15.

The voltage delivered by the rectifier is given by the general formula:—

$$V_g = V_{g_0} - \Delta V = \frac{E_m m}{2\pi}(1+\cos\alpha) - \frac{m}{2\pi}I_g\omega L_1$$
$$= \frac{E_m m}{2\pi}\left(1+\cos\alpha - \frac{I_g\omega L_1}{E_m}\right) \quad . \quad (30)$$

Since the maximum rectifier short-circuit current, i.e. the short-circuit current which may flow at the earliest possible starting angle α, is given by

$$I_k = \frac{mE_m}{\omega L_1}$$

we may rewrite equation (30) in the form

$$V_g = \frac{E_m m}{2\pi}\left(2\cos^2\frac{\alpha}{2} - m\frac{I_g}{I_k}\right) \quad . \quad . \quad . \quad (31)$$

Formulæ (29), (30), and (31), are at first sight rather unexpected. In zero-anode circuits, anode currents are overlapped twice as often as in simple parallel circuits; for instance, in the 3-phase parallel circuit we have only three angles of overlap during one alternating-current cycle, whereas in 3-phase zero-anode circuits there are six angles of overlap. One would therefore expect the voltage drop in zero-anode circuits to be higher than that in simple parallel circuits. On the contrary, in zero-anode circuits the duration of the anode-current overlap is shorter than in simple parallel circuits. This is accounted for by the fact that the delay in transferring current from anode valve to controlled valve, or vice versa, is caused by the inductance present in only one of the anode transformer phases. In simple parallel circuits, however, where the current is transferred from one controlled valve to another, the angle of overlap is defined by the total inductance present in the circuits of two overlapped phases. In zero-anode circuits the doubling of the number of the angles of overlap is therefore completely compensated by the decrease in the value of each. The voltage drop thus has the same value as in simple parallel circuits.

In polyphase rectifier circuits, with increase of the load I_g the angles β_1 and β_2 may increase to such a degree that the number of valves operating ("firing") simultaneously is more than two. This type of rectifier operation is readily investigated by means of the above-mentioned analyses of the variable firing action of one

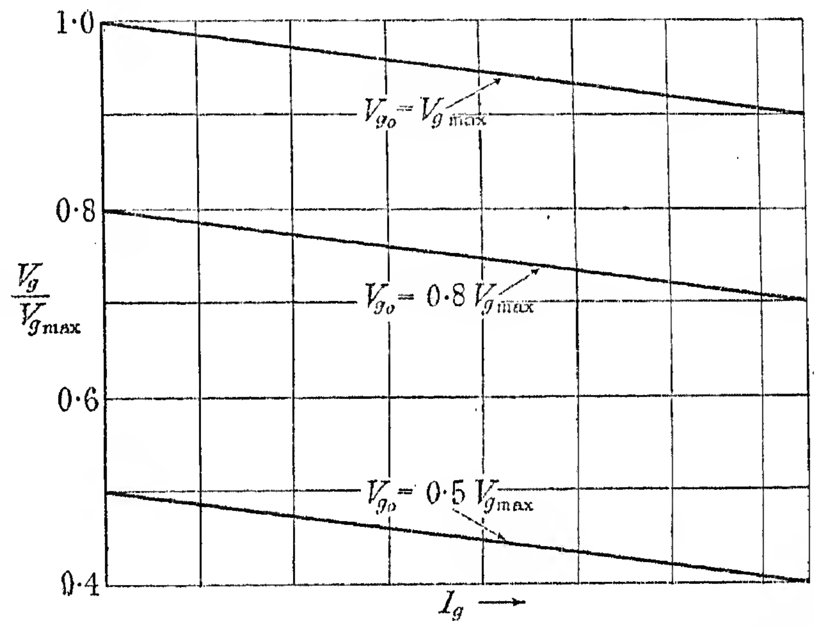


Fig. 15.—Voltage-regulation curves for zero-anode rectifier circuit.

or two anodes, and also by the well-known principles of the usual or non-controlled rectifier circuits.

The anode-current overlap affects the power factor of the rectifier installation, as well as the phase-displacement angle and distortion factor. The presence of the angle of overlap ensures that the wave-forms of the anode currents are not rectangular (as we have supposed them to be when deriving the formulæ involving K_1, K_2, and $\cos\phi$), and consequently formula (8) will not give the effective anode-current values. To obtain these values it is necessary to calculate the r.m.s. current curves (see Fig. 14). As a result of this calculation we should find that in order to obtain the effective anode-current value it is necessary to multiply the value corresponding to formula (8) by some correction factor less than unity. This factor will be a function of two independent variables—the load current (I_g) and the controlled-valve starting angle (α)—so that its calculation and use will be rather difficult.

Actual calculations show that the effective anode-current values under normal conditions are less than the values deduced from formula (8) by only 4–8 per

cent. It is therefore quite possible to use formula (8) in all technical calculations.

As regards the displacement factor ($\cos\phi$) of rectifier installations operating on a.c. mains, owing to the presence of the angle of overlap this factor will be somewhat higher than would have been expected from formula (19). The correction to be applied is, however, quite negligible.

(11) Efficiency of the Grid-Controlled Rectifier.

Rectifier-installation efficiency depends on the losses in the power transformer and the valves. Exact efficiency values can be determined only from the results obtained in experimental investigations and calculations made for special installations. In this section some common qualitative considerations will be given with regard to the relative efficiencies of zero-anode and ordinary controlled rectifier installations.

The efficiency of a rectifier may be expressed by the formula

$$\eta = P_g/P_L \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

where P_g is the power output and P_L is the power input.

Now P_L is equal to P_g plus the sum of the losses in the power transformer and valves. Power-transformer copper losses are proportional to the square of the effective current value, while the iron losses at a given alternating voltage value are constant, irrespective of the values of V_g and of the rectifier load. Let us denote the copper losses by $aI^2_{eff.}$ and the iron losses by c. The valve losses are proportional to the rectified current, and may therefore be represented by bI_g.

Then
$$P_L = P_g + aI^2_{eff.} + bI_g + c$$

Thus the efficiency is given by

$$\eta = \frac{P_g}{P_g + aI^2_{eff.} + bI_g + c} \quad . \quad . \quad . \quad . \quad (33)$$

In the light of this formula, let us analyse the nature of the changes of the rectifier efficiency with decrease of the rectifier voltage V_g. Let us suppose that the rectifier current I_g remains constant as V_g decreases. For simple parallel circuits we shall have

$$\eta = \frac{P_g}{P_g + c_0}. \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where c_0 denotes the total losses.

As V_g decreases, P_g will decrease in proportion; the losses, however, will remain constant as $I_{eff.}$ does not change. Consequently the numerator in formula (33) will decrease more than the denominator, and the efficiency will drop with decrease of V_g. The efficiencies obtained in zero-anode circuits are relatively much better; in such circuits $I_{eff.}$ decreases with decrease of V_g, as is seen from formula (8), and therefore only the losses in the valves and the power-transformer iron are constant and independent of V_g. The transformer copper losses, however, decrease with decrease of V_g; that is, c_0 in equation (34) decreases also with V_g. In zero-anode circuits, therefore, as the value of V_g falls the efficiency decreases more slowly than in the case of simple parallel circuits.

DISCUSSION ON

"THE APPLICATION OF A GAS-COOLED ARC TO CURRENT CONVERSION, WITH SPECIAL REFERENCE TO THE MARX-TYPE RECTIFIER."*

NORTH MIDLAND CENTRE, AT LEEDS, 6TH NOVEMBER, 1934.

Mr. W. E. French: I think it may prove useful to state as briefly as possible the working principle of the Marx rectifier. The fundamental rectifying event is, of course, the arc. The arc itself is a flow of current in a dielectric, in this case air. In order that the dielectric may become conducting, it must be ionized.

Ionization can take place in several ways. The temperature of a cathode can be raised to so high a temperature that dissociation of its material sets in, resulting in electron emission from the hot cathode. If an electrostatic field exists between the anode and the cathode, owing to an electric potential between these electrodes, the free negative electrons will travel to the positive anode. If a large potential is applied, the velocity of the electrons is increased and sufficient energy is imparted to enable them, on impact with intervening neutral gas molecules, to split negative electrons from the latter, leaving positive ions. These return in small part to the cathode, but mostly remain in the inter-electrode space. Thus the gap is ionized, and the presence of electrons and ions produces the conditions which make the gap conducting. If now the source supplying electrical energy to the gap is large and the appropriate potential exists at the electrodes, a power arc will appear between them, and this can be maintained provided the conditions conducive to gaseous conduction, as just described, are maintained. This method of ensuring gaseous conduction in a gap is used in the thermionic valve and in the mercury-arc rectifier.

Another method of producing gaseous conduction is by employing electrostatic fields of great intensity between the electrodes, by applying great potentials between them, which detach electrons from the cathode.

Under the influence of the large voltage gradients the electrons assume great velocities, again increasing the inter-electrode space charge by impact with the neutral molecules and thus leading to breakdown of the intervening dielectric, say air, and so to gaseous conduction. This phenomenon is well known to electrical engineers in the shape of corona effects and the flash-over arc.

We are quite familiar with the process of rectification in the mercury arc where practical current conduction can only occur during that part of the cycle when the anode is electropositive with respect to the cathode, owing to the electron flow to the anode—which becomes inert in the negative half wave—selecting the positive half-waves to form a unidirectional voltage and current. The working principle of rectification in the Marx rectifier is not at once apparent from the paper. We know that the arc between the contacts of a circuit breaker can only be removed with considerable difficulty after many cycles have elapsed. The rectifying arc in the Marx rectifier is closely akin to the arc in a breaker, with the added disadvantage that the contacts of the latter are stationary.

One can obtain a better appreciation of the operation of the Marx rectifier by studying the principle of the compressed-air circuit-breaker. In this device, synchronously with the lift of the moving contact from the conical seating of the contact nozzle, compressed air is forced into the ensuing gap, driving the opening-arc outwards. The object of using compressed air is, first of all, to accelerate the recovery of the dielectric properties of the gap by accelerating the removal of the ionized gas; secondly, by the removal of the heat generated, and of possible hot gas paths, to prevent re-ionization of the gap; and thirdly, by the introduction of compressed gas, or air, to improve the electric strength of the air between the switch contacts, since it is known that the disruptive strength of gaseous dielectrics increases with their density. Oscillograms of compressed-air breakers show that rupture occurs smoothly, and certainly within less than half a cycle.

This, then, also represents one of the phases in the cycle of operations of the Marx rectifier, as I conceive it. Air under a definite pressure is forced into the arc chamber, to effect a rapid dielectric recovery of the gap. Further, the air density, together with suitable distances between the electrodes, ensure that the electric strength of the gap is maintained on voltage recovery during the idle half-cycle of the gap. At the proper instant, i.e. shortly after the zero passage of the positive half-wave of the voltage, determined by the setting of the revolving spark-gap as shown in Figs. 5, 6, or 20, a high-frequency high-voltage wave train is introduced between the gap electrodes. The resulting pilot spark ionizes the gap and so paves the way for the rectifying arc; scavenging with compressed air, and dielectric recovery, combined with zero passage of voltage and current waves, extinguish the rectifying arc; and the rectifying operation with reference to a particular voltage wave is then complete, to recommence after the idle half-cycle. Thus the device is able to select successively the half-waves required for the production of a unidirectional voltage and current.

I should now like to add a few comments on the Marx rectifier itself. Most of the electrical ignition devices appear to be of a somewhat complex nature: there are

* Paper by Dr. W. G. THOMPSON (see vol. 75, p. 608).

the Tesla groups, capacitances, inductances, and valve rectifiers. From this point of view mechanical ignition appears to offer the very considerable advantage of simplicity, in spite of the necessary phase advancer, and it would therefore, I imagine, have a greater reliability. Further, in the case of the electrical ignition system, a voltage considerably in excess of the normal value must be employed to break down the gap rapidly. It therefore becomes necessary to insulate the apparatus against the ignition voltage rather than against the normal voltage of the rectifier itself. A recent article by R. Strigel* describes a split spark-gap (Fig. B) in which two almost hemispherical electrodes converge against each other, forming a rather small gap, and are mounted opposite to the main electrode, forming with them the main gap. If the terminals of the split electrodes are put across the terminals of the reactance L, when a very steep-fronted impulse wave is sent into that coil a big potential difference will occur across the split electrodes, which will force a spark across the subsidiary gap. This pilot spark will then ionize the main gap and so initiate the main discharge over the main gap. Although there is a slight ignition delay of some microseconds, Strigel claims that the apparatus

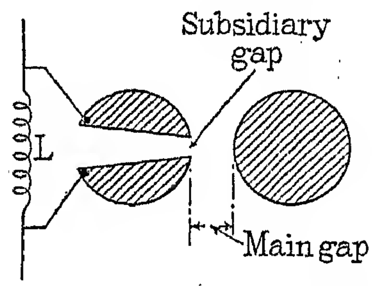


FIG. B.—Split-electrode spark-gap.

responds to comparatively low voltages across the main gap. It occurs to me that Strigel's apparatus might be usefully embodied in the Marx rectifier; owing to the small length of the subsidiary gap, the ignition voltages could be held at considerably lower values.

The efficiency of the rectifying arc in the Marx rectifier is very remarkable indeed, but I should like to know something about the overall working efficiency of the whole apparatus. It would also be interesting to know the dimensions and overall space requirements in relation to the output of the rectifier.

With reference to the tests carried out at the Zschornewitz power station, I confess some difficulties in following the author's statement. He mentions that a voltage of 90 kV (max.) was applied to the gap, which he describes on the same page as a gap of 10 mm at an air pressure of 2·5 atmospheres; and he further states that the rectifier was subjected to a heat run at 500 volts and 300 amperes. Now, as far as I can see, in the first place the gap would be much too small to negotiate the voltage, and in the second case it would be much too large.

Finally, I come to the part of the paper which interests me most, namely Section (10), dealing with inversion. In my opinion, the future of all rectifying apparatus is closely bound up with this problem. Regarding inversion as energy return, there are two ways in which this can be accomplished. First, by current reversal; and secondly, by voltage reversal. Not until the second method was fully realized did the grid-controlled mercury-arc invertor come into being. The first method may be regarded as a perfectly natural event. The energy return between a.c. and d.c. systems, or vice versa, becomes merely a matter of relative magnitude of the voltages. So far, the mercury jet-wave rectifier or invertor has been typical of this method. The second method, of which the mercury-arc grid-controlled rectifier or invertor is typical, is forced and artificial, since the energy return depends entirely on the action of the grid. Remembering then that there is only a one-way passage for the current in this device, and that the anodes are held at a positive potential by the d.c. current of the invertor, the grid must introduce an ignition delay of about 180° in the invertor as compared with that of the straight rectifier, in order to produce the relative and required voltage reversal. If I am correct it appears that the Marx rectifier comes within my definition of a natural invertor, i.e. one which provides inversion by current reversal. The advantages of the natural invertor are great, and I have serious reasons to believe that all is not well with the grid-controlled invertor, in spite of the profuse Continental literature on this subject.

Mr. J. E. Calverley: On reading of the Marx rectifier and regarding it only as a spark-gap which is broken down by a high-voltage discharge, I was struck by its wonderful simplicity; but after seeing the author's slides and hearing of the need for water-cooling and the use of the magnetic field I am disappointed to find that it is developing away from simplicity and tending to become complicated.

I should like a little more information about the possible use of this type of rectifier for operation at high frequency, and should be obliged if the author would tell us whether the residual ionization can be cleared away more rapidly than in a mercury-arc rectifier. The maximum frequency which can be used on a mercury-arc rectifier is dependent on the time taken to de-ionize the arc path, and I imagine the same limits will apply to rectifiers of the Marx type.

Mr. French has suggested that all is not well with the inverted mercury-arc rectifier. I can assure him, however, that it is coming along normally and is now being installed in railway substations where regeneration is required, following experimental work and trials which have taken place abroad. It will be some considerable time before inversion of the arc becomes general, owing to problems on the a.c. side relating to power factor and wave-form, but all these difficulties apply equally to rectifiers of the Marx type. In this respect the Marx rectifier belongs to the same group as the mercury-arc rectifier, as it is a rectifier and not a convertor. Inversion does not take place naturally, because the arc when struck is a d.c. arc and has no inherent tendency to commutate itself or become extinguished. The back-e.m.f. of the transformer must be used to extinguish the arc, and therefore the transformer must be energized from a separate a.c. system of the correct frequency and the ignition maintained in synchronism with it. The wave-form of current will be

* *Archiv für Elektrotechnik*, 1934, vol. 28, p. 586.

rectangular, as it is in the mercury-arc rectifier, and it would be of interest if the author would enlarge on this side of the matter.

The mechanical form of ignition described in the paper strikes me as having very interesting possibilities. It would appear that a rotating block of insulating material would possibly be quite as satisfactory as a block of metal, and might result in more rapid ignition. In the case of very high voltage when the gap is highly stressed, corona would be formed, followed by immediate spark-over and ignition of the arc.

There is a further point about the Marx rectifier which worries me a little, and that is the possibility of the high-frequency ignition voltage finding its way through to the d.c. side or to the a.c. side. Apart from other dangers, it would tend to cause surges and the possibility of insulation troubles.

Mr. R. D. Ball: The basic idea of the Marx rectifier, which has not yet been touched on in this discussion, is that one of the electrodes is a point while the other is a plate. Therefore, there is a definite potential gradient dependent on the radius of curvature of the sharp point, and as a rule when the point is positive the gap breaks down at one-third of the voltage required to break down the gap in the reverse direction. The real theory underlying the Marx rectifier is not explained in the paper.

Is rectification in the Marx-type rectifier as complete as in the mercury-arc type? Is inverted working of the former as simple as the grid-control system used with the mercury arc? I think that inversion will have a very great future, because it seems the only way of speed-regulating large a.c. slip-ring induction motors, using two cascade rectifiers, as described in British Patent No. 397768. Perhaps the author would tell us whether the same sort of thing can be done with the Marx rectifier.

Prof. E. L. E. Wheatcroft: I have had some experience of air-blast rectifiers working with point and plate. With this electrode system there is a natural rectifying action, in that the striking voltage is different in the two directions. The ratio of these two voltages is dependent upon the spacing and can be made as high as $2\frac{1}{2}:1$. With a pure resistance load $2\frac{1}{2}:1$ is sufficient, but when the load is smoothed with a condenser a value of more nearly $5:1$ is needed, on account of the back voltage maintained on the condenser. Similar considerations apply to polyphase operation.

We have worked also with two points, and in this case we found that the air blast gave a natural rectifying action, giving a striking-voltage ratio of about $1\frac{1}{2}:1$. Marx obtains his rectifying ratio by an external ignition system, and it seems to me that this is a great advance if a suitable system can be devised.

The ratio of static striking voltages is not, however, the only problem in rectifier operation, although it is the first essential. Given the necessary static difference a rectifier will operate on light loads, but it will eventually break down as the load is increased. To improve this dynamic condition is the function of the air blast. For example, a point-plate rectifier will carry without air blast a current of about 1 mA per mm^2 of point section, and the effect of the blast is to raise the maximum current to about 40 mA.

Given the external ignition arrangements, the rectifier problem is only that of dynamic breakdown. The Marx rectifier uses the blast to scavenge the air and raise the breakdown point: it also uses magnetic rotation of the arc. It is not absolutely essential to have both: what is their relative importance?

I should like the author to tell us what method of ignition is likely to be adopted in the Marx rectifier. The Tesla coil is, I take it, of no practical use, and there must be great objections to mechanical systems. If a pilot spark is used, how much power must it provide? One of the author's slides seemed to show ignition coils larger than the rectifier itself.

[The author's reply to this discussion will be found on page 412.]

North-Eastern Centre, at Newcastle, 12th November, 1934.

Mr. J. A. Harle: The author describes an interesting Continental development which, in the event of high-voltage d.c. transmission materializing, may meet the need for a very cheap device for both converting and inverting. The need for such a device in this country will become acute if it is ever necessary for the grid transmission lines to be put underground. There are certain aspects of the device on which I should like further information. While I can visualize the device working very well under normal load conditions with air flow, cooling, etc., properly adjusted for this condition, the question of short-circuit conditions must be carefully considered. Naturally this will depend on the reactance of the transformer supplying the rectifying unit, which can vary from 5 to even 20 per cent depending upon the percentage regulation that can be tolerated. This means that short-circuit currents from 20 to 5 times normal full-load current must be catered for, and the question arises whether the rectifier can deal with these conditions rapidly enough without damaging the electrodes by forming arc craters. For example, is the air supply adjusted to suit possible normal conditions, or is it adjusted to suit possible short-circuit conditions?

Another point is that the action of the arc discharge on the air will be to cause combinations of oxygen and nitrogen, forming gases such as nitric oxide, which may require to be dealt with. Accumulations of such gases are very destructive to metal fittings, and even small traces in the air result in rapid corrosion. For example, under suitable conditions up to 2 per cent of the air passed through an arc can be combined, so that for large-power units the possibility of such combination would require to be considered. Naturally if the quantity of gas combined were sufficient it might be possible to use the gases for chemical processes, and I should appreciate information as to whether any tests regarding the percentage of nitric oxide have been carried out.

The next point I should like the author's views on are in respect to the arc-back voltage. For example, he suggests (vol. 75, page 613) the use of electrode spacings

given by $2E/30\,000$, where E is the d.c. voltage. While this will be satisfactory in laboratory tests, will it be adequate for service? In d.c. work, switching surges frequently give voltages above $2E$, and in addition atmospheric surges are to be considered. Consequently I think that spacings given by $4E/30\,000$ or $5E/30\,000$ will probably be necessary, resulting in an increase of arc voltage and a lowering of efficiency.

Mr. James Dickinson: Dealing with the paper first and turning to Fig. 2, the current wave does not seem to be as distorted as one would expect. In the case of an ordinary a.c. arc, with a series steadying resistance, an interesting distortion is brought about in both the current and the terminal-voltage waves. At the beginning the supply voltage must rise to a certain value to strike the arc. The current wave, under the conditions assumed of negligible inductance and capacitance, is flat until the arc is struck. Then the current increases rapidly, causing an appreciable drop in voltage across the steadying resistance. This lowers the arc potential difference, as shown in Fig. 2, which remains approximately level owing to the increasing current causing a proportionately increasing drop of voltage across the steadying resistance. The falling current causes a small extinction peak, and then the current suddenly drops to zero. This peculiar state of affairs gives an arc power-factor of roughly 0·6 with hard carbons. Under these conditions, since the author has compared the principle of the static rectifier with that of a circuit breaker, it might be justifiable, theoretically, to expect that some form of transient would be produced both in the a.c. and in the d.c. networks.

The operation of the high-frequency ignition is equivalent to closing the circuit breaker, while the action of the air-blast and associated apparatus is similar to opening the breaker. The effect of striking the arc is the same as inserting at a given time a voltage equal and opposite to that between the arc electrodes. The effect of breaking the arc at a given time is the same as suddenly inserting at that time a voltage which produces in the branch of the network under consideration a current equal and opposite to the current which would exist in the branch in the absence of the open circuit. By considering the problem in this way, i.e. by considering the response of the network to a suddenly impressed e.m.f., we are led to suspect that for impressed voltages other than sinusoidal voltages there will be difficulties in the network arising from travelling waves, a fruitful source of which is switching.

The inductance, being a variable element in the circuit, and a factor causing the current to lag on the voltage, may considerably modify the possibilities associated with relatively fixed parameters.

I should be interested to know whether any investigations have been made to assess the behaviour of a complete network with which this rectifier is likely to be connected.

In a short paper by E. Kobel* experiments are described and oscillograms explained which show that, contrary to recent experience with mercury-vapour rectifiers, it is now possible to quench an anode current by means of a polarized grid, even when the anode voltage is positive. To do this the grid must be of suitable dimensions and position in relation to the anode, and the mercury vapour-pressure in the space between the anode and the grid has to be maintained within certain determined limits. Precautions have to

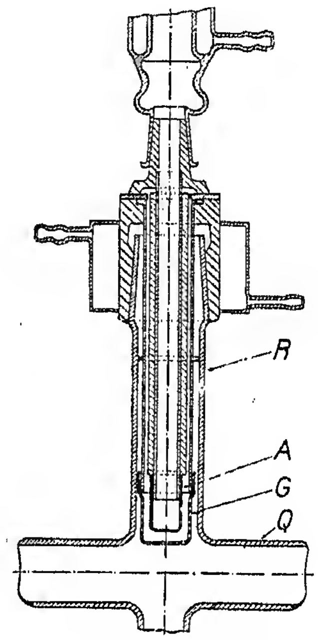


FIG. C.—Section through anode and grid of control unit.

A. Anode. G. Grid.
Q. Quartz tube. R. Glass tube.

be taken against the voltages set up inductively by the sudden extinction of the anode current. Condensers connected between the anode and cathode will deal with these voltages satisfactorily, or, alternatively, a second or leakage grid may be mounted between the anode and cathode and connected through a resistance to the anode. These two means may also be used simultaneously.

This ability to extinguish an arc can be utilized in several outstanding problems, e.g. automatic conversion of direct current to polyphase alternating current without employing an alternating voltage, voltage regulation

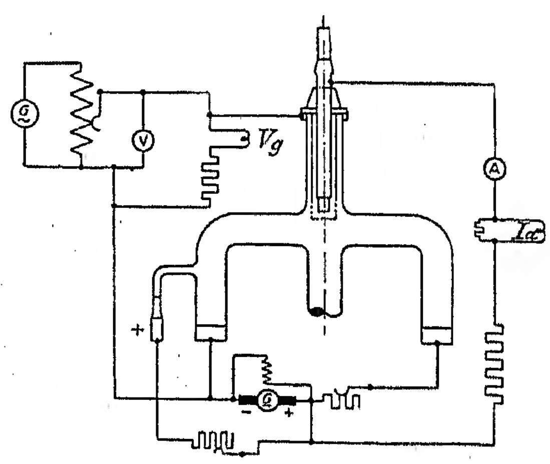


FIG. D.—Diagram of connections.

V_g. Oscillograph loop for grid voltage.
I_a. Oscillograph loop for anode current.

without spoiling the power factor on the primary side of the transformer, phase compensation, etc. The following is a short description of the essential features of the apparatus developed by Herr Kobel after 2 years' work in the research laboratories of Messrs. Brown, Boveri and Co., of Baden, Switzerland.

I have had prepared four slides (Figs. C, D, E, and F)

* "The Interruption of Arcing Anode Currents (A.C. or D.C.) with a Grid in Mercury Vapour," *Association Suisse des Électriciens, Bulletin*, 1933, vol. 24, p. 41.

from Kobel's paper to explain the essential features both of the apparatus and of the oscillograms.

Fig. C shows the design of the anode and grid. A direct-current arc is established between the exciting anodes in the horizontal quartz tube (Q). The anode (A) and grid (G) are housed in the vertical glass tube (R).

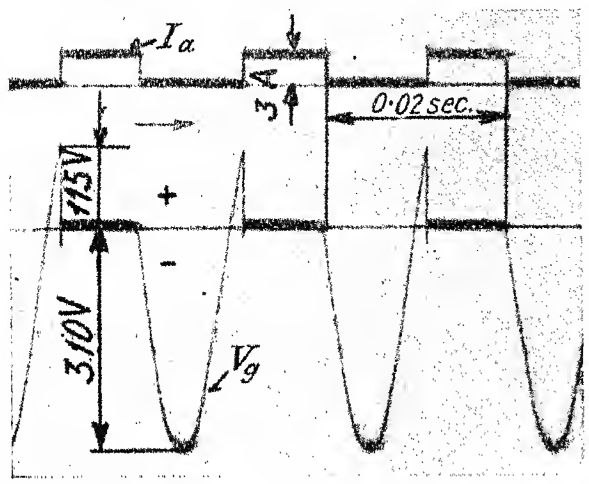


FIG. E.—Oscillogram showing the breaking of a continuous anode current by means of a 50-cycle alternating grid voltage.

I_a = anode current (1 mm ≡ 0·91 A).
V_g = grid voltage (1 mm ≡ 14·5 V).
Mercury vapour-pressure = 2×10^{-3} mm Hg.
Mercury temperature = 25° C.

The anode is hollow, 0·35 mm thick, and is mounted on a hollow iron tube terminating at the top end in a glass cooler fitted with an inspection window, to enable the temperature of the bottom of the anode to be taken optically. The quartz tube and the iron tube are each separately connected to the same high-vacuum pump through valves. The sides and end of the anode are surrounded by a control grid, also mounted on a thin iron tube enclosing the anode tube. The control grid is like a sieve, being perforated with many small holes.

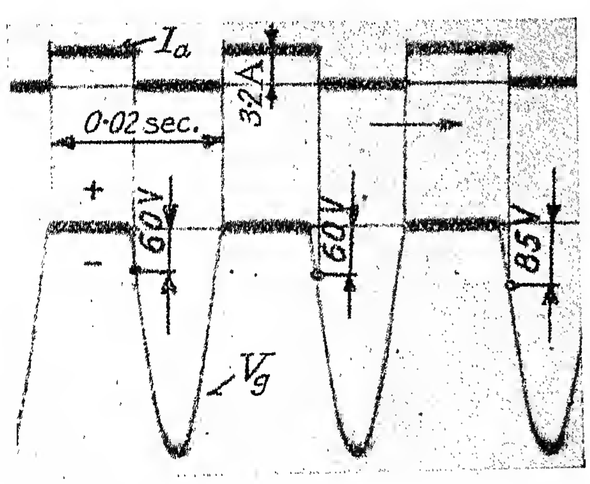


FIG. F.—Oscillogram showing the breaking of a continuous anode current by means of a 50-cycle alternating grid voltage.

I_a = anode current (1 mm ≡ 0·91 A).
V_g = grid voltage (1 mm ≡ 14·5 V).
Mercury vapour-pressure = 10×10^{-3} mm Hg.
Mercury temperature = 46° C.

Fig. D shows the electrical connections for taking the oscillograms shown in Figs. E and F. The anode is connected to the positive pole of a d.c. supply, and the control grid to a source of alternating current. The oscillograms show how the value of the negative grid voltage, required for interrupting the direct current flowing through the anode, depends upon the mercury vapour-pressure. Fig. E shows the breaking of a continuous anode current I_a by means of a 50-cycle alternating grid voltage V_g. The mercury vapour-pressure is 2×10^{-3} mm of mercury, and the negative grid voltage is between 10 and 20 volts at break. Fig. F, with the same symbols as those of Fig. E, shows that for 10×10^{-3} mm of mercury 60–85 volts are required on the grid to quench the direct-current arc. For a vapour pressure of 28×10^{-3} mm of mercury the corresponding voltage is about 290 volts. These values refer to a grid having holes of 0·9 mm diameter, distant 4 mm from the anode, and with a total grid open surface equal to 40 per cent of the anode surface. It is clear from these oscillograms that the breaking of the direct current at low mercury vapour-pressures occurs with smooth regularity. With higher vapour-pressures, of the order 28×10^{-3} mm of mercury, the breaking is irregular. In this case the negative grid-potential required to break the current is of the same order as the peak value of the applied voltage, and quenching becomes intermittent. The ignition voltage of 115 volts in Fig. E is due to the fact that there was no exciting arc in the horizontal limb. With an exciting arc in this limb this peak vanishes as shown in Fig. F. The interruption of the arc occurs with extreme rapidity.

Kobel develops his experiments on single-phase and 3-phase 50-cycle tubes; he discusses the conduction of electricity within the tube, and describes the lay-out and design of a 3-anode tube for phase compensation, and for the automatic conversion of direct current to 3-phase alternating currents of 120 amps. at 400 volts were interrupted satisfactorily at their maxima. His paper is well illustrated with oscillograms and circuit connections.

Kobel has determined that this grid of special dimensions can regain control of the arc upon being made negative, and that it is thus possible completely to control the instants of starting and stopping of the anode currents. The grid in general use at present can only prevent the initiation of an anode current, when negative, and release the current on again becoming positive, without having any action on the current after it has been started. It acts as a trigger for starting the anode during each cycle, as in the case of thyratrons.

I have called attention to Kobel's paper because, so far as I know, there has been, in Britain, only a short abstract* of it, and in America only a brief reference at the end of a paper by Didier Journeaux on "Voltage Control of Vapour Rectifiers."† In this latter paper it is pointed out that, with this type of grid interruption, it would be possible to draw a leading current from the line.

Mr. E. J. Williams: The advantages of the transmission of large amounts of power over long distances by means of high-voltage direct current as compared with the present a.c. system, have been much canvassed in recent years. Opinions differ, however, both as to the actual economies to be expected and as to the details of the apparatus best suited for this purpose.

Prof. Marx‡ in 1932 characterized the rectifier which

* *Science Abstracts*, B, 1933, vol. 36, Abs. No. 1197.
† *Electrical Engineering*, 1934, vol. 35, p. 976.
‡ *Elektrotechnische Zeitschrift*, 1932, vol. 53, p. 737.

is the subject of the present paper as "a new rectifier for very high voltages and powers." As it is agreed that the advantages of direct-current over alternating-current transmission will only be realized by the employment of very high voltages for very large amounts of power, the Marx rectifier would seem to be the apparatus for which the protagonists of d.c. transmission have been waiting. If, therefore, the predictions of the experts come true, the engineer may find himself called upon in the not-distant future to design, manufacture, or operate, this piece of apparatus. Hence, although the present paper makes it clear that the Marx rectifier is still in the development stage, it is of interest to try to picture what form this conversion apparatus will take in actual practice. The following questions will immediately present themselves.

The figure of 1·5 is mentioned in the paper as the factor of safety of the valve gap. Does the author consider that this figure is high enough? A more usual figure for the majority of electrical apparatus is 3 or 4. It would be interesting to know what considerations led to the adoption of this figure.

Again, in common with most other rectifiers, this arrangement gives a d.c. voltage which is not steady. Harmonic filters will therefore be necessary, especially in conjunction with long transmission lines. The provision of such filters for the very high voltages involved will entail a serious addition to the cost of the apparatus and to the floor space required.

The interesting tests carried out in 1932 at Zschornewitz by Prof. Marx show what the single-phase rectifier is capable of. In practice, polyphase rectification would be adopted. Have any experiments on a similar scale been carried out with polyphase arrangements?

Mr. M. Waters: It is noticeable that quite a low gas pressure is used in the rectifiers which the author describes, and it would appear that a considerable spacing must be used between the electrodes to obtain a reasonable safety factor for the reverse voltage. It would be interesting to know what advantage in size would be obtained by the use of gas pressures up to 15 atmospheres, and whether high-voltage rectifiers could be worked with advantage up to this pressure.

Mr. H. V. Field: This rectifier is characterized by an extremely high efficiency which, according to the equation quoted on page 613 (vol. 75), is more or less independent of the operating voltage. I presume that this applies only to high-voltage rectifiers, and not to those operating at low voltage. What is the lowest voltage at which tests on this class of rectifier have been carried out, and what is the estimated lowest voltage at which it is commercially practicable? Is it necessary to vary the air flow to suit the load, or can this be fixed at a satisfactory value for all loads? Are the electrodes designed to give (*a*) streamline or (*b*) turbulent air flow? The former would seem desirable for minimum arc loss, and the latter for maximum de-ionization rate after arc extinction. On page 616 the question of transferring wattless power is raised. I presume that this only applies to possible conditions of unbalanced a.c. load after inversion. Have any tests been carried out on complete systems, with conversion from alternating current to direct current, followed by inversion from direct current to alternating current, using these rectifiers? Have misfires been known to occur, and, if so, what effect is produced in the system?

Mr. F. C. Winfield: The author describes the development of the circular-orifice type of electrode with magnetic control and water cooling in which the arc foot is deliberately caused to move round the lip of the orifice in order to prevent local burning and destruction of the contact. He later states, however, that the baffle plate has now become the main electrode, and it would seem that the very troubles which he has overcome by his orifice design must reappear in the baffle-plate electrode. Perhaps the author would explain how this is dealt with. I should be glad to know whether the material of the electrode has any influence on the problem, and whether other materials, besides copper, have been tried out. Mr. Harle has remarked that he sees no apparent application for this rectifier at present, and suggests that its chief function is for extra-high-voltage work. In reading the paper I did not get the impression that this rectifier was suited to high-voltage work only, but rather that one of its attractive features was its apparent universality of application and that its possibility of application directly to very high-voltage work was a specially notable feature. The apparatus appears to me to be more simple and direct as compared with the mercury-arc rectifier when its auxiliaries are included, and the absence of difficult vacuum or pressure conditions is very attractive. Now since the mercury-arc rectifier has been developed to a stage of sound commercial application in the lower-voltage traction and similar ranges, it would be interesting to know whether the author considers that the Marx rectifier can compete with the mercury-arc rectifier within this range, and whether he can give any direct comparison between the two devices up to, say, 3 000 volts at 5 000 kW.

The Author's Reply to the Leeds and Newcastle Discussions.

Dr. W. G. Thompson (*in reply*): The interest with which the paper has been received and the many excellent points raised by the speakers in the discussions are much appreciated. I am much indebted to Mr. French for his lucid explanation of the processes involved in establishing the conducting path between the electrodes in the various types of rectifiers. In the early designs of the Marx rectifier the method of arc extinction was very similar to that employed in some types of air-blast circuit breakers. One of the basic principles of the more recent forms of the rectifier is that the arc is maintained as short as possible, instead of being blown out into a loop as in a circuit breaker. A short arc ensures a minimum release of arc energy and the quiet extinction of the arc at the current-zero. Following exhaustive tests, the mechanical method of ignition was discontinued because of the air-flow disturbances involved. In the latest arrangements the necessity for extra insulation on account of the ignition system has been avoided by using an auxiliary arc directed by the air stream to initiate the power arc. The principle is slightly different from that referred to in Fig. B.

At the proposed working voltages the efficiency of the Marx rectifier is very high. The power consumption of the auxiliaries is largely independent of the size of the rectifier units, and for the large outputs intended the overall efficiency should exceed 99 per cent. The minimum amount of floor space would probably be obtained by mounting the valves on a tank containing the auxiliaries, a 6-phase unit giving a 50-kV d.c. output then requiring about 40 sq. ft. and standing about 8 ft. high.

With regard to the tests at Zschornewitz, the air pressure should have been quoted as 2·5 atmospheres gauge, instead of absolute. The reverse breakdown strength of the electrode gap of a Marx valve under load is roughly 25 kV per cm per atmosphere, and at 10 mm spacing and 2·5 atmospheres gauge pressure 90 kV would be just within the limits of the valve in question. On the heavy-current test the arc voltage was considerably less than the applied voltage of 500 volts, so that the electrode spacing of 10 mm was not too great. The Marx valve has no polarity limitations and thus permits of inversion by current reversal, but, since the quenching action of the electrode gap only becomes effective at the current-zero, the valves must be arranged in a circuit that permits of phase commutation for invertor work.

Replying to Mr. Calverley, at the time that the paper was written there were tendencies to complication in design of the valve. Fortunately the introduction of permanent magnets for moving the arc, the raising of the upper limits for the air-cooled designs, and the modification of the ignition system, have done much to enhance the simplicity of the design. So far there has been no direct attempt to operate the valve at frequencies other than about 50 cycles per sec., but the de-ionization has been found to be effective even with circuits where certain forms of electrode designs tend to quench the arc and give rise to voltage-recovery oscillations of considerable amplitude and of frequencies of about 1 000 cycles per sec. The practically instantaneous cut-off of the arc luminosity at the end of the arcing period which is revealed by stroboscopic observation, and the experimental results from tests of arc-gap recovery, suggest that the de-ionizing time for the Marx rectifier is extremely short, but until the opportunity presents itself for a comparative test with standardized measuring methods and circuit conditions it is difficult to make a true comparison between the Marx and the mercury-arc rectifier. The Marx valve itself is, strictly speaking, an arcing device which is independent of electrode polarity and takes on the function of rectifier or invertor according to the dictates of the external circuit and the timing of the arc ignition. Those problems inherent in the invertor circuit still remain, but with the gas-cooled arc equipment the ignition of the incoming phase in a polyphase invertor may be advanced as desired to meet commutation requirements, and the reversal of power flow may be effected without the use of an additional set of valves.

The main difficulty with mechanical ignition is the disturbance of the air flow by the rotating arm. Corona ignition is an interesting suggestion, but it seems that there would have to be an undue delay in igniting a given phase unless the running clearance was very fine. The original Tesla ignition was open to some of the criticisms Mr. Calverley puts forward, but these are avoided in the more recent ignition using an auxiliary arc, generated at one electrode only, to bridge the main electrode gap.

In reply to Mr. Ball, the point-plane rectifier was merely the start of the development of the Marx rectifier, and its characteristics are much removed from the operating conditions of the symmetrical electrode designs referred to in the paper. The real theory underlying the Marx rectifier is a very fascinating study, but it unfortunately involves a considerable amount of abstract physics associated with the conduction of electricity in gases and therefore hardly comes within the scope of an engineering paper. In the type of valve under discussion rectification is fully complete, no reverse current flow taking place. Provided the economic conditions can be satisfied there seems no reason why the gas-cooled arc convertor should not be used for motor control as far as the current-carrying capacity permits.

Prof. Wheatcroft's experience with the point-plane rectifier is very interesting; with the early design of point-plate electrode adopted at Brunswick it was found possible to operate with currents up to 3 amperes (peak) for about 15 minutes before overheating occurred. Regarding the relative importance of the magnetic field and air blast, the former is advisable for currents exceeding 20 amperes, to reduce electrode wear. The air blast is essential in all cases for scavenging the products of arcing. The energy requirement for an e.h.t. ignition spark would be about 0·5 kW per phase. In the practical case some form of auxiliary arc ignition would be used instead of the Tesla coils. Those coils shown in the slides were generally existing laboratory apparatus, and not specially designed to be of the minimum size required for a given valve.

Mr. Harle raises the question of the behaviour of the Marx valve under short-circuit conditions and points out the need for dealing with currents up to 20 times full load. The problem is one of assessing the rating of the given rectifier electrode arrangement. In the first place, some of the electrode arrangements have been found capable of successfully interrupting currents of over 10 000 amperes, functioning as a switch. The rating of the rectifier is dependent upon the maximum continuous load-carrying capacity of the electrode material, and in general it has been found that the short-time rating, limited by the arc-quenching ability of the device, is considerably in excess of the working range. It must be remembered that the arc length is fixed in the present electrode design; even a considerable increase in current would not mean a disproportionate arc-core diameter compared with the electrode dimensions, and moreover the cathode and anode foot-points are shielded. Thus it is that an electrode system designed for about 100 amperes (peak) continuous rating was able to carry a current of 1 000 amperes (peak) for 1 minute before arcing-back started. It appears to be a case of maintaining the arc energy at a minimum and using sufficiently massive, well cooled, electrodes to prevent an instantaneous temperature-rise. So far the air has been

adjusted to suit normal conditions; increasing the air beyond the optimum value merely results in an undesirable increase in arc energy.

Since the arc is not caused to flare, the quantity of gas actually associated with the arc is comparatively small, and chemical tests made with a view to investigating the safety of operating the valves in closed rooms showed only the slightest traces of chemical action.

Various suggestions have been made for dealing with surges; actually any reasonable arcing-back-voltage safety factor can be easily obtained by increasing the air pressure in the valve, while arresting devices may be associated with the d.c. line. One view put forward is that since the valve is free from polarity limitations the surge will be transmitted through the arcing phase of the rectifier, so that the voltage-rise will not take place at the rectifier itself.

In reply to Mr. Dickinson, Fig. 2 is diagrammatic and is intended for clear illustration and definition purposes. In actual practice the current wave is as he suggests. The example of the carbon arc, restruck every half-wave, is hardly a true comparison with the power rectifier wherein effective phase commutation materially assists the maintenance of a reasonable power factor. With well-designed electrodes, forced quenching of the arc is avoided and a quiet extinction of the arc takes place at the current zero; the virtual voltage referred to will therefore also be zero, and no surging takes place. This has been confirmed with the oscillograph. Large inductances are a common feature of rectifier circuits and should introduce no new difficulty in the present case. So far as the complete network is concerned there is nothing inherent in the Marx rectifier to differentiate its influence on the network from that of any other rectifier. The mercury-arc innovation described in Mr. Dickinson's communication seems promising, and its development will be awaited with interest.

Mr. Williams mentions the safety factor of 1·5 referred to in the paper. The position was that most of the tests at Brunswick were carried out at about atmospheric pressure; to attain the practical factors of safety would have meant an increased electrode spacing and a larger arc voltage that would have been misleading. The factors of safety suggested are easily obtainable by raising the air pressure. The economics of the harmonic-filter problem should not prove too great an obstacle. In the first place the kilowatts of electrical power transmitted at the high voltages will be considerable, the apparatus will be mounted out of doors, and the capital cost of the rectifier installation will be small compared with the rest of the system and the possible revenue for energy transmitted. Polyphase work has had to be carried out on a much smaller scale than at Zschornewitz, as the cost of load-absorbing equipment for that voltage and power would be prohibitive.

Replying to Mr. Waters, compressed-gas equipment for pressures up to 15 atmospheres has already been used in connection with condensers for Schering-bridge work. So far as I am aware, the high-pressure Marx valves already tried out have proved satisfactory under laboratory conditions. For very high-pressure work the circulating gas may require to be free from dust, necessitating the use of simple filters. Using high-pressure valves, the advantage of size reduction would be limited by external flash-over considerations.

As Mr. Field remarks, the very high efficiencies are obtained at the higher voltages. Small rectifiers of the Marx type have been run off 240-volt mains for experimental purposes. Without an analysis of the proposed operating conditions, a detailed knowledge of the capital charges and running expenses governing the minimum yearly operating cost, and an assessment of any advantages peculiar to the given rectifier, it becomes difficult to stipulate a lower commercial working voltage limit with any degree of exactness. It is practically impossible to obtain streamline flow for the whole of the arc length, but so long as churning-up of the arc does not take place the air flow is probably satisfactory. Inverted operation, and also the coupling of a.c. and d.c. supplies, have been carried out in the laboratory. Misfires have occurred when probing the limits of the control settings, but in practice protection would be required against continued invertor misfire.

In answer to Mr. Winfield, the adoption of the central main electrode enabled a much improved magnetic-field system to be used for producing a strong radial field over the working face. The actual electrode part is cheap and easy to replace and the water or air cooling is simplified. A much smaller arc loss also results, and heating of the electrodes is minimized. Most of the available metals have been tried out; but copper, with its high melting-point, good thermal conductivity, and resistance to burning, is the most suitable for electrodes. 3 000 volts is well within the working range of the Marx rectifier, but comparison with other types is difficult within the short space available, for the reasons given in the reply to Mr. Field.

INVESTIGATION OF VALVE PERFORMANCE BY AN ELECTRO-DYNAMOMETER METHOD.*

By D. A. Bell, B.A., Graduate.

(*Paper first received* 1*st August, and in final form* 18*th October*, 1934.)

Summary.

An electrodynamometer is employed to pick out the component of fundamental frequency from the output of a valve working under non-linear conditions but with sinusoidal input. By this means effective values of valve parameters can be deduced for working conditions at large amplitudes.

Table of Contents.

(1) Introduction.

The characteristics of thermionic valves are usually specified by any two of the parameters:—

Anode resistance, $\rho = (\partial V_a/\partial i_a)_{V_g=\text{const.}}$
Mutual conductance, $G = (\partial i_a/\partial V_g)_{V_a=\text{const.}}$
Amplification factor, $\mu = (\partial V_a/\partial V_g)_{I_a=\text{const.}}$

These parameters would be constant in an idealized valve whose characteristics were straight lines, but in actual valves, whose characteristics are more or less curved, their values will vary from point to point of the surface which represents in three dimensions the relations between anode current I_a, anode potential V_a, and grid potential V_g. A closer approximation in the specification of the valve is therefore obtained by plotting graphs of the parameters for various operating conditions, e.g. graphs of μ or ρ against I_a. The anode current is probably the most suitable quantity in terms of which to plot the variation of the parameters, since experience shows that the lumped characteristic, namely

$$I_a = f(V_a + \mu V_g),$$

is reasonably closely obeyed provided that the grid is not at a positive potential comparable with that of the anode; differences depending upon the ratio of V_a to V_g for a given value of I_a are expected to be only second-order effects. Such graphs, however, show only the values of parameters which are defined in terms of limitingly small changes in the variables about certain points; the equivalent parameters for large amplitudes are not readily deducible by taking averages over the operating cycle, owing to distortion of the wave-form (or formation of harmonics) which arises from the curvature of the characteristics. For many purposes the useful output of a valve is only that component which is of the same wave-form as the input, so that in general the large-amplitude parameters should be defined in terms of fundamental components only. For example, the effective mutual conductance at large amplitudes (G')† may be defined as the ratio of the fundamental-frequency component of alternating anode current to the grid voltage when the latter is of sinusoidal form.

Of the various methods available for picking out the fundamental component from the distorted wave-form of the anode current, the one adopted by the author (working at a frequency of 50 cycles per sec.) was to pass the anode current through the moving coil of a dynamometer whose fixed coil was excited from the source which supplied the grid and anode potentials to the valve. By integration over one cycle of the fundamental frequency, it will be seen that the mean value of the couple $\sin(pt)\sin(npt)$ or $\sin(pt)\cos(npt)$ due to any harmonic of the nth order is zero; consequently the deflection of the instrument is proportional to the current of fundamental frequency only. A practical difficulty arises from the direct current flowing in the anode circuit, which if allowed to flow through the moving coil of the dynamometer will cause a small deflection by reaction with any external field, such as the earth's field. For work of the highest accuracy, therefore, it might be preferable to interpose a transformer between the anode circuit and the dynamometer, so as to eliminate direct current from the instrument. As the work described below was intended to show the scope of the method and the general phenomena to be expected, however, rather than the detailed performance of any particular valve, it was considered sufficient to reduce the relative importance of the couple due to direct current by using the maximum possible current in the field coils of the instrument; actually the power dissipated in the field of the most sensitive instrument employed was some 20 watts. The error was further reduced by the fact that when the alternating current was small the direct current was likely to be constant, and therefore would produce only a shift of the zero. The various types of measurements carried out will now be considered individually.

* The Papers Committee invite written communications, for consideration with a view to publication, on papers published in the *Journal* without being read at a meeting. Communications (except those from abroad) should reach the Secretary of the Institution not later than one month after publication of the paper to which they relate.

† The symbols ρ', G', and μ' will be employed to represent the effective values at large amplitudes of the quantities corresponding to the zero-amplitude parameters ρ, G, μ.

(2) EFFECTIVE RESISTANCE OF THE DYNATRON.

If a dynatron valve be represented by a negative resistance $-\rho'$, then the circuit of Fig. 1(*a*) is electrically equivalent to that of Fig. 1(*b*), where $L/(RC)$ has been written for the dynamic resistance of the tuned circuit at resonance. It is clear that the condition for steady oscillation, i.e. that an alternating current of appropriate frequency should be able to circulate indefinitely in the circuit without gaining or losing amplitude, is simply $\rho' = L/(RC)$. The effective value of ρ' is that for the fundamental frequency of oscillation, or in other words the effective value is equal to the ratio of the fundamental-frequency components of anode voltage and current; it follows that if a curve be available of the effective value of ρ' plotted against amplitude, then it is possible to read off the amplitude of oscillation which will be generated if the valve be connected to any circuit of given dynamic resistance. Oscillation will naturally be impossible in regions of the curve where ρ' is positive, and for a tuned circuit of given dynamic resistance it will only be possible if the negative value of ρ' reaches an equally low (numerical) value. When the characteristic is of the type illustrated in Fig. 4B, there may be two possible amplitudes, but only one is stable, namely the one for which an increase in amplitude results in an increase in the numerical value of ρ': it will be seen that this is the larger amplitude (the one on the right-hand descending branch of the curve in Fig. 4B). The dynamometer makes it possible to obtain such a curve with ease, and since the direction of its deflection indicates whether the phase of the anode current is similar or opposite to that of the applied voltage, there is no ambiguity between positive and negative values of resistance.

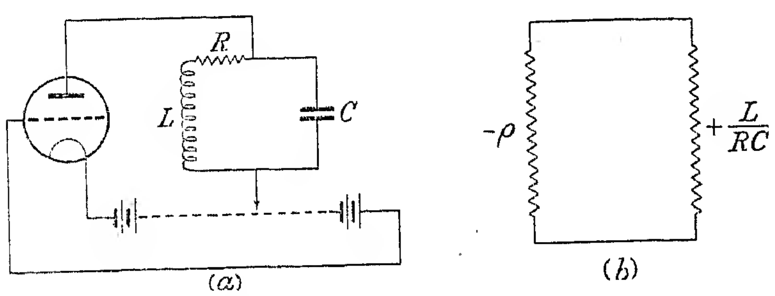


FIG. 1.

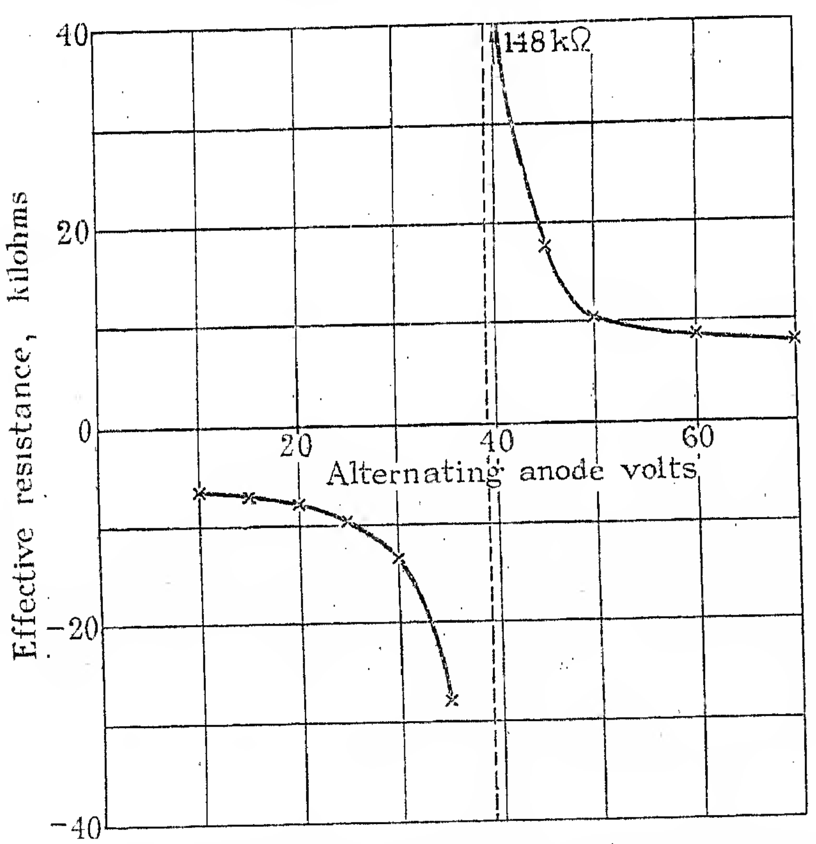


FIG 2.—Results obtained with LS5 valve, $V_a = 78$, $V_g = 162$, $V_f = 4 \cdot 0$.

A typical curve for a triode dynatron (LS5) with anode battery potential within the region of negative slope is given in Fig. 2. Referring to the static characteristic of this valve (Fig. 3) it will be seen that, if X is the operating point, then the negative resistance should remain fairly constant until the amplitude is so large that the cycle extends beyond the regions A and B; part of the cycle then occupies a region of positive resistance, so that the negative alternating current grows less rapidly with rising voltage—the effective resistance becomes larger—until ultimately with the cycle extending to some such points as C and D there is no net current flow at the fundamental frequency, and the effective resistance has become infinite. Further increase in the amplitude results in a positive current, indicating a gradually decreasing positive resistance which should be asymptotic to the value in the regions OCA and BDE; this limiting value would be reached when the region AB of negative slope occupied a negligible portion of the whole cycle. (In practice the positive resistance may reach a minimum at a value greater than this and then rise again, owing to the limitation of the positive characteristic at $V_a = 0$ and at saturation.)

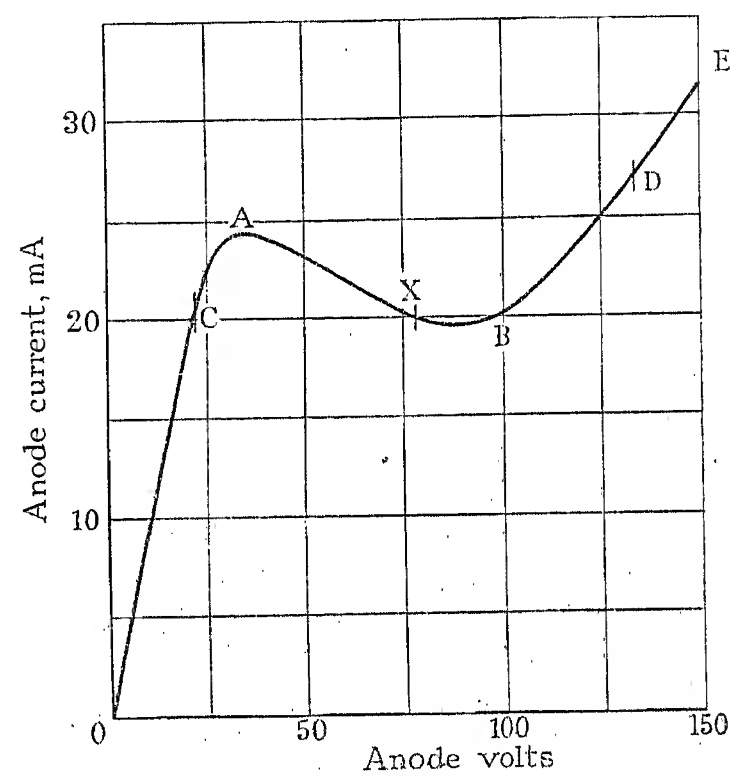


FIG. 3.

Fig. 4A gives a number of curves for another triode (Mazda AC2/HL) with various values of anode battery potential; all, however, within the region of negative slope. As might be expected, the amplitude falls if the anode battery voltage is either raised or lowered from

the value corresponding to the mid-point of the region of negative slope: for given values of resistance, the curve for $V_a = 80$ volts shows larger amplitudes than the curves for either 104 or 60 volts. It is well known that certain valves will continue to oscillate, after they have once been started, if the anode potential is made zero or even negative; in other words, if the operating point is moved from X in Fig. 3 progressively through A and C to the origin, or even farther to the left. Under such circumstances the valve's resistance is evidently positive for small amplitudes, but may become negative over a certain range if the slope AB is sufficiently pronounced to outweigh the positive region OA. Ultimately

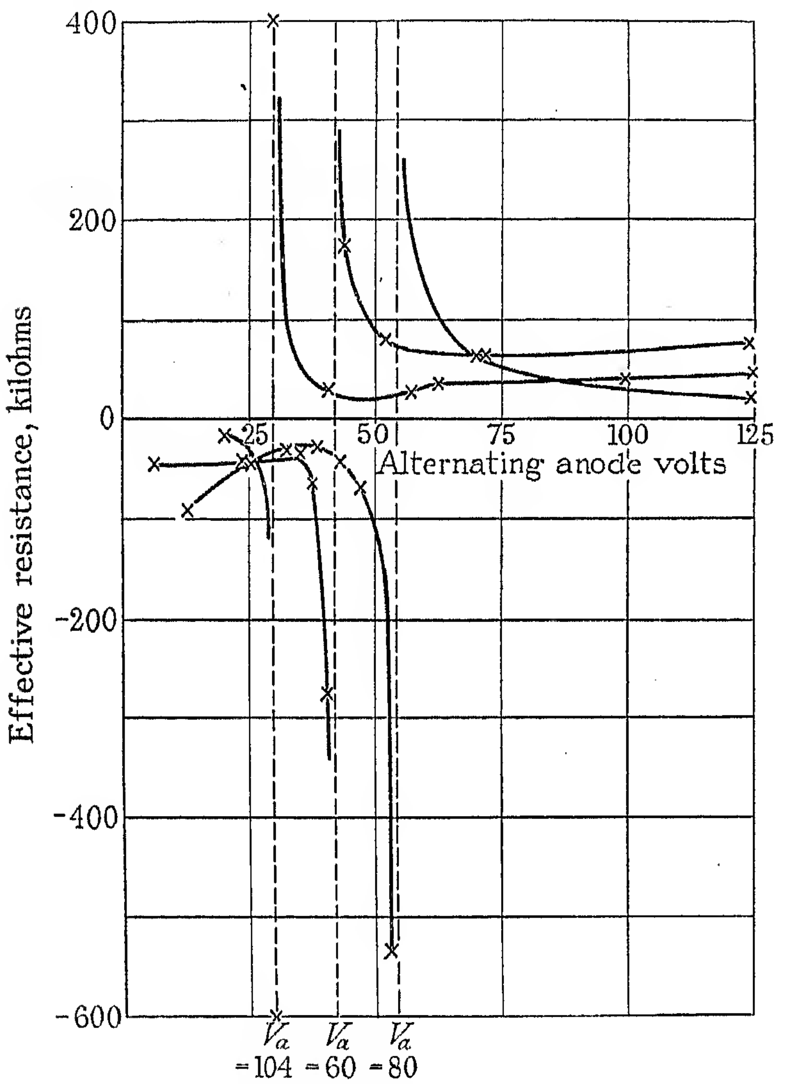


FIG. 4A.—Results obtained with AC2/HL valve, $V_g = 126$.

the resistance must again become positive, when the amplitude is sufficiently large. A qualitative curve for the Mazda AC2/HL at zero anode potential is given in Fig. 4B; compared with Fig. 4A, the amplitude is larger (say 95 volts as against 45 volts, at 200 kilohms), but the minimum negative resistance is also increased (of the order of 150 kilohms in place of 25). Adjustment of the anode voltage is therefore a possible means of matching the dynatron to different values of load resistance.

(3) AMPLIFICATION FACTOR OF VALVES WORKING AT LARGE AMPLITUDE.

(a) Triodes.

As the author was interested in the possible effect of variation in the effective value of amplification factor on the frequency of a simple valve oscillator, the dependence of μ' on amplitude was investigated by the following method. The alternating anode current indicated by the dynamometer having been noted for a given alternating grid voltage, the a.c. source was transferred to the anode circuit and the voltage adjusted to produce the same dynamometer reading as was observed at first. The ratio of the voltages required in the anode and grid circuits respectively is the effective amplification factor for the particular set of operating conditions and for the amplitude of alternating anode current indicated by the dynamometer deflection. (This is, of course, virtually a successive determination of G' and ρ', from which μ' is obtained by division.) By analogy with the

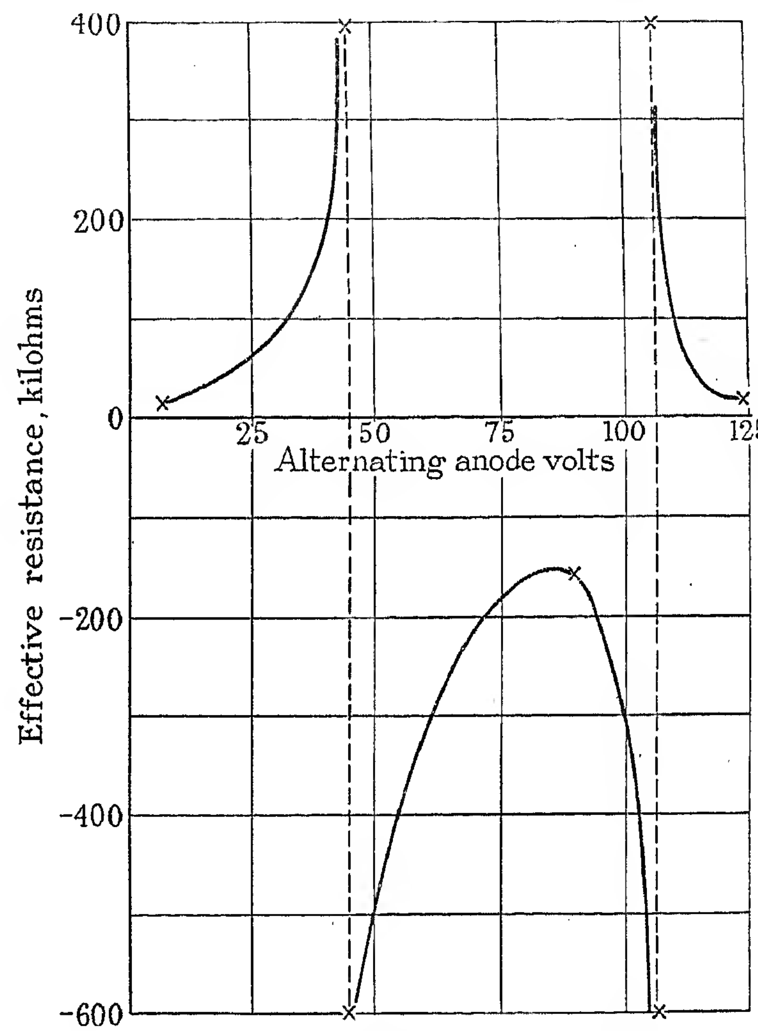


FIG. 4B.—Results obtained with AC2/HL valve, $V_g = 126$, $V_a = 0$.

static lumped characteristic, the values of μ' are plotted not against the amplitudes of grid or anode voltage but against the value of alternating anode current.

The factors whose effect on μ' might be studied are: (i) Amplitude of alternating anode current. (ii) Initial operating point, i.e. value of anode current (d.c.) for zero input, or, if even greater detail is required, values of anode and grid potentials separately. (iii) Anode load impedance. The effect of (ii) cannot well be discussed without (iii), for, while (ii) fixes the position of the centre of the cycle, (iii) partly determines the regions of the characteristic into which the peaks extend.

It will be appreciated that in order to measure both input and output voltages of the valve, a voltmeter is required which shall cover a very wide range, from the smallest grid voltages to the largest anode voltages;

and since μ' is determined as a ratio, accuracy of the small readings and consistency of scale throughout are essential. The simplest means of meeting these requirements was found to be a valve voltmeter with resistance potential-divider. The chief source of inaccuracy is the presence of the impedance of the a.c. source in the anode circuit during one only of the two measurements; this error is to be minimized by keeping the impedance of

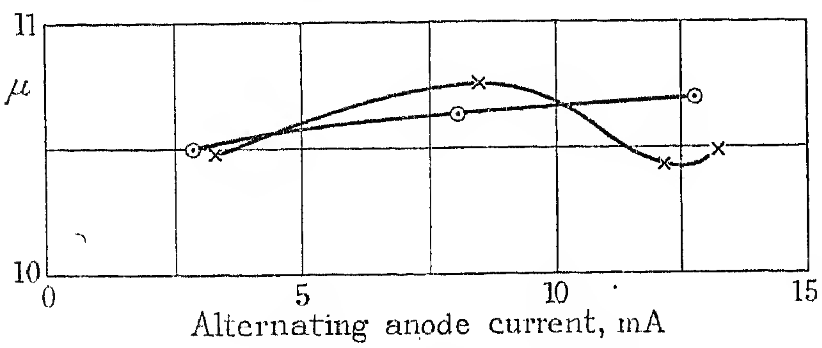


FIG. 5.—Results obtained with L.F. amplifying valve at 50 cycles per sec.

⊙ Load = 29 ohms.
× Load = 3 029 ohms.

the source small compared with that of the valve plus anode load. The residual error will be in the direction of a large value of μ'. A specimen measurement is shown in Fig. 5; the variation is seen to be surprisingly small when it is realized that the valve was biased to somewhere near the bottom bend of its anode-current/grid-voltage characteristic, so that the mean anode current rose from 6·2 to 12·6 mA over the range

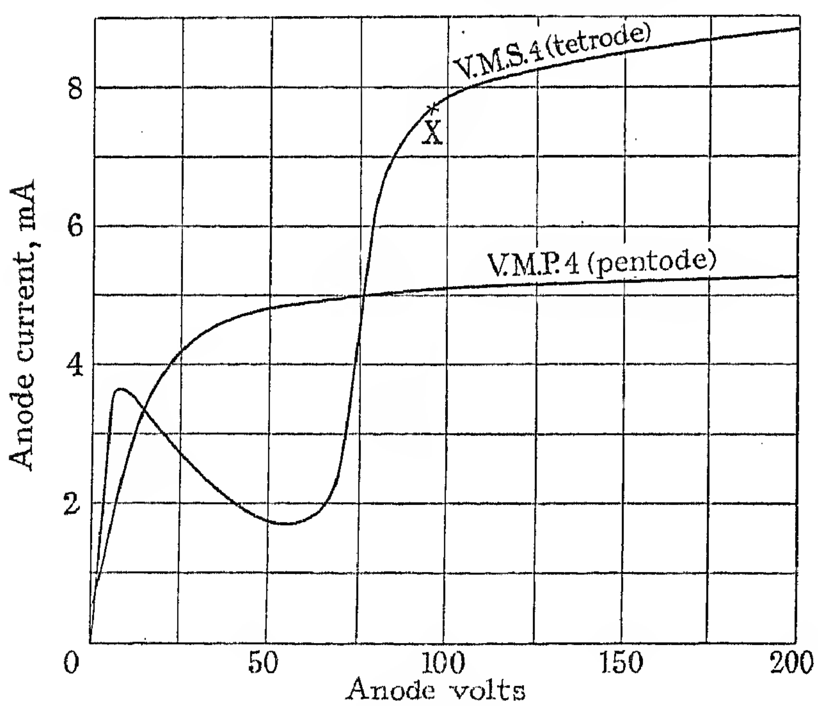


FIG. 6.

of the curve which was taken with little anode load. The relative independence of amplitude is probably due to the fact that whereas the negative half-cycles extend into a region of low anode current where μ is likely to decrease, the positive half-cycles extend to regions of higher anode current where μ is probably increasing. The effect of load is to modify the region into which the cycle extends, on both half-waves, so that the net result depends upon the particular valve and operating conditions, and no general rule can be given. The two curves in Fig. 5 for different load resistances suggest, however, that the effect of load impedance is small.

(b) *Screen-grid and High-frequency Pentode Valves.*

Measurements on screen-grid valves are of interest owing to the large variation of amplification factor with amplitude reported in connection with the determination of their anode-grid capacitance* by means of the "Miller effect." Using the same method as with triodes, graphs were plotted for dynamometer deflection against alternating grid and anode voltages in turn, with zero anode load, giving curves from which μ' was obtained by division of corresponding ordinates. It was found that G' remains constant up to quite large amplitudes of alternating anode current, but that ρ' at first remains constant and then decreases. This is entirely in accord

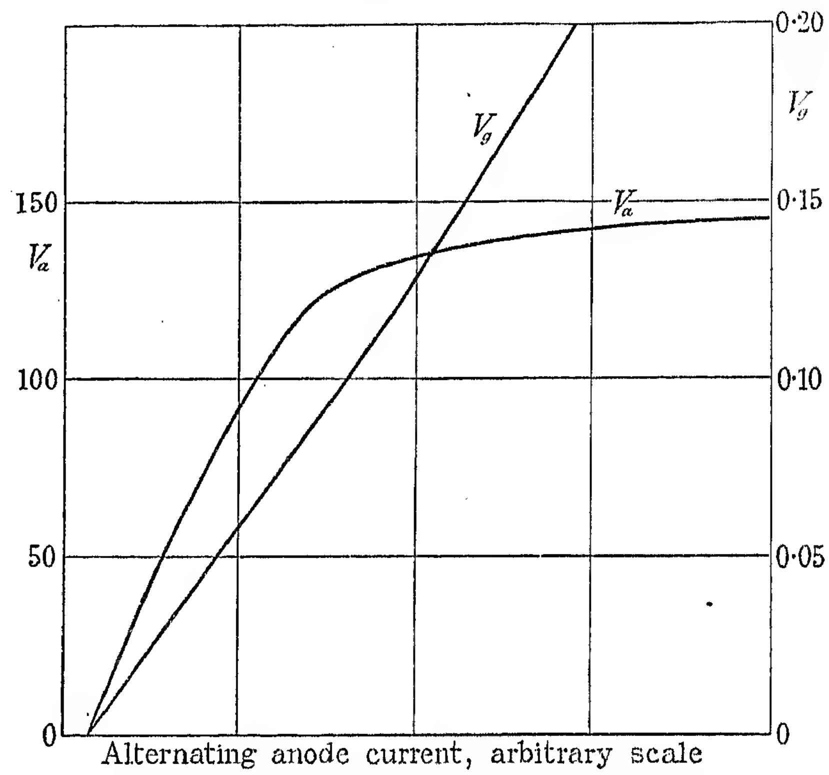


FIG. 7.—Results obtained with VMP4 valve.

with the static characteristics; for it is well known that the effect of the screening grid is to leave G relatively unchanged, but to increase very greatly ρ and hence μ, provided that the anode potential sufficiently exceeds that of the screen. Secondary emission, however, causes the anode-current/anode-potential curve to represent only a relatively low resistance for smaller values of anode potential. It follows that if the swing of anode voltage is sufficient to bring an appreciable part of the cycle into the region below the point X (see Fig. 6) the value of μ' must fall. In the dynamometer method used here to measure μ', the limit within which the value is expected to remain constant is quite clear: the peak a.c. voltage applied to the anode must not greatly exceed OX, where O is the operating point. If $V_a = 200$ volts and $V_s = 80$ volts, to take typical values, OX may be of the order of 100 volts; and if the valve has an anode resistance of 0·5 megohm then the maximum alternating current for constant μ' will be 0·2 mA (peak). A sensitive dynamometer was therefore employed, which

* T. IORWERTH JONES: "The Measurement of the Grid-Anode Capacitance of Screen-Grid Valves," *Journal I.E.E.*, 1934, vol. 74, p. 589.

would give full-scale deflection for a current of the order of $0 \cdot 05$ mA (r.m.s.), and shunted as required.

A typical pair of curves for alternating anode current against grid and anode voltages respectively is given in Fig. 7, referring to a high-frequency pentode valve (Marconi VMP4). Since it is known that it is the anode voltage which is the limiting factor, and not the amplitude of anode current, the curves for μ' have been plotted in terms of anode voltage. The four valves represented in Fig. 8 are two pentodes (Marconi VMP4 and MSP4) and two tetrodes (Marconi VMS4 and Cosmos AC/S). (The Cosmos AC/S is one of the earliest indirectly heated tetrodes, and must not be confused with the more recent types of Mazda AC/S.) The cross on each curve marks the point at which the minimum

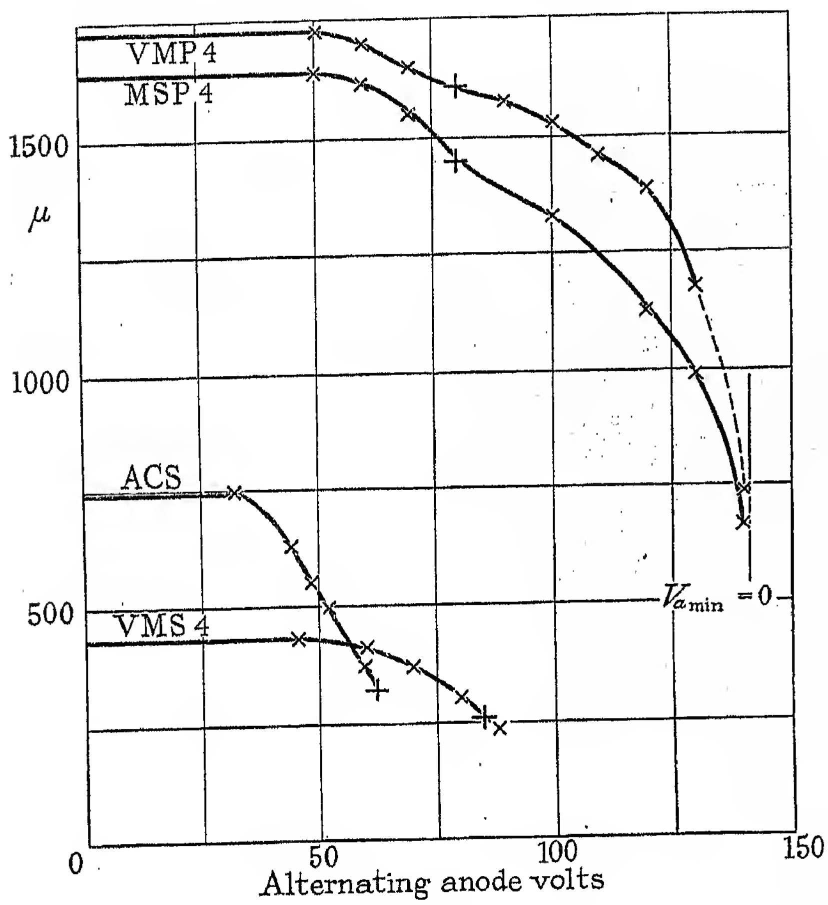


FIG. 8.

anode potential, at the negative peak of the cycle, is just equal to the screen potential, while the lines opposite the ends of the pentode curves indicate the points at which the minimum anode potential would be zero. Inspection of the curves shows that when the minimum anode potential is just equal to the screen potential, the proportions of the initial amplification factor which are retained by the various valves are as follows: AC/S, 43 per cent; VMS4, 60 per cent; MSP4, $88 \cdot 5$ per cent; VMP4, $93 \cdot 5$ per cent. The superior performance of the pentodes shows that the designers have achieved their object, and by means of the additional electrode have eliminated the effect of secondary emission. (The extra grid presumably has little effect on the emission of secondary electrons, but creates a potential gradient which makes it impossible for the secondary electrons to leave the neighbourhood of the anode.)

Owing to the very small input voltages involved, the mutual conductance is perhaps more difficult to measure than the anode resistance, and the author was anxious to be sure that there was no anomaly which had been overlooked owing to its occurring with extremely small amplitudes. No such effect was observed within the limits of the sensitivity of the apparatus, and the mutual conductance of the Cosmos AC/S valve was found to be constant down to an input of $2 \cdot 5$ millivolts on the grid.

The falling-off in amplification factor is of course accompanied by rectification, but the change of anode current is small; with the MSP4, for instance, the change of anode current was of the order of 5 per cent only when the alternating anode voltage was 120 volts, at which point the amplification factor had fallen to about 69 per cent of its initial value. Change of anode current cannot, therefore, be relied upon as a warning that changes are occurring in the characteristics of tetrode and pentode valves.

(4) Measurements of Efficiency.

Since the dynamometer measures the fundamental-frequency component of anode current, the true efficiency is readily deducible if the anode load resistance and the d.c. current and potential are known. An interesting point that came to the author's notice is that the efficiency of a valve may be much less if the anode load is a pure resistance than if it is a resonant circuit tuned to the desired frequency. The particular case examined was a triode, biased back approximately to cut-off, in

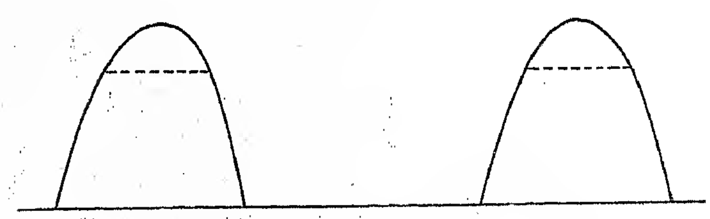


FIG. 9.

which condition an oscillator efficiency of $78 \cdot 5$ per cent is theoretically obtainable. The observed efficiencies with a resistance load were, however, found to be always less than 40 per cent, a result which is explained by the analysis given below.

In the ideal valve, having a linear characteristic and sharp cut-off, the anode current under the conditions we are considering would be a series of half sine-waves, as shown in Fig. 9. It can be shown by Fourier analysis* that if i is the instantaneous peak value of such a wave-form of anode current, then the peak value of the fundamental component of the current is $i/2$, so that the corresponding r.m.s. value is $I = i/(2\sqrt{2})$. If the anode load has a resistance R, the useful output is $I^2R = i^2R/8$, while the mean value of anode current (d.c.) is i/π. It remains only to fix the value of applied anode potential V, which is governed by the condition that at maximum current the instantaneous anode potential must not fall below zero, or $V = iR$. The d.c. power is therefore

$$iR\,\frac{i}{\pi} = \frac{i^2R}{\pi}$$

and the efficiency

$$\left(\frac{i^2R}{8}\right)\left(\frac{\pi}{i^2R}\right) \times 100 = 39 \cdot 3 \text{ per cent}$$

* See, for example, E. B. Moullin: "Radio Frequency Measurem p. 116.

If, on the other hand, the load is a tuned circuit, which at the fundamental frequency behaves as a resistance R_e, but has negligible impedance to all harmonics, the maximum voltage which can be developed across it is R_e times the fundamental current, or $iR_e/2$. It is thus possible to halve the d.c. anode voltage when the load is a tuned circuit in place of a simple resistance, and the efficiency is correspondingly doubled. The increased efficiency with the tuned-circuit load is due to the elimination of harmonic power; if the anode voltage is practically pure fundamental, although the anode current may contain large harmonic components there will be no loss of power since there is no voltage in phase with them. If the load is a pure resistance, however, every current, of whatever frequency, must be accompanied by a dissipation of energy I^2R.

The author was particularly interested in the possible efficiency of valve oscillators in which the grid is maintained negative throughout the cycle, so as to prevent the flow of grid current; but it is then impossible to work to the conditions adopted in the calculation of efficiency above. For if v_a, i_a, and v_g, represent peak values of alternating anode voltage, anode current, and grid voltage, and B_a and B_g are anode-battery and grid-battery potentials, then the condition for the desired efficiency is $v_a = B_a$, which is equivalent to $Ri_a = B_a$, where R is the anode load resistance. Now

$$i_a = \frac{\mu' v_g}{R + \rho'}$$

so that the condition becomes

$$\frac{R\mu' v_g}{R + \rho'} = B_a$$

whence $$v_g = \frac{B_a}{\mu'}\left(1 + \frac{\rho'}{R}\right)$$

If, however, the valve is biased back to the point at which anode current is just cut off, then $B_g = B_a/\mu'$ (approximately). Consequently v_g is greater than B_g in the ratio

$$\left(1 + \frac{\rho'}{R}\right) : 1$$

It follows that if a valve is to be worked at the highest efficiency attainable in the absence of grid current, the anode load must be of as high a resistance as possible.

The Table shows a set of efficiency measurements taken with various values of anode load resistance, but always with the input limited so that the grid current was negligible (a fraction of a microampere).

TABLE.

R	i_0	i	P	Efficiency	$V_{min.}$
ohms	mA	mA	watts	per cent	volts
7 000	1·75	9·5	0·98	23·3	170
7 000	4·9	11·2	1·26	27·7	192
7 000	0·8	8·6	0·86	24·4	140
7 000	2·5	10·8	1·18	28	142
10 000	1·35	7·6	0·94	30·9	124
10 000	2·8	8·7	1·08	31·6	109
15 000	2·3	6·5	0·87	35·3	55
17 000	3·0	6·25	0·88	38	22
20 000	2·1	5·2	0·74	38·5	33

The symbols employed are: i_0 = anode current with zero input; i = anode current under working conditions; P = a.c. power output; $V_{min.}$ = calculated value of minimum value to which the anode potential falls during the cycle; R= anode load resistance.

It will be seen that as the anode load is increased, the efficiency rises from 23·3 to 38·5 per cent (cf. the ideal, 39·3 per cent) while the value of $V_{min.}$ in general falls. If the condition of zero grid current is not imposed, the efficiency with this valve can be raised to about 44 per cent; or with another valve, which gave an efficiency of about 35 per cent without grid current, the efficiency rose to 50·3 per cent under the following conditions: R = 10 000 ohms, i_0 = 0·7 mA, i = 13 mA, P = 2·19 watts, $V_{min.} = -$ 87 volts. The calculated value of $V_{min.}$ has now become an appreciable negative quantity, which means that in fact the peak of the current wave must have been cut off as suggested by the dotted lines in Fig. 9. This type of wave-form results in a twofold gain of efficiency here; for firstly the efficiency when the anode is feeding a tuned circuit will be slightly greater,* and secondly the ratio of the peak of the fundamental component to the peak of the whole wave is greater, so that, as has been explained in the above analysis, the relative loss when a resistance load is used in place of a tuned circuit is much less.

(5) ACKNOWLEDGMENTS.

The author desires to thank Mr. E. B. Moullin for the interest he has taken in this work, which was carried out in the Engineering Laboratory, Oxford; and also the Council of the Institution, since this is an item of the work which has been made possible by the Swan Memorial Scholarship.

* E. B. MOULLIN: *loc. cit.*

DISCUSSION ON

"SOME CONSIDERATIONS IN THE DESIGN OF HOT-CATHODE MERCURY-VAPOUR RECTIFIER CIRCUITS."*

Mr. D. M. Duinker (Holland) (*communicated*): It may be of interest to point out that the minimum value for the inductance L (see Fig. A) in order that the current i should always be greater than zero may be arrived at in the following simple way, which holds good for any number of phases and avoids the use of the Fourier series. I shall make only one assumption in addition to those introduced by the author, namely that the ripple voltage across the load R is negligible compared with the ripple voltage between the points A and B in Fig. A. In practice the former voltage will never exceed a small percentage of the latter. This implies that at the terminals A and D of the choke there is the same ripple voltage as between A and B. Now when the current i flows without becoming zero and the voltage-drop in the transformer and tubes can be neglected, the oscillogram of the potential difference

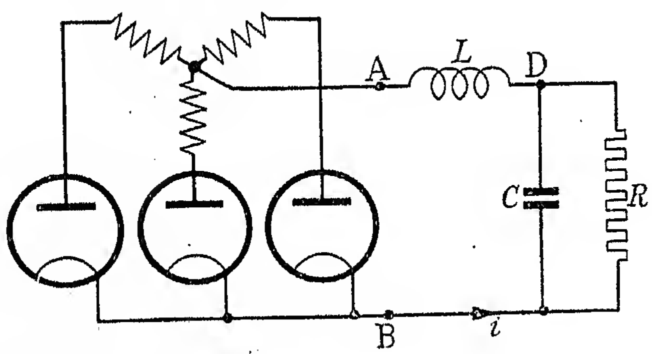


Fig. A.

between A and B consists of a series of tops of sine waves (Fig. B), of which there are m in one supply cycle (m being the number of secondary phases). The mean value of this voltage is given by the well-known formula

$$v_m = E\frac{m}{\pi}\sin\frac{\pi}{m} = EM \quad . \quad . \quad . \quad . \quad (1)$$

the symbol M being introduced in order to shorten the expression.

During the interval $\frac{\pi}{2} - \frac{\pi}{m} < \omega t < \frac{\pi}{2} + \frac{\pi}{m}$ the voltage across the choke is

$$v_L = E\sin\omega t - v_m = E(\sin\omega t - M) \quad . \quad . \quad (2)$$

This gives rise to an alternating current, $i_{a.c.}$, determined by

$$L(di_{a.c.}/dt) = v_L = E(\sin\omega t - M) \quad . \quad . \quad (3)$$

the solution of which is

$$i_{a.c.} = (E/X)(K - M\omega t - \cos\omega t) \quad . \quad . \quad (4)$$

where $X = \omega L =$ reactance of the choke (for the supply frequency), and $K =$ an integration constant, which can be found from the condition that the mean value of $i_{a.c.}$ should be zero and which turns out to be

$$K = \tfrac{1}{2}\pi M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The total current flowing through the choke consists of an a.c. component

$$i_{a.c.} = (E/X)(\tfrac{1}{2}\pi M - M\omega t - \cos\omega t) \quad . \quad . \quad (6)$$

and a d.c. component

$$i_m = \frac{v_m}{R} = \frac{EM}{R} = \frac{E}{X}\cdot\frac{X}{R}\cdot M \quad . \quad . \quad . \quad (7)$$

giving a total current

$$i = i_m + i_{a.c.} = \frac{E}{X}\left[M\left(\frac{X}{R} + \frac{\pi}{2}\right) - (M\omega t + \cos\omega t)\right] \quad (8)$$

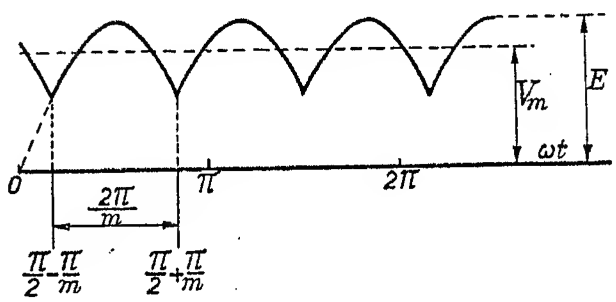


Fig. B.

As the current in the rectifier cannot have a negative value, the term between [ ] must be ≥ 0 for any value of ωt within the interval under consideration (viz. $\frac{1}{2}\pi - \pi/m \rightarrow \frac{1}{2}\pi + \pi/m$).

We now see that if

$M(X/R + \frac{1}{2}\pi) >$ max. value of $(M\omega t + \cos\omega t)$, current i will always be > 0;

$M(X/R + \frac{1}{2}\pi) <$ max. value of $(M\omega t + \cos\omega t)$, current i will be zero for a definite time, m times per cycle;

$M(X/R + \frac{1}{2}\pi) =$ max. value of $(M\omega t + \cos\omega t)$, current i will just become zero for a moment, m times per cycle.

Now the maximum value of $(M\omega t + \cos\omega t)$ within the considered interval amounts to

$$M\arcsin M + \sqrt{(1 - M^2)}.$$

The condition that the current i shall always be greater than zero thus becomes

$$M(X/R + \tfrac{1}{2}\pi) > M\arcsin M + \sqrt{(1 - M^2)}$$

or

$$\frac{X}{R} > \frac{1}{M}\sqrt{(1 - M^2)} - \arccos M \quad . \quad . \quad (9)$$

* Paper by Mr. C. R. Dunham (see vol. 75, page 278).

By introducing an auxiliary angle μ defined by

$$\left.\begin{aligned} M &= \cos\mu \\ (1/M)\sqrt{(1-M^2)} &= \tan\mu \end{aligned}\right\} \quad . \quad . \quad . \quad (10)$$

condition (9) can be simplified to

$$\left.\begin{aligned} X/R &> (\tan\mu) - \mu \\ \text{or} \qquad L &> \frac{(\tan\mu)-\mu}{2\pi f}R \end{aligned}\right\} \quad . \quad . \quad . \quad (11)$$

where
$$\mu = \text{arc}\cos\left(\frac{m}{\pi}\sin\frac{\pi}{m}\right) \quad . \quad . \quad . \quad . \quad (12)$$

and f = supply frequency.

For the usual numbers of phases (m) we then get the following table:

m	M	μ radians	$\tan\mu$	$(\tan\mu)-\mu$	$L_{min.}$
2	0·637	0·881	1·211	0·330	0·0527 R/f
3	0·827	0·597	0·680	0·083	0·0132 R/f
4	0·900	0·450	0·483	0·033	0·0053 R/f
6	0·955	0·301	0·311	0·010	0·0016 R/f
12	0·989	0·151	0·152	0·001	0·00016 R/f

When condition (11) is fulfilled, equation (8) is applicable; from which we see that the total current i reaches its maximum value when $(M\omega t + \cos\omega t)$ is a minimum. From this it may be shown that this maximum value amounts to

$$i_{max.} = \frac{ME}{X}\left(\frac{X}{R} + \tan\mu - \mu\right) = \frac{ME}{X}\left[\frac{X}{R} + \left(\frac{X}{R}\right)_{crit.}\right] \quad (13)$$

Thus when

$\frac{X}{R} >> \left(\frac{X}{R}\right)$ critical, $i_{max.}$ hardly exceeds $\frac{ME}{R}\ (= i_m)$;

$\frac{X}{R} = \left(\frac{X}{R}\right)$ critical, $i_{max.} = 2\frac{ME}{R_{crit.}}$;

$\frac{X}{R} < \left(\frac{X}{R}\right)$ critical, $i_{max.} > 2\frac{ME}{R}$;

equations (6), (7), and (8) are no longer valid.

For biphase rectification ($m = 2$) we take from the above table (X/R) critical $= (\tan\mu) - \mu = 0\cdot330$, which corresponds to the value to which curve A in Fig. 7 of the paper asymptotically approaches for large values of $y(=\omega CR)$. This should be the case, as we started from the assumption of a negligible ripple across R, which implies a large value of C.

For a large number of phases ($m = 6$ or 12) the overlap interval between the succeeding anode currents, due to the hitherto neglected transformer impedance, should be taken into account (in the present theory as well as in the method followed by the author). Therefore the values of $(\tan\mu) - \mu$ and $L_{min.}$ stated in the above table for $m = 6$ and 12 should only be considered as a first approximation. When, however, a high degree of smoothing is required, the lowest value of L, as given by (11), would imply in many cases an impractically large value of C,* so that for economical reasons one is compelled to use larger values of L, often even multiples of the computed minimum. Thus it seems hardly necessary to try for higher accuracy in such cases.

Mr. C. R. Dunham (*in reply*): The additional assumption introduced by Mr. Duinker, namely that the ripple across the load R is negligible, so simplifies the problem that the mathematics required by the original method, in this special case, becomes trivial. Thus the conditions are that the inductance L has to withstand the whole of the alternating voltage, and the peak value of the alternating current through it has to be less than the d.c. output of the rectifier.

As in the paper, the alternating current produced by the harmonic voltage of lowest frequency alone need be considered; therefore we have the inequality

$$\frac{v_{a.c.}}{\omega L} < \frac{v_{d.c.}}{R}$$

or

$$\frac{\omega L}{R} > \frac{v_{a.c.}}{v_{d.c.}}$$

referring to the harmonic of lowest frequency. Introducing the coefficient from the relative Fourier series this inequality may be written:

$$\frac{2\pi f L}{R} > \frac{2}{m(1-m^2)}$$

where f is the supply frequency and m is the number of phases, giving the results of column 5 of Mr. Duinker's table.

* The degree of smoothing, (ripple voltage between A and B)/(ripple voltage across R), is roughly equal to $m^2\omega^2 LC - 1$.

SOME PRINCIPLES UNDERLYING THE DESIGN OF SPACED-AERIAL DIRECTION-FINDERS.

By R. H. Barfield, M.Sc.(Eng.), Associate Member.

[From the National Physical Laboratory.]

(*Paper first received 7th February*, 1934, *and in final form* 20*th February*, 1935; *read before the* Wireless Section 5*th December*, 1934.)

Summary.

The paper constitutes a theoretical and experimental investigation of the principal forms of the spaced-aerial or Adcock direction-finder. The analysis is concerned mainly with the determination of two important properties of the systems, namely their performance under the influence of downcoming waves and their efficiency as receivers of wireless energy. These characteristics are reduced to quantitative form by introducing the two factors named respectively "standard-wave error" and "pick-up factor," and the various aerial-system modifications are studied with the object of showing how these factors may be predicted for any given system and how they vary with the dimensions, wavelength, and other characteristics, and also with the constants of the ground on which the aerials are erected. The four principal aerial systems dealt with are the "U" type, the elevated type, the coupled type, and the balanced type.

The "standard-wave error" of the type of direction-finders which is based on the principle of the closed loop or coil is first derived theoretically; it is found to be approximately 35° for a large range of wavelengths, and to be independent of ground conductivity and dimensions of the loop within wide limits. Experimental confirmation of this is obtained on short waves (20 to 50 m).

The theory of the "U" type is then considered and the method of calculating the standard-wave error for any given case is worked out. The agreement between values obtained in this way and the experimental results is satisfactory. The effect of screening the horizontal members of the system is discussed, but no quantitative formula for calculating the effect of the screen is found to be obtainable. A description is next given of a series of experiments with the "U" type on short waves, the system being tested by means of locally-generated downcoming waves of variable polarization. This method was employed to measure the effect of screening and burying the horizontal limb of the "U" aerial, as well as to determine the standard-wave error of the unscreened system.

Turning to the elevated type, after a brief theoretical discussion, experiments with the rotating form of this system are described; these include determinations of the standard-wave error by means of an elevated transmitter of variable polarization, an investigation of the effect of shortening the lower limbs of the dipoles, and an investigation of the effect of increasing the height of the apparatus above the ground. A working formula is obtained for calculating the standard-wave error for a system of this type.

The next modification dealt with is the coupled type, the various forms of which are described and illustrated. A description is given of a medium-wave direction-finder which was constructed to work on this principle. This was tested by means of downcoming waves from a kite transmitter and also by a local injection method. A formula for calculating the performance of the coupled type of aerial is derived from theory based on these experiments.

The balanced type, which is next considered, is described in detail and the results of the tests made with the kite transmitter are given.

A combination of the coupled and balanced systems, named the balanced-coupled type, is then described. This system was found to have a standard-wave error too low to be measured.

Practical tables are given for each system, showing the standard-wave errors for various wavelengths, aerial-system dimensions, and soil conductivities. These tables show that the systems vary very greatly in performance with the conditions and with the details of their design.

The pick-up factors of the various systems are dealt with in a separate section. The case of the loop or coil aerial is first dealt with and is used as a standard of reference for the other systems. Each of the spaced-aerial systems is then examined in turn with regard to its properties in this respect.

The paper concludes with a comparative table of standard-wave errors and pick-up factors for the various systems.

Table of Contents.

TABLE OF CONTENTS—*continued.*

LIST OF SYMBOLS.

E = intensity of electric field of single incident ray or of ground ray.
H = intensity of magnetic field of single incident ray or of ground ray.
V = potential difference across specified output terminals of aerial system.
X = intensity of total horizontal electric force in plane of propagation.
Y = intensity of total horizontal electric force perpendicular to plane of propagation.
Z = intensity of total vertical electric force.
a = area of coil or loop receiver.
c = velocity of light.
f = frequency.
h = height of vertical aerial.
h_e = effective height of vertical aerial.
h' = height of horizontal members above ground.
$j = \sqrt{(-1)}$.
l = length of half-dipole of elevated system.
n = number of turns of coil aerial.
p = pick-up factor.
s = spacing between vertical aerials of Adcock system.
u = e.m.f. representing unwanted pick-up of direction-finding system.
w = e.m.f. representing wanted pick-up of direction-finding system.
α = intensity of total horizontal magnetic field in plane of propagation.
β = intensity of total horizontal magnetic field perpendicular to plane of propagation.
γ = intensity of total vertical magnetic field perpendicular to plane of propagation.
ϵ = polarization error.
ϵ_s = standard-wave error.
θ = angle of incidence.
κ = dielectric constant of ground.
λ = wavelength.
ρ_v, ρ_h = reflection coefficients of ground for vertically and horizontally polarized waves respectively.
σ = conductivity of ground, in electrostatic units.
ϕ = phase angle, due to path difference, between incident and reflected ray at a given height above ground.
ψ = angle of polarization (the angle between the electric field and the vertical plane of propagation).
$\omega = 2\pi \times$ frequency.

(1) INTRODUCTION.

The closed-loop direction-finder remained a satisfactory instrument only as long as it was restricted to the reception of vertically polarized waves. The growing use of this type of direction-finder under circumstances where this condition was found not to hold, even approximately, e.g. over long ranges at night over land (night effect), or for reception from aeroplanes with trailing aerials, or for short waves in general, drew attention to the need for an improved type of instrument not subject to this limitation.

(a) *Previous Work.*

In 1916, Adcock* suggested the use of spaced open vertical aerials and followed it up by tests with an arrangement embodying this principle† (referred to in this paper as the "U" type). Later, the work was continued by Dr. Smith-Rose and the author working for the Radio Research Board, when a second embodiment of the Adcock principle, referred to as the elevated or "H" type, was developed.‡ A third modification, the screened "U" type, was shortly afterwards described by T. L. Eckersley,§ and a fourth

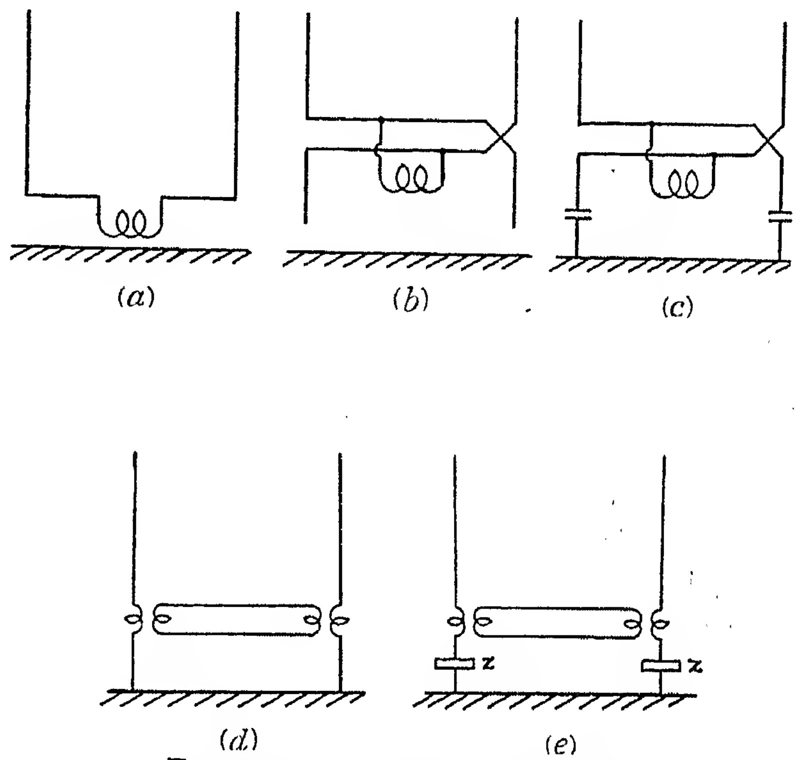


FIG. 1.—Types of Adcock systems.
(a) "U" (unscreened) type. (b) Elevated type. (c) Balanced type. (d) Coupled type. (e) Balanced-coupled type.

and fifth by the present author in 1930|| (the balanced and rotating types). Other modifications have been suggested since then, and one of these—the coupled type—is described in some detail in this paper.

All these justified the original Adcock principle in that they to some extent removed the limitation suffered by the loop type, but on the other hand all fell short of the ideal of a perfect direction-finder giving true azimuths for all types of waves. Further work therefore remained to be done. It appeared, as this work went on, that real progress would be aided more by a general study of the whole problem on a quantitative basis than by concentration on a single type. For this reason the work was diverted to an investigation of the principles determining the behaviour of the various modifications, with the particular object of providing some sort of guide to the design of these types of direction-finders for given practical circumstances. With this object in view, it has been found convenient to divide the paper into

* British Patent No. 130490.
† *Journal I.E.E.*, 1926, vol. 64, p. 837.
‡ *Ibid.*, 1926, vol. 64, p. 831.
§ *Ibid.*, 1929, vol. 67, p. 992.
|| *Ibid.*, 1930, vol. 68, p. 1052.

sections each dealing with one of the spaced-aerial modifications that have been investigated. Each section contains an account of the experimental work on the system in question, together with a theoretical discussion for which the experimental results serve as a guide and a check. The nature of each of these types is illustrated in Fig. 1.

(b) *Definitions of "Standard-wave Error" and "Pick-up Factor."*

The greater part of the work has been devoted to the investigation of the directional characteristics of the systems when under the influence of waves of any given angle of incidence or state of polarization. In order to sum up the performance of a given system in this respect into a single figure instead of a whole family of curves for every angle of polarization, it has been found convenient to select, as an arbitrary standard of comparison, the error for one particular angle of incidence and angle of polarization and to use this as a figure of reference or performance. Hence the wave for which both these angles are 45°* is referred to as the "standard wave" and the error produced thereby in the system is called the "standard-wave error" of the system.

The other factor to which special attention has been paid is the reception efficiency of the system. This, from the nature of the aerial arrangement, has a tendency to be low in comparison with that of an open single aerial or of a closed loop of comparable dimensions, particularly on long wavelengths and when the angle of incidence is small. To express this property of the aerial system, a figure known as the "pick-up factor" has been introduced; it is defined as the ratio of the potential difference (in volts) obtainable across the output terminals of the aerial system to the strength (in volts per metre) of the vertical electric field of the incoming wave. The variation with frequency of the above factor largely determines the wavelength range of the system. Other characteristics of the system, such as instrumental accuracy under practical conditions of usage, and directional characteristics for ground rays, are treated as subsidiary and are not dealt with.

(c) *Nature of Polarization Errors in Direction-finding Systems.*

The error or "unwanted pick-up" produced in a direction-finding system by a downcoming wave is caused by the action of the horizontally-polarized portion of the wave; while what may be called the "wanted pick-up" of the system is produced by the vertically-polarized portion. The ratio (unwanted pick-up)/(wanted pick-up) determines the magnitude of the error, which may conveniently be termed the "polarization" error.

Let the total electric force at the surface of the ground due to the combination of the incident and reflected waves be resolved into three components, X, Y, and Z, where X is the horizontal component in the plane of propagation, Y is the horizontal component perpendicular to this plane, and Z is the vertical component.

* For simplicity it is assumed that the e.m.f.'s induced in the system by the horizontally and vertically polarized components are in phase. The "standard wave" would therefore be not linearly but elliptically polarized, and should thus be defined as having equal horizontally and vertically polarized components with a suitable phase relationship to fulfil the above condition.

The unwanted pick-up is caused by the Y component acting on the horizontal members of the system; the wanted pick-up is caused by the space phase-difference between the Z components at each of the two aerials forming a pair. Their ratio is vitally affected by the angle of incidence of the wave. As this decreases, the Z component—owing to the steeper inclination of the wave—becomes less, while the space phase-difference, for the same reason, also decreases. On the other hand, owing to a decrease in the reflection coefficient of the ground, the Y component increases, since it is the residual difference between the incident and the reflected ray. Hence the ratio (unwanted pick-up)/(wanted pick-up), which determines the error, increases very rapidly with decreasing angle of incidence. Under most practical circumstances the variation is, in fact, approximately proportional to $\cos\theta/\sin^2\theta$, where θ is the angle of incidence, and thus the error for $\theta = 20°$ will be approximately 6 times greater than the error for $\theta = 45°$.

Both wavelength and soil conductivity also affect the error, but not so drastically. The unwanted and wanted pick-ups both decrease with increasing wavelength, the former owing to increasing reflection coefficient and therefore less residual Y component, the latter owing to decrease in the space phase-difference of the Z component. The resultant variation of the error is proportional to the square root of the wavelength. Variation of ground conductivity, within practical limits, has little effect on the Z component but causes a variation of the Y component (and therefore of the error) inversely proportional to the square root of this variable.

In contrast to this, we find that the error of a closed-loop direction-finder is almost entirely unaffected by changes of conductivity or wavelength, while the angle of incidence only varies the error in proportion to $\cos\theta$. It thus comes about that all imperfect spaced vertical-aerial systems, while they may be considerably superior to a loop under some conditions, may, under other conditions, lose much of their superiority or actually become inferior to the loop.

(d) *Standard-wave Error of Loop.*

The standard-wave error of a closed-loop direction-finder and the general expression for its polarization error at any angle of incidence may be fairly simply obtained. Assuming a single, plane-polarized downcoming wave, let θ = angle of incidence, ψ = angle of polarization, ρ_v, ρ_h = coefficients of reflection of vertically and horizontally polarized components respectively, ϵ = error, $E = H$ = field strength of incident ray, β = total horizontal magnetic field of vertically polarized component, α = total horizontal magnetic field of horizontally polarized component. Then

$$\tan\epsilon = \alpha/\beta$$

and, since the reflection of both horizontal magnetic components takes place without change of sign,

$$\alpha = H\cos\theta(1+\rho_h)\sin\psi$$
$$\beta = H(1+\rho_v)\cos\psi$$
$$\therefore \tan\epsilon = \tan\psi\,\frac{1+\rho_h}{1+\rho_v}\cos\theta \quad . \quad . \quad . \quad . \quad (1)$$

For a standard wave we have, by definition, $\psi = 45°$, $\theta = 45°$,

$$\therefore \tan \epsilon_s = \frac{1+\rho_h}{1+\rho_v} \cdot \frac{1}{\sqrt{2}}$$

Also for $\theta = 45°$ the value $(1+\rho_h)/(1+\rho_v)$ may be taken as approximately $1 \cdot 0$ for all wavelengths above $\lambda = 15$ m, provided that the field under consideration is sensibly at the surface of the ground. Therefore

$$\text{Standard-wave error of loop} = \text{arc tan } (1/\sqrt{2}) = 35 \cdot 3°$$

Thus the standard-wave error of a rotating-loop direction-finder, or of any direction-finding system depending on the directional characteristics of a closed loop, may be taken as about 35° for all wavelengths provided that the dimensions of the loop and its height above the ground are both small compared with the wavelength.

The calculation of this error for the various modifications of the spaced vertical-aerial direction-finder is, however, not so simple. The elevated, balanced, and coupled types, as will be shown later, all involve factors which are very difficult or impossible to estimate accurately. The "U" type, however, in its simplest form is much less complicated than the others. For this reason it is in this paper considered first, both theoretically and experimentally. Further, since much of the technique used in testing it is common to the others, and as its performance under varying conditions of angle of incidence, ground reflection-coefficient, wavelength, etc., is in many ways typical of spaced-aerial systems in general, it has been found advisable to devote somewhat more attention to it than to the others.

(2) THE "U" TYPE OF AERIAL SYSTEM.

(a) *Theory*.

The "U" system was first employed by Adcock in 1916 in his experiments with aeroplanes in France.* It is perhaps the simplest form which the spaced-aerial direction-finder can take. It consists of two pairs of spaced vertical aerials, which are exactly analogous to the crossed loops of a Bellini–Tosi direction-finder; the two opposing aerials forming a pair are joined together at their lower ends by a horizontal conductor on, near, or below the ground which contains the field coil of the goniometer (see Fig. 2).

From a theoretical point of view it has the advantage that, given the electrical constants of the ground on which it is erected, its performance can be quite accurately calculated. It can be shown, for instance, that on an "ideal" site with a perfectly conducting surface its standard-wave error would be zero, and if it is erected over sea water or on soil of very high conductivity this factor may still be satisfactorily low for suitable wavelengths and aerial heights. On ground of normal conductivity, however, owing to residual horizontal fields it will not in general give a much better performance in this respect than the loop direction-finder.

Attention was therefore turned in the early stages of the work to the possibility of reducing or eliminating the effect of these residual fields by providing the horizontal member with an open-ended metallic shield.* This experiment gave no detectable improvement and, as a consequence, the experimental work was diverted to the study of other types. Later, however, T. L. Eckersley modified this form of screened "U" system by earthing the screen at each end. Employing such a system on short waves, he found he was able to obtain directional effects on many stations which gave no bearings on a loop direction-finder. This type has more recently been developed for practical direction-finding on long waves. Attempts have also been made to improve the performance of the system by covering the ground beneath the aerials with an earth mat or thin conducting screen, in order to produce, in the locality of the apparatus, a perfectly conducting surface.

Investigations of the behaviour of these modifications and of the general case of the unscreened "U" system are described below.

(i) *Polarization errors.*—The expression for the polarization error produced in an unscreened "U" system such as that shown in Fig. 2 is derived in the following manner. Let u = e.m.f. constituting the unwanted pick-up, w = e.m.f. constituting the wanted

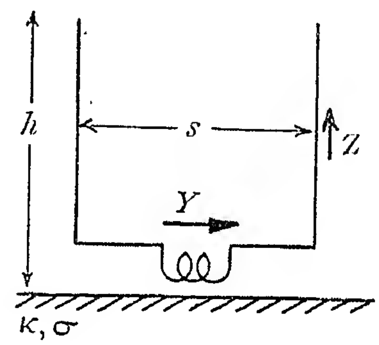


FIG. 2.—"U" type.

pick-up, s = spacing between the aerials, h_e = effective height of the aerials, θ = angle of incidence of the wave, ϵ = error produced by the wave, and let Z and Y have the meanings defined on page 425. If, then, u and w are in phase, the magnitude of the error is given by $\tan \epsilon = u/w$. The phase relation between u and w depends on the phase between the horizontally and vertically polarized components of the wave and may therefore in practice have any value. The assumption that they are cophasal, however, is justifiable not only for the sake of simplicity but also because it gives the maximum possible error for a given ratio of horizontal to vertical polarization. We then have

$$u = Ys\dagger \qquad w = Zh_e \sin\left(\frac{2\pi s \sin\theta}{\lambda}\right)$$

$$\tan \epsilon = \frac{u}{w} = \frac{Y}{Z} \cdot \frac{s}{h_e \sin\left(\dfrac{2\pi s \sin\theta}{\lambda}\right)}$$

If s is small compared with λ, we get

$$\tan \epsilon = \frac{1}{2\pi} \cdot \frac{Y}{Z} \cdot \frac{\lambda}{h_e} \operatorname{cosec} \theta \quad . \quad . \quad . \quad (2)$$

* *Journal I.E.E.*, 1926, vol. 64, p. 837.

* *Journal I.E.E.*, 1927, vol. 65, p. 600.

† This formula neglects the effect of the X component of the wave on the horizontal members and is therefore only a first approximation. The correction for this component is a small one for most practical cases but the simplified formula is unsuitable for cases where ϵ, as found by equation (2), is large (> 20°), or where the waves arrive at nearly vertical incidence (θ < 30°).

which, for a standard wave ($\theta = 45°$), becomes

$$\tan \epsilon_s = 0\cdot 225 \frac{Y}{Z}\cdot\frac{\lambda}{h_e} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The term Y/Z is obtained from simple electromagnetic theory by the method shown in the Appendix. It involves a knowledge of the electrical constants of the ground and assumes a sharp reflecting boundary coincident with the earth's surface. The experimental values of the standard-wave error on medium and short waves agree sufficiently well with the calculated values, however, to show that this method of obtaining Y/Z is a reliable one.

The above analysis shows that the performance of a "U" system is closely bound up with the ratio wavelength/(effective height). The factor s (aerial spacing), provided it is small compared with the wavelength, does not affect the error. Further, since Y/Z, as a first approximation, is proportional to $\cot \theta$, the error increases very rapidly as θ decreases; namely, as $\cos \theta/\sin^2 \theta$.

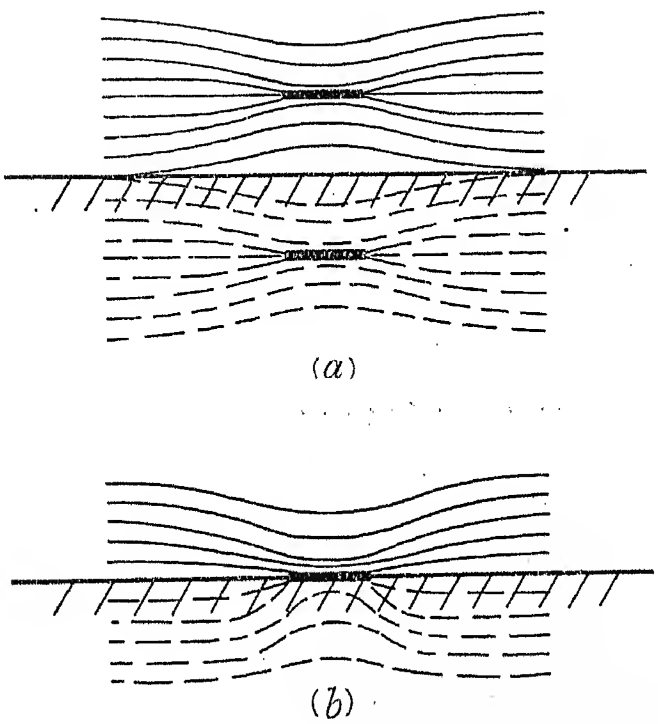


FIG. 3.—Field distortion due to screen.

The standard-wave error is seen from (3) to be proportional to λ/h_e and to the ratio Y/Z. The latter increases with the effective refractive index of the ground, $\frac{1}{\sqrt[4]{[\kappa^2+(4\sigma^2/f^2)]}}$, so that the standard-wave error depends on the site on which the apparatus is erected. The behaviour of the "U" type in these respects is typical of that of the other spaced-aerial systems described, for the residual error is, in each case, produced by the same fundamental cause, i.e. that of pick-up of the Y component by the horizontal members of the aerial system.

(ii) *Screened "U" systems.*—The theory of the effect of a screening tube enclosing the horizontal members, or of an earth mat beneath them, is not susceptible to simple analysis and it has not been found possible so far to derive anything which can be applied, even approximately, quantitatively. The object of the screening tube or earth mat is to provide a secondary field of such a nature as to counterbalance exactly the primary horizontal field (which causes the errors) but which shall not, at the same time, introduce secondary vertical components of appreciable magnitude. Alternatively we may put it that the resultant field distortion produced locally at the receiving system by the presence of the screening tube must be confined to horizontal planes. The unearthed screen is a conductor totally immersed in a non-conducting medium, and the resultant field around it in the presence of horizontally polarized waves is of the type shown in Fig. 3(*a*). Presumably the screen actually at the boundary of the two media and in contact with both will have some such field as that depicted in Fig. 3(*b*). In each case, the distorted field contains vertical components which prevent the screen from perfectly fulfilling its desired function, though the all-important question of the degree of the prevention cannot be solved by this qualitative survey. It can only be said that it forms an explanation of the comparatively small effect which the experiments show the screen to give rise to.

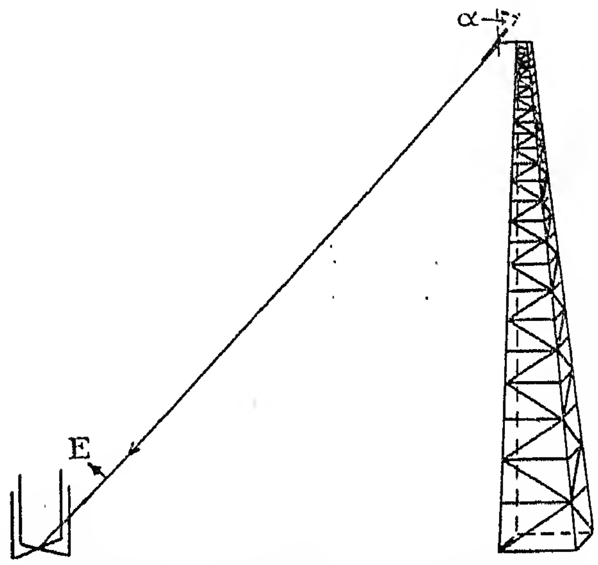


FIG. 4.—"U" type: elevated-transmitter tests.

Since in designing this type of "U" system it is at present impossible to calculate the effect of the screen, all that can be done is to take the results of the experiments of the particular cases as described and apply them to any general case under consideration. The results of the three sets of experiments investigating the properties of the screened "U" system are in fair agreement with each other. They agree in showing that the screen with its ends free has no perceptible diminishing effect on the circulating current set up by a horizontal field, and that the screen with its ends earthed has a diminishing effect of from 0·5 to 0·75 on the unwanted pick-up.

(*b*) *Experiments.*

The following experiments were carried out to determine the standard-wave error of the "U" system and its modifications, and as a general check on the theory.

(i) *Short waves.*—The arrangement tested is shown in Fig. 4. It consisted of a 4-aerial "U" system with radiogoniometer, the height of the aerials being 5 m and the spacing between them also 5 m. The testing transmitter was situated at the top of a mast, at a height of about 25 m above the ground. The angle of incidence

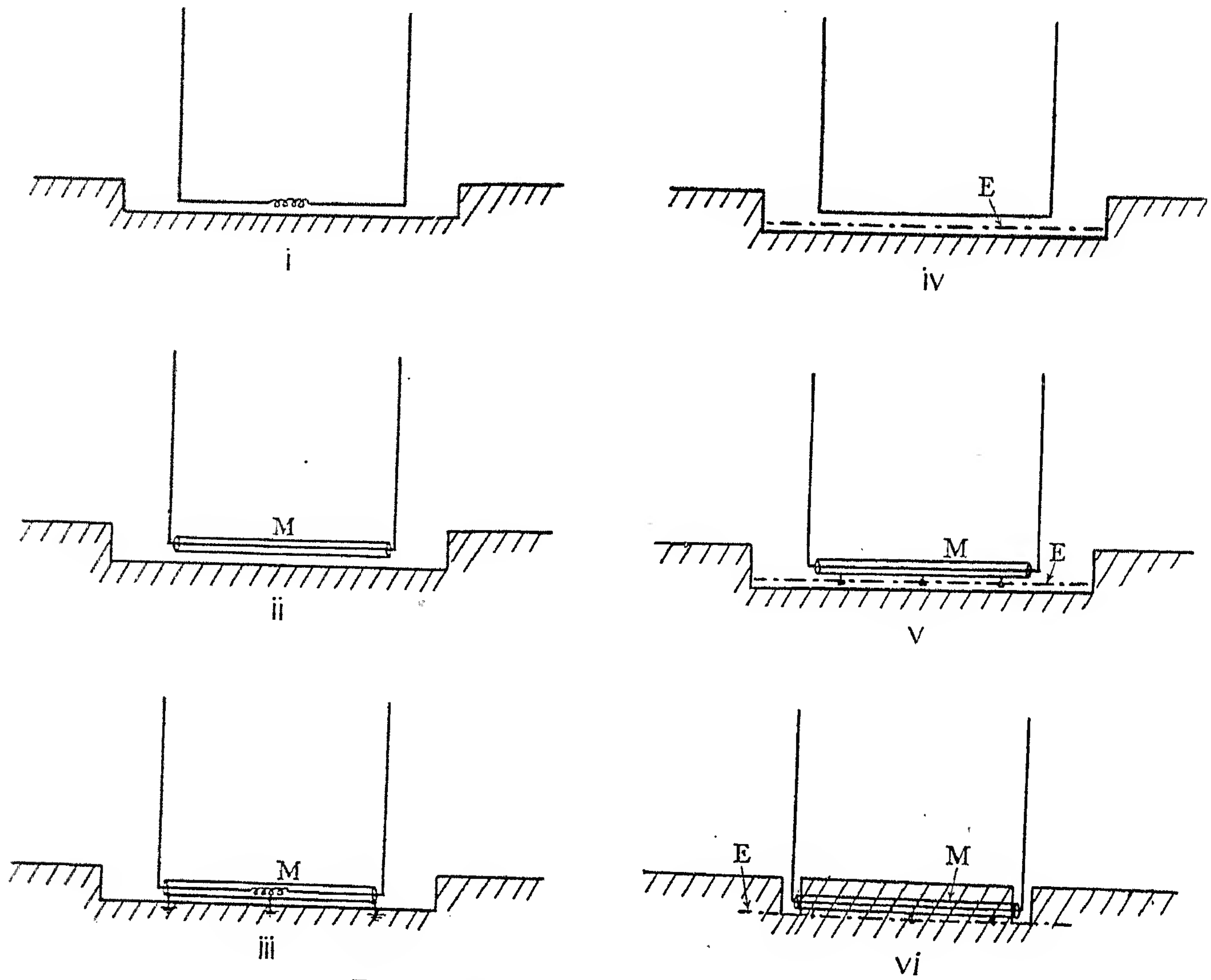


FIG. 5.—Types of "U" aerials tested.
M = metal pipe; E = earth mat.

of the waves at the receiver was approximately 45°. The transmitting aerial was an aluminium dipole 5 m long which could be rotated to any desired angle in the vertical plane at right angles to the plane of propagation. An observation of the apparent bearing of the transmitter was made on the goniometer with the aerial rotated through 45° from the vertical, first in a clockwise and then in a counter-clockwise direction. The instrumental error due to the polarized downcoming wave is then half the angular difference between the two bearings so obtained. To deduce the standard-wave error, however, this angle must be multiplied by a correction factor to allow for the fact that the transmitting aerial rotates about a horizontal axis and not about an axis in the direction of travel of the wave.

The modifications of the aerial system which were tested (see Fig. 5) were as follows: (i) Horizontal members unscreened and 7·5 cm above the surface of the ground. (ii) Horizontal members screened with brass tubes (M), with ends of screen not connected to earth. (iii) Horizontal members screened with brass tubes, with ends of screen connected directly to earth. (iv) An 8-metre square earth mat (1·1-cm mesh wire netting) on the ground beneath the aerial system, with the horizontal members unscreened. (v) The same arrangement as (iv), but with the horizontal members screened and connected to the mat throughout their length. (vi) The same arrangement as (v), but with the earth mat buried 15 cm deep so that the earth just covered the screening tubes.

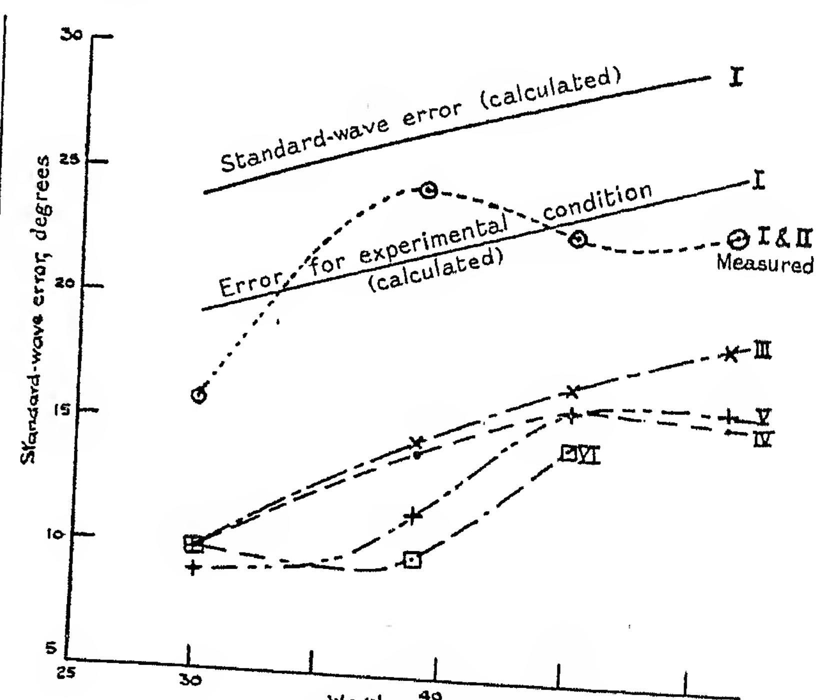


FIG. 6.—"U" type: curve of standard-wave errors for earth-mat and screen tests.

The results obtained with the various modifications are indicated as follows:—
I and II, ⊙
III, ×
IV, ●
V, +
VI, ⊡

Each of these arrangements was tested over a wavelength range of from 30 to 60 m. The results are shown in the curves of Fig. 6, and the average effect is sum-

marized in Table 1, which shows the error reduction-factor of the various screening arrangements.

As appears later, the value of the standard-wave error obtained from (i)—the only calculable arrangement—agrees satisfactorily with theory. The unearthed screen of (ii) has no detectable effect; the earthed screen alone gives an error reduction factor of about $\frac{2}{3}$ on the average, while the value for the mat alone is about $\frac{1}{2}$. Finally, there is no improvement on combining the mat and the screen or on burying the combined arrangement.

These results were confirmed by a slightly different method. The same transmitter was employed, but the aerial system consisted of a single pair of aerials 2·5 m high and spaced 2·5 m apart. If such an arrangement is placed in the vertical plane containing the transmitter, it will give maximum response to a vertically polarized wave and zero response to a horizontally polarized wave. If it is in the vertical plane at right angles to this, the reverse will be the case. To make it capable of responding to both vertically and horizontally polarized waves at the same time the system was therefore, for the purpose of the experiments,

TABLE 1.

System	Mean standard-wave error	Mean reduction factor, with respect to system (i)
	degrees	
(i)	26	—
(ii)	26	0·0
(iii)	16·5	0·65
(iv)	13	0·50
(v)	13	0·50
(vi)	13	0·50

aligned in an intermediate vertical plane. Measurements were made on the received current at the centre of the horizontal member with the transmitting aerial first vertical and then horizontal. This gives the pick-up of the system with respect to the horizontally polarized wave in terms of its pick-up with respect to the vertically polarized waves and enables the standard-wave error, which is an expression of the same ratio under other conditions, to be very simply calculated. Systems similar to (i), (ii), and (iii) were tested by this method and the results were found to agree closely with those of the previous tests. In addition, an arrangement was tested in which the screening tubes were buried at a depth of 1 m. This arrangement was found to give the same error reduction factor as was given by the system with the screen unburied.

An arrangement with an earth mat extended well beyond the aerials was also tested by this method. The mat consisted of a strip of wire netting 30 m long extending 15 m on either side of the centre of the system and parallel to the horizontal member. The mat was laid on the ground, well trodden down, and connected to buried earth pins at its end and centre. The system under these conditions was found to have a reduction factor of about $\frac{2}{3}$.

(ii) *Medium waves.*—A test of a "U" system was carried out on a wavelength of 380 m by a method closely resembling the first series of tests. The elevated transmitter was in this case, however, raised by means of a kite in the manner described in an earlier paper.* The dimensions of the system were: height of vertical members, 24 m; spacing, 15 m; height of horizontal members above ground, 5 m. The horizontal members were provided with a removable screen which consisted of a "tube" of 1·3-cm mesh wire netting of box section 23 cm square. This screen was connected at the centre to the netting screen surrounding the operating hut. It could be connected at its extremities to earths consisting of vertical cylinders of tinned iron plate 0·4 m in diameter and 1·0 m long, sunk with their tops 15 cm below the surface of the ground. The high-frequency resistance of the earth connections was of the order of 20 ohms.

Three modifications of this aerial system were tested: (*a*) unscreened, (*b*) screened but not earthed, and (*c*) screened and with the screen earthed at its ends and centre. The standard-wave error obtained from these tests was found to be of the order of 13° for all three types. The error reduction produced by the screen was therefore not detectable. The measurements, however, were not of a very high order of accuracy, owing to the difficulty encountered in making accurate simultaneous determinations of the angles of elevation and polarization of the transmitting aerial, together with its true and apparent azimuths, all of which were rapidly changing with the movements of the kite. It is probable that a definite error reduction was produced by the screen, but the factor can certainly not have been less than 0·75.

(*c*) *Calculation of Errors.*

The formula (3) may now be used to determine the theoretical values of the standard-wave error of those cases dealt with in the experiments to which it can be applied, namely the system (i) (see Fig 5) on short waves and the system described above on medium waves. The correlation between theory and experiment for the former is shown in Fig. 6. The agreement is probably as good as can be expected when account is taken of the uncertainty of the factors of effective height, soil conductivity, and position of the effective reflecting surface caused by the ground. In the medium-wave case the measured value was 13° and the calculated value 15°. In both cases the following assumptions were made: $\sigma = 1\cdot5 \times 10^8$, $\kappa = 15$, $h_e = \frac{1}{2}$ (actual height). The details of the calculation for the second case are given below, as an example of the method.

Actual height of aerial = 24 m; effective height = 12 m; $\lambda = 300$ m. Thus, taking the approximate relationship found in the Appendix,

$$\frac{Y}{Z} = \frac{1}{\sqrt[4]{[\kappa^2 + (4\sigma^2/f^2)]}} = 0\cdot058$$

and substituting in equation (3), we get

$$\tan \epsilon_s = 0\cdot33, \quad \text{or} \quad \epsilon_s = 18°.$$

This assumes equality of phase between the e.m.f.'s due to Y and to Z, whereas, in the tests, the phase difference

* *Journal I.E.E.*, 1930, vol. 68, p. 1058.

was approximately 45°. By correcting for this, the value $\epsilon = 15°$ is obtained.

Fair agreement between theory and experiment having thus been found in both cases, the expression (3) can be used, with some confidence, to calculate the standard-wave error of an unscreened "U" system of any given dimensions, and Table 2 gives some calculated values of this factor for various wavelengths and soil conductivities with systems of appropriate dimensions.

type). An impedance Z is connected across the horizontal members at their centre. This impedance may consist of an inductance and variable condenser in a tuned system, or it may be a fixed inductance in the case of an untuned system. It is connected directly through a transformer to the first stage of the amplifier in the case of the rotating-pair type, while in the case of the 4-aerial type it constitutes one of the field coils of the radiogoniometer.

TABLE 2.

*Standard-wave Errors of "U" System (unscreened).**

Wavelength	Total height of aerial	Standard-wave error (degrees)				
		Sea ($\sigma = 10^{10}$)	Very wet ground ($\sigma = 5 \times 10^8$)	Normal soil ($\sigma = 10^8$)	Dry ground ($\sigma = 5 \times 10^7$)	Very dry ground ($\sigma = 10^7$)
metres	metres					
10 000	50	6	18	25	33	37
1 000	30	3	13	21	24	34
300	30	2	8	14	17	27
100	15	1	5	10	13	21
30	5	3	12	21	25	34
10	1·5	7	21	28	32	38

* Standard-wave error for loop = 35°.

In this table the effect of the X component has been taken into account, where necessary.

The analysis has shown that, as long as the spacing s is kept small compared with the wavelength, it does not affect the standard-wave error. This dimension is therefore not included in the table. If the spacing is too large for the approximation of equation (1) to be made, however, the tendency will be for the error to be increased.

The figures given in Table 2 for the standard-wave error of the unscreened "U" system may, as a first approximation, be converted to represent those for the screened "U" system by multiplying them by $\frac{2}{3}$, which is the mean value of the reduction factors obtained from the experiments described in Section 2(b).

(3) THE ELEVATED OR "H" TYPE OF AERIAL SYSTEM.

In this type the vertical spaced aerials are in the form of dipoles, of which the lower halves are clear of the ground. The horizontal leads coupling them together are situated at the centre of the system and therefore at a height above the ground of at least half the total height of the aerials. The simplest form is that in which the horizontal members are directly connected to the aerials as shown in Fig. 7, and it is this type which has been investigated. Other forms, however, are possible, e.g. a system in which transformers are inserted between the aerials and the connecting members. The advantages and disadvantages of such modifications will be more clearly appreciated after all the types have been dealt with.

The pair of aerials illustrated in Fig. 7(a) may rotate about a vertical axis (rotating-pair type), or it may form half of a 4-fixed-aerial system, being thus analogous to one of the closed loops in a Bellini–Tosi system (4-aerial

Though the system is entirely free from errors when sufficiently remote from the earth's surface, the proximity to the ground in which it normally operates introduces an asymmetry. This asymmetry gives rise to errors when abnormally-polarized downcoming waves are being received. Unlike that for the "U" system, the calculation of the error due to this effect is very complicated.

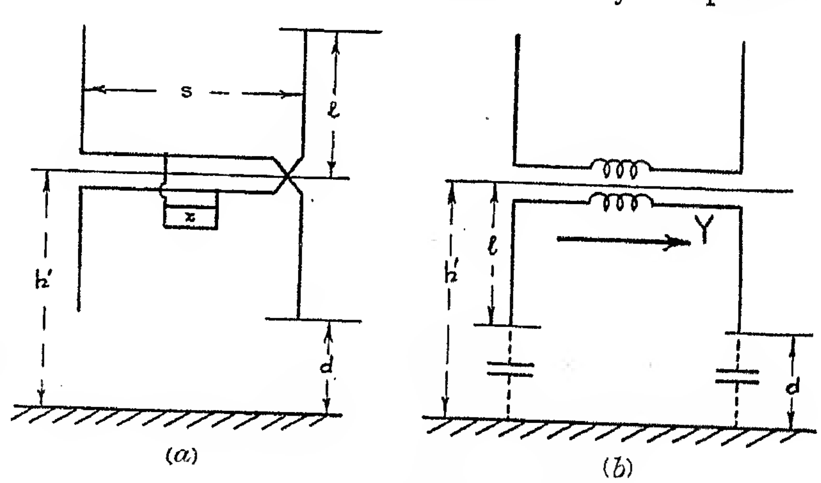


FIG. 7.—Elevated type.
(a) Simplified form.
(b) Nature of asymmetry effect.

For this reason there was a considerable need for an experimental investigation of the system which could form the basis of an empirical formula for calculating its performance.

(a) *Theory.*

It is possible to redraw the circuit, for the purpose only of examining the asymmetry effect, in the manner shown in Fig. 7(b), where, for the sake of simplicity, the upper and lower halves of the aerials are separated by the device of showing the loading inductance, Z,

twice over. It will at once be clear that a horizontal field, such as Y, acting on the horizontal conductors of the system, will have an effect on the circuit containing the lower halves of the dipoles which will differ from that on the circuit containing the upper halves. For the lower circuit approaches to a closed loop, while the upper, having only three sides, is an open circuit. This will not be affected in any way by enclosing the horizontal members in a metallic tube, as the effect of the screen is merely to transfer the action of the field from the horizontal to the vertical members. It is only when the height of the system above the ground becomes so great that the capacitance to earth of the upper and lower halves is sensibly the same that complete equality of action is obtained and the system becomes free from error.

Apart from the factor of asymmetry thus introduced, the remaining factor governing the magnitude of the

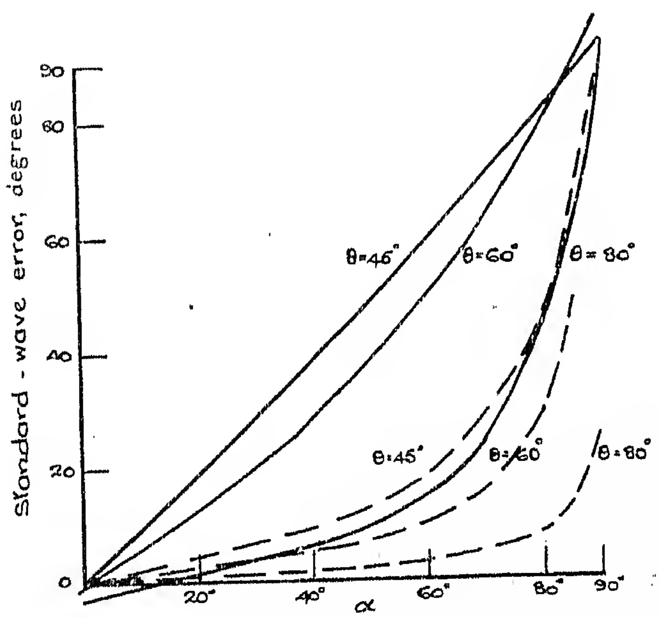


FIG. 8.—Elevated type: variation of error with α and θ.
———— Closed-loop direction-finder.
— — — Adcock direction-finder.

error occurring with the elevated type will be the same as for the " U " type, so that we may write:—

$$\tan \epsilon = \frac{Y_h}{Z_h} \cdot \frac{\lambda}{h_e} \cdot \frac{1}{2\pi \sin \theta} F \quad . \quad . \quad . \quad . \quad (4)$$

where Y_h/Z_h is the value of Y/Z at the height above ground of the centre of the system, and where F may be defined as the " asymmetry factor " of the system. F will depend on the dimensions and height above the ground of the latter. The calculation of this factor is very difficult, but the experimental results may be employed as a means of obtaining an empirical curve for it.

(b) *Experiments.*

The principal tests were carried out on an apparatus of the rotating-pair type; some of these tests have already been described.* The apparatus consisted, in this case, of two vertical dipoles of total length 3 m, spaced 3 m apart on a rotating horizontal tube containing the horizontal connecting leads. At the centre of the tube the leads are brought out and connected to the primary of a transformer which rotates with the arm. The secondary of this transformer is connected to the amplifier via another coupling coil. The height of the horizontal arm above the ground is normally 1·8 m, and consequently the ends of the lower half-dipoles are 0·3 m above the ground. The wavelength range over which the system has been tested is from 20 to 130 m.

The tests were made with locally-produced downcoming waves from the same elevated transmitter as was used in the tests of the " U " system (see page 427). They consisted, firstly, in measuring the error produced on various wavelengths with waves of different angles of incidence and degrees of polarization, secondly in measuring the effect of altering the height of the apparatus above the ground, and thirdly in measurements of the change in error caused by making the upper and lower

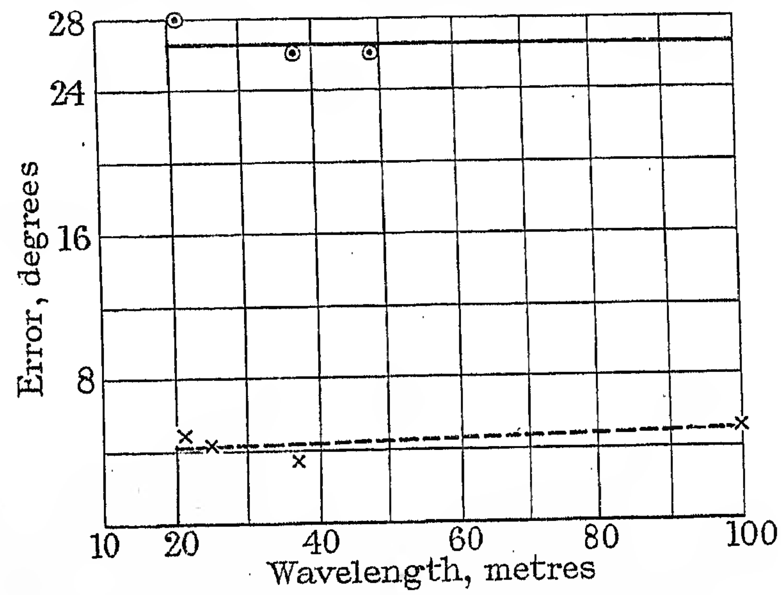


FIG. 9.—Variation of error with wavelength.

Elevated Adcock (calculated) — — — —
Elevated Adcock (observed values) × ×
Coil direction-finder (calculated) ————
Coil direction-finder (observed values) ⊙ ⊙
Angle of incidence = 60°. Angle of polarization = 45°.

halves of the dipoles of unequal length, which forms a means of restoring electrical symmetry to the system.

(i) *Effect of varying* λ, θ, *and polarization.*—The first series of experiments, carried out on a wavelength of 24 m, produced results of which Fig. 8 is a typical example. It shows the error plotted against the angle of polarization of the downcoming ray for angles of incidence of 80°, 70°, 60°, and 45° (angle measured from the vertical), while on the same graph are plotted the errors for a loop direction-finder. These curves conform to theory in that the error is proportional to the tangent of the angle of polarization.

Further tests were made with the values of the angles of incidence and polarization both fixed at 60° and 45° respectively, to determine the variation of the error with wavelength. These results are shown in Fig. 9.

(ii) *Effect of varying height above ground.*—In the second series of tests the effect of altering the height h' (Fig. 7) of the system above ground was examined. The results are shown in Fig. 11(*a*), the error being plotted against the ratio l/d, where l is the length of half the dipole and d is the height above the ground of the bottom of the lower dipole. The error is seen to decrease rapidly as the

* *Journal I.E.E.*, 1930, vol. 68, p. 1052.

height is increased, falling to 1° for a height of 4 m, as compared with 6° at a height of 2 m. These results can, by making certain assumptions, be generalized into a basis for predicting the standard-wave error of an elevated aerial of any dimensions, wavelength, height above ground, and soil conductivity.

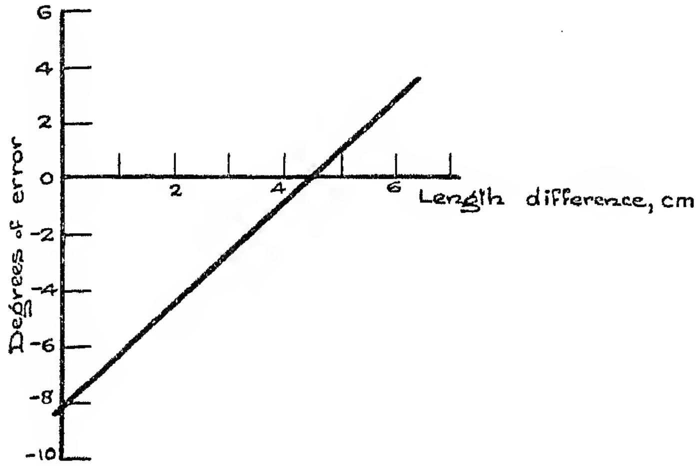


FIG. 10.—Effect of altering length of lower limbs: elevated type.

(iii) *Effect of shortening lower limbs.*—The third series of tests consisted in making error measurements with the upper and lower halves of the dipole of unequal length. It had been noticed that the error could be altered in this way, and in fact could be reversed in sign, if the tops of the dipoles were longer than the bottoms. It was therefore clear that, other things remaining constant, a certain difference of length could be found which would give zero error. Tests made on a wavelength of 24 m with an angle of incidence of 60° showed that the balanced arrangement was achieved when the length-difference was 5 cm. When, however, the angle of incidence was varied it was found that the critical length-difference was altered. The possibility of utilizing this as a perfect method of balancing the aerial system is therefore ruled out, but as a partial remedy it could probably be introduced in practice with considerable advantage. The variation of error with length-difference for one particular angle of incidence is shown in Fig. 10.

(c) *Calculation of Errors.*

The results of the second series of tests, combined with equation (4), may now be used for the purpose of obtaining a general formula for the calculation of the standard-wave error of the elevated type of aerial for various wavelengths, dimensions, and soil constants, which might occur in practice. Fig. 11(*a*) shows the experimental relationship obtained between ϵ and l/d in these tests and, by substituting in (4) the experimental values of ϵ together with the appropriate numerical values of the other known or calculable terms in the expression (viz. Y_h/Z_h and λ/h_e), the derived curve (Fig. 11*b*) connecting F with l/d has been obtained. The relationship thus obtained between F and l/d can now be used in conjunction with equation (1) to predict

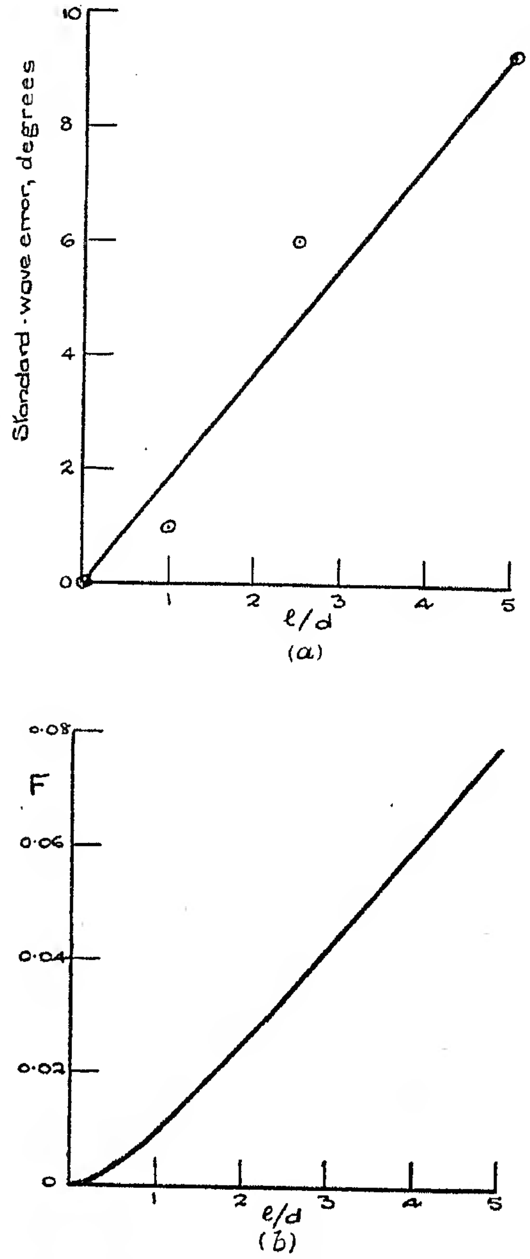


FIG. 11.—Elevated type: effect of varying height above ground.

TABLE 3.

Standard-wave Errors of Elevated Type for Practical Cases.

λ	l	h'	l/d	Standard-wave error (degrees)			
				$\sigma = 10^{10}$	$\sigma = 5 \times 10^8$	$\sigma = 10^8$	$\sigma = 10^7$
30	2	3	1·5	3	3·25	3·33	5
300	8	13	1·5	2	2·25	2·5	3·5
1 000	12	16	3	4	4·5	5·25	8
10 000	15	20	3	4·25	5·5	7·5	15

values of the standard-wave error for any given elevated system. The assumption underlying this reasoning, that the relationship between F and l/d is entirely independent of such variables as wavelength and ground constant, is admittedly not axiomatic, and further experiments would be necessary to substantiate this hypothesis. Since, however, it appears to be the most rational assumption available, it has been adopted in the calculation of the predicted standard-wave errors for the various values given in Table 3. It will be noted, on comparing the Table with the corresponding Tables (2 and 4) for the U and coupled types, that changes in ground conductivity have comparatively little effect on the standard-wave error of the system. This is because the horizontal field, Y, which produces the errors is, in this case, at some distance above the ground. Its magnitude, as will be seen from the Appendix, is therefore chiefly governed by the phase angle between the incident and reflected rays due to path difference, rather than by the conductivity of the ground.

(4) The Coupled Type of Aerial System.

(a) Theory.

The essential feature of this type is that there is no conductive connection between the two spaced aerials

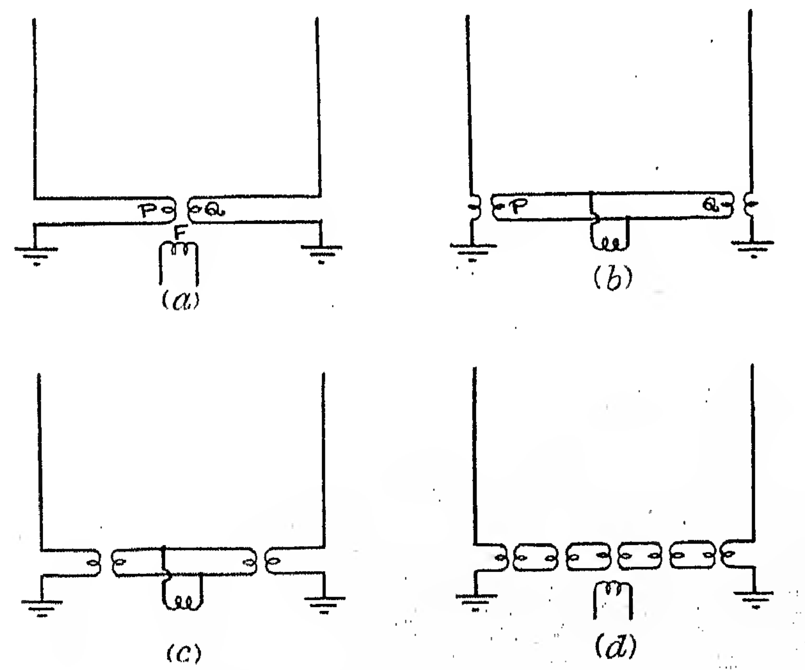


Fig. 12.—Variations of coupled type.

which form the directional pair, the horizontal feeders being connected to the aerials through transformers by magnetic coupling through mutual inductances. The type varies according to the number and disposition of these transformers in each feeder, and some variations are shown in Fig. 12. The object of the transformers is to increase the impedance between the horizontal and vertical members so as to reduce that current flowing into the aerials which is produced by the action of the horizontally-polarized wave on the feeder. To make this impedance high, either the capacitance between the transformer windings must be made as low as possible or a large number of transformers must be used, in the manner of the links of a chain.

In all these systems we have the wanted pick-up produced by the differential action of the vertical electrical field on each of the vertical aerials forming a pair, and the unwanted pick-up produced by the action of the horizontal field on the horizontal members. The action of the former is easily calculable, while that of the latter is greatly complicated by the fact that the network in which the resulting currents flow includes the earth on the one hand and the open circuit between the aerials on the other. This complication has made it impossible to predict theoretically the behaviour of a coupled system without making simplifying assumptions which cannot safely be accepted in the absence of some experimental check. A description of some of these experiments is now given.

(b) Experiments.

(i) *Medium waves.*—The first coupled system to be tested was designed for medium-wave working; it is illustrated in Figs. 13 and 14. The details of the system are as follows: height of aerial (h), 24 m; spacing between aerials, 15 m; length of centre link (l), 10 m; wavelength range, 70 to 1 000 m; transformer primary inductance, 40 μH; transformer secondary inductance, 130 μH; transformer mutual inductance, 32 μH; capacitance between primary and secondary, 40 $\mu\mu$F; goniometer field-coil inductance, 90 μH; goniometer search-coil

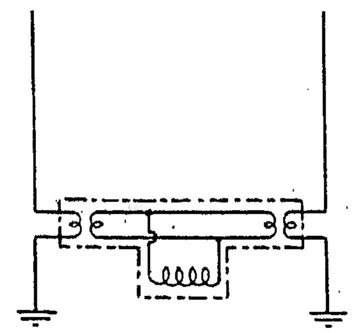


Fig. 13.—Circuit diagram of medium-wave coupled type.

inductance, 195 μH; goniometer mutual inductance, 37·5 μH. The circuit diagram is given in Fig. 13, and a sketch of the aerial system in Fig. 14.

Measurements of the standard-wave error of the system were made by means of a kite-elevated transmitter, by the method described in an earlier paper,* on a wavelength of 300 m, and this error was found to be approximately 8°.

The problem now arises as to the exact cause of this error. A consideration of the problem shows that there are several possible causes, namely, interaction between the isolated horizontal portions and the vertical aerials, unbalanced currents induced in the aerial circuits by the "lumped" capacitance coupling of the transformer windings, unbalanced currents induced in the secondary circuit by currents flowing across the inter-transformer capacitance, and direct pick-up in the closed circuit containing the transformer secondaries. Of these the possibility of the last occurring is eliminated by the use of the screening tubes, and the possibility of unbalanced currents in the secondary circuit is eliminated by the fact that the presence or absence of an inter-winding screen (connected to the tube) was found to have no

* *Loc. cit.*

effect. It was decided that the quickest way to find out which of the remaining two was the cause of the error, or how each contributed to it, was to make an experimental investigation. The method applied to this case may be called a "local injection method."

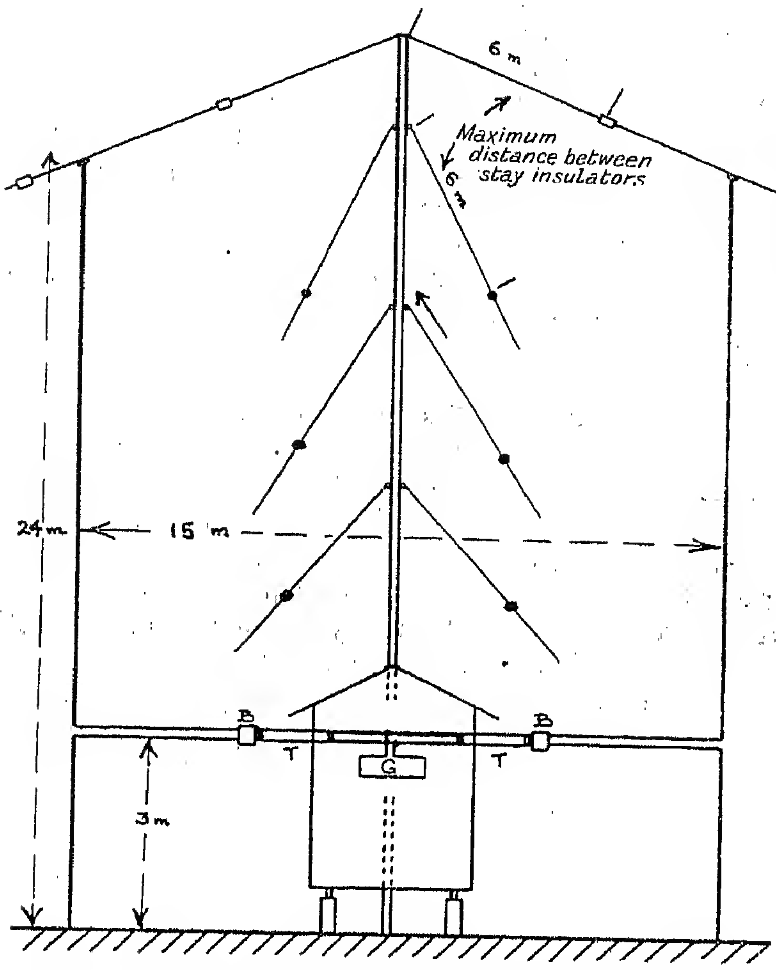


FIG. 14.—Medium-wave coupled type: sketch of aerial arrangement.

B. Box containing transformer.
G. Goniometer.
T. Brass tube.

(ii) *Local injection method.*—The general arrangement is shown in Fig. 15A. The method is to generate a localized e.m.f. of known value in the horizontal members to simulate the action of the horizontal electric field of a wave, and to measure the circulating current produced

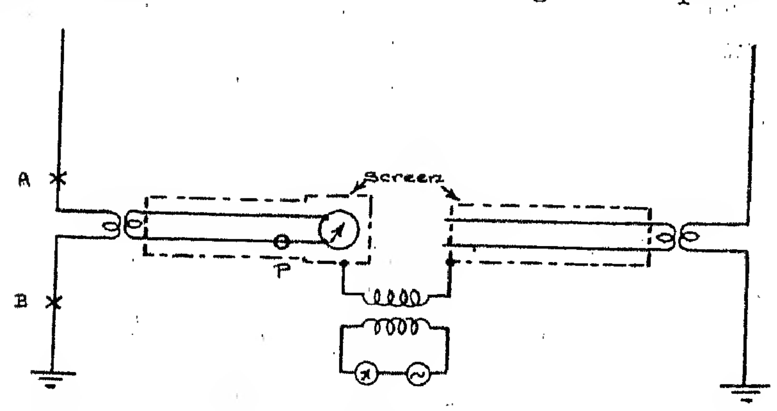


FIG. 15A.—Local injection method of determining standard-wave error.

by this e.m.f. in the secondary circuit of the system, i.e. the current which determines the apparent bearing. It will be realized that this secondary current is produced by a primary current flowing in the aerial circuit, the primary current being in its turn produced by the coupling of the horizontal limbs to the vertical aerial, resulting from the inter-winding capacitance of the transformers.

An alternative method, which was employed as a check, is to measure the primary current at some point in the aerial circuit such as A or B in Fig. 15A. The disadvantage of this is that the current is not uniform and is found, in practice, to be widely different at these two points. To estimate the effective primary current from such measurements it is necessary to make assumptions with regard to its distribution which are not easy to check. The overall method of measuring the secondary current resulting from the locally induced e.m.f. has the advantage that it provides an accurate measure of the equivalent primary current without any assumptions having to be made.

A value for this secondary current in terms of the local e.m.f. induced having been obtained, it is possible to relate it to the current which would be induced in the system by the vertical field (Z) of a "standard wave" of such intensity that its horizontal field produces an effect equivalent to that of the local source. The ratio of these two currents is equal to $\tan \epsilon$, where ϵ is the standard-wave error, and it can be shown that

$$\tan \epsilon = \frac{Y}{Z}\frac{\lambda}{h_e}\frac{L_s}{M}\frac{\sqrt{2}\,I_sZ_A}{E_h} \quad . \quad . \quad . \quad . \quad (5)$$

where Y/Z = ratio (horizontal field)/(vertical field) of

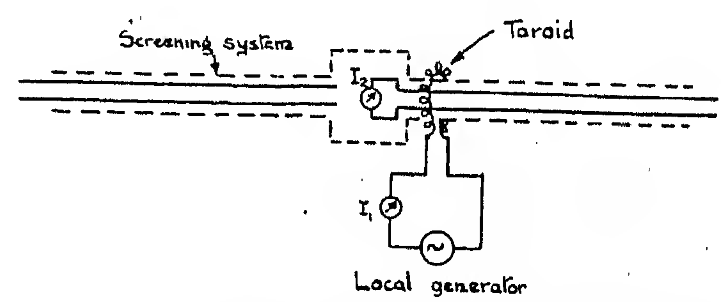


FIG. 15B.—Alternative method of injecting local e.m.f. into horizontal members.

standard wave, h_e = effective height of aerial, L_s = inductance of transformer secondary, M = mutual inductance of transformer, I_s = secondary current as measured, E_h = locally-induced e.m.f., and Z_A = aerial-circuit impedance. The method of inducing the local e.m.f. and of measuring the resultant secondary current may be most readily grasped by a reference to Fig. 15A. It will be noted that the e.m.f. is induced in the outer screening tube surrounding the secondary circuit, as it is here that the Y component of the field must act.

The standard-wave error determined was found to be approximately 9°, and this value is in agreement with the elevated-source test (see page 427). When the capacitance coupling between the primary and secondary circuits was reduced to one-quarter of its original value, however, the standard-wave error as determined by (5) was found to decrease to 2°, i.e. approximately in the same ratio as the capacitance reduction. It therefore appeared fairly certain from this that capacitance coupling between primary and secondary was the main cause of the error which had been found by actual measurement.

(c) Calculation of Errors.

Since it was now apparent that the lumped capacitance between the transformer windings was the only factor of importance in determining the amount of error to which the system was subject, it was possible to calculate the standard-wave error in the following manner.

Considering only a single aerial of the pair, assume that the whole of the current passing through the mutual capacitance (C_T) of the transformer traverses the whole of the transformer primary. In this case we have, for the aerial current due to the horizontal field,

$$I_y = \tfrac{1}{2} Y s \omega C_T$$

and, for the effective or differential component of the aerial current due to the vertical component,

$$I_z = \frac{Zh_e}{Z_A} \sin \frac{\pi s \sin \theta}{\lambda}$$

where Z_A is the aerial impedance, and θ the angle of incidence. Thus I_z is half the circulating current for the pair of aerials. The error is therefore given by

$$\tan \epsilon = \frac{Y s \omega C_T Z_A}{2 Z h_e \sin \left(\frac{\pi s \sin \theta}{\lambda}\right)} \quad . \quad . \quad . \quad (6)$$

If it is a standard wave, $\theta = 45°$, and, assuming that both ϵ and $\pi s/\lambda$ are small, we have approximately

$$\epsilon = 0 \cdot 225 \frac{Y}{Z} \frac{\lambda}{h_e} Z_A \omega C_T \quad . \quad . \quad . \quad . \quad (7)$$

The factor Y/Z is determined by the method given in the Appendix, provided the ground constants are known. It increases approximately with the square root of the frequency and also with the height of the horizontal members above the ground.

The aerial impedance, Z_A, is calculated in the usual way from a knowledge of the total capacitance and inductance of the aerial circuit; at frequencies less than the resonant frequency of the aerial it varies inversely as the frequency, and at frequencies greater than resonance directly as the frequency. Hence it appears that at frequencies considerably below the natural frequency of the aerial circuit the standard-wave error decreases with $\sqrt{f}$, i.e. the longer the waves the less the standard-wave error, but that at frequencies greater than the natural frequency the standard-wave error increases as $f^{3/2}$.

It will also be noted that the aerial spacing s does not affect ϵ as long as $\pi s/\lambda$ is small. When this becomes large, however, the vertical pick-up does not increase so fast with s as the horizontal pick-up, so that ϵ then tends to increase with s. It will further be noted that ϵ is directly proportional to the factor λ/h_e.

It will be of interest to calculate some values of this factor for the coupled system, the details of which have been given on page 433, and for which the various dimensions and constants have all been measured or calculated. Thus $h_e = 12$ m, $C_T = 11 \cdot 3\ \mu\mu$F, $Z_A = 640$ ohms, and, for $\lambda = 300$ m, $Y/Z = 0 \cdot 12$ at height of horizontal leads (3 m). Then $\lambda/h_e = 25$, $\omega C_T = 7 \cdot 1 \times 10^{-5}$, and hence $Z_A \omega C_T = 0 \cdot 045$. Hence

$$\begin{aligned} \epsilon &= 0 \cdot 25 \times 0 \cdot 12 \times 25 \times 0 \cdot 045 \\ &= 0 \cdot 035. \end{aligned}$$

Thus the standard-wave error is about 2°, and this agrees with the experimentally determined value. For the same system with capacitance between the primary and secondary circuits 4 times as large, we find the standard-wave error is approximately 7°. It was with this arrangement and on approximately this wavelength that the kite tests were made. A reference to these experiments will show that there is agreement between the calculated and experimental values.

In Table 4 the values of standard-wave error for the coupled type of aerial have been calculated for various other wavelengths and soil conductivities, taking the most favourable values for transformer constants which may be obtainable in practice.

TABLE 4.

*Standard-wave Errors of Coupled Type of Aerial System.**

Wavelength	Total height of aerial	Z_A	C_T	Standard-wave error (degrees)			
				Sea ($\sigma = 10^{10}$)	Very wet ground ($\sigma = 5 \times 10^8$)	Normal soil ($\sigma = 10^8$)	Very dry ground ($\sigma = 10^7$)
metres	metres	ohms	farad				
10 000	50	1 000	10×10^{-12}	0·02	0·1	0·15	0·4
1 000	30	1 000	10×10^{-12}	0·1	0·3	0·7	2·2
300	30	1 600	10×10^{-12}	0·2	0·9	2·0	6·4
100	15	1 000	5×10^{-12}	0·2	1·0	2·2	6·9
30	5	1 000	3×10^{-12}	0·7	3·2	7·2	21·8

* Calculated from equation (7).

(5) THE BALANCED AERIAL SYSTEM.

(a) Description, and Early Experiments.

This modification of the spaced-aerial direction-finder is illustrated in Fig. 16(*a*). It will be seen that it consists of two vertical aerials connected by twin horizontal leads in opposition across a common inductance or impedance which may form the field coil of the radio-

goniometer. Balancing impedances Z_B are inserted in the earth leads and these impedances are adjusted to have such a value that, assuming the aerial system to be divided into two as in Fig. 16(*b*), the bottom half has the same impedance as the top half. The system under working conditions, and methods of determining experimentally the value of the balancing impedance, have already been described.* In this case the impedances were condensers, and therefore the balancing held good strictly at one wavelength only. It was found, however, that, over the wavelength range for which the system was used, the balancing held as a first approximation and that further accuracy would have been without advantage owing to other second-order factors which then became of dominating importance.

This apparatus was tested by means of the kite for its polarization errors, and the standard-wave error was found to be of the order of 8°. The dimensions of the system were as follows: aerial height, 25 m; aerial spacing, 16·5 m; wavelength range, 200–1 000 m; height of horizontal members above ground, 3·5 m;

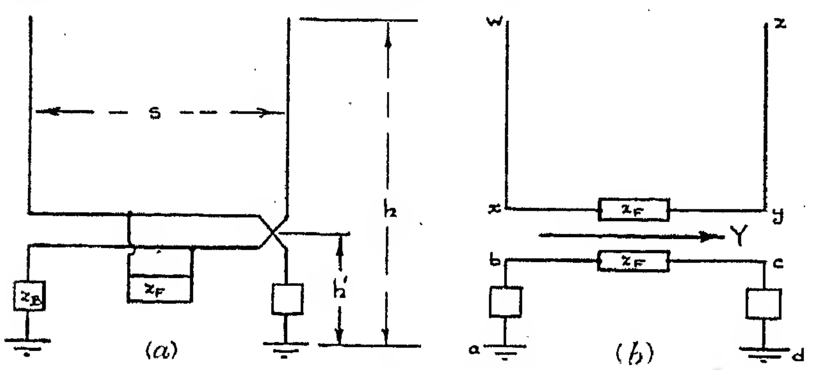


FIG. 16.—Balanced system.

value of balancing condensers, 120 $\mu\mu$F; and standard-wave error, 8° (on 300 m). The aerial system was untuned.

(*b*) *Theory.*

The theoretical aspects of this system have been discussed in an earlier paper.† In a perfectly balanced system the current set up by a given field Y in the lower circuit *abcd* (Fig. 16*b*) would be equal in phase and magnitude to the current set up in the upper circuit *wxyz*. This would involve equalization of the impedances of these two circuits with respect to an e.m.f. induced in each circuit. Exact equalization of impedance over a frequency band of practical width would involve substituting for condensers composite impedances of inductance and capacitance. Further, since the inductance and capacitance of the aerial are distributed, while these quantities in the balancing impedances are "lumped," it is probable that even then exact balancing would not be attainable.

Equalization of e.m.f., though at first sight already achieved, on closer consideration is seen not to be present. This was pointed out in the paper referred to above. It was then shown that a fundamental asymmetry of the two circuits is introduced by the fact that the lower halves are connected to earth. This can be seen from several aspects. In the first place the lower half may be regarded as a closed loop, the e.m.f. in which is calculated from the total magnetic field linking it. Though for a perfectly conducting earth this will be found to be the same as the e.m.f. induced by the single electric force acting on the upper limb, in the case of an imperfectly conducting earth the finite resistivity of the earth introduces a differential effect which cannot be neglected.

Another way is to consider the electric forces only. That inducing current in the top circuit consists of the resultant field Y' at the height h' above the ground, while that inducing current round the bottom circuit is the difference between Y' and Y, the electric force at ground level.

Yet a third way is to argue that (apart from the e.m.f. induced in limb *ba*) the earth currents parallel to and set up by the horizontal field Y are partly shunted through the lower circuit containing the limb, there being no equivalent current component in the top half. All are doubtless aspects of the same phenomenon, but all point to a fundamental limit to the principle of balancing.

It appears from the first two of these rough modes of looking at the problem that bringing the horizontal member as near as possible to ground level should greatly improve the exactitude of balancing attainable. A limit to this is set by the capacitance to earth of the limbs which is thereby introduced; this would result in an uneven current distribution in the members of the pair, consequently setting up a new source of error. As already pointed out, it appears worth while to study, say by the local injection method, the smallest possible value of h' at which successful balancing could be achieved, and then to investigate, by means of the elevated transmitter, the standard-wave error of the system for different wavelengths and angles of incidence.

Thus the balanced system might well repay closer study. Only a separate series of experiments by the elevated-transmitter method combined with the local injection method of balancing (see Section 4), and alternatively or in addition a complete mathematical investigation of the problem, can settle to what extent the art of balancing can be carried out successfully. For these reasons the calculation of the standard-wave error of a balanced system, or the design of such a system for a given standard-wave error, is not possible. If the balancing be assumed perfect then the standard-wave error is zero, whereas, as shown above, the nature and extent of departure from perfection cannot at present be determined. All that can at present be said is that experiments on medium and short waves show that, with due precautions, a system having a standard-wave error of the order of 8° can be obtained.

It is possible that further study will reveal that this figure can be considerably improved upon without difficulty. If so, the system will be a valuable one, as it possesses the advantage of having a high pick-up factor over a relatively large wavelength range.

(6) COMBINATION OF BALANCED AND COUPLED AERIAL SYSTEMS.

A combination of the balanced and coupled systems is shown in Fig. 17. It is the outcome of an extension of the experiments with the local injection method,

* *Journal I.E.E.*, 1930, vol. 68, p. 1052.

† *Loc. cit.*, p. 13.

which showed how the horizontal pick-up currents flowing through the inter-winding capacitance to aerial and earth could be equalized by means of a series condenser in the earth lead. The experiment further showed that the resultant current flowing through the primary winding could be reduced in this way to a value too small to be measured. This probably reduces the error-producing factor of inter-winding leakage to the level of other error-producing factors hitherto ignored. It might at first appear that, by balancing in this way, tightly coupled transformers of high inter-winding capacitance could be used, giving an increased pick-up factor, but in doing this the risk would be run of encountering the balancing difficulties set forth above, which arise from the fact that the art of attaining perfect balancing has not yet been sufficiently studied and may be impossible. It appears rather that the combination of the two imperfect systems in the most perfect forms obtainable in practice will produce the best results.

Calculation of standard-wave error is not possible for such a system, owing to lack of knowledge of balancing phenomena. All that can be said is that a "balanced coupled" system will be many times better in this respect than the corresponding unbalanced system,

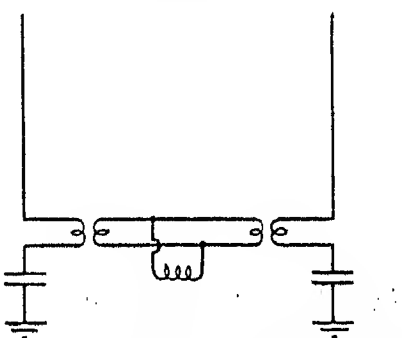


FIG. 17.—Balanced-coupled system.

for which the standard-wave error can be calculated by the method already given. Thus an arrangement of the coupled chain system (see page 433) had its standard-wave error reduced from 8° to less than 1° by balancing in this way.

A more detailed study of the system will enable it to be ascertained to what degree of exactitude the balancing need be carried. As far as experiments have gone, everything tends to show that, given an unbalanced system in which standard-wave error has been reduced as low as possible by proper transformer design, the insertion of a fixed condenser in the earth lead of a value roughly equal to that of the upper half of the aerial will reduce the standard-wave error to the order of one-tenth of its previous value, provided that the wavelength is greater than the natural wavelength of the aerial system. No further refinement of balancing is necessary.

(7) PICK-UP FACTOR.

The pick-up factor of a receiving-aerial system has already been defined as the ratio of the output potential difference (at specified output terminals) to the strength of the field acting on the aerial system, assuming that the latter refers to that component of the electromagnetic field which the aerial is designed to receive. The field strength must be specified in the same unit of potential difference per unit length as that employed to specify the output potential difference. Thus if a vertical aerial of effective height h_e is acted upon by a vertical field of E volts per metre and the voltage resulting therefrom on the grid of the first valve is V volts, the pick-up factor is given by $p = V/E$. It will be observed that p is not a numerical ratio but has the dimension of length. This is unavoidable, since the field strength is a potential gradient and the output is a potential difference. To obtain a numerical ratio it would be necessary to express the output potential difference in terms of the e.m.f. induced in the aerial (i.e. the product of field strength and effective height). The quantity so obtained would then have the advantage of being independent of the units used in the calculation, but would, unfortunately, lose most of its

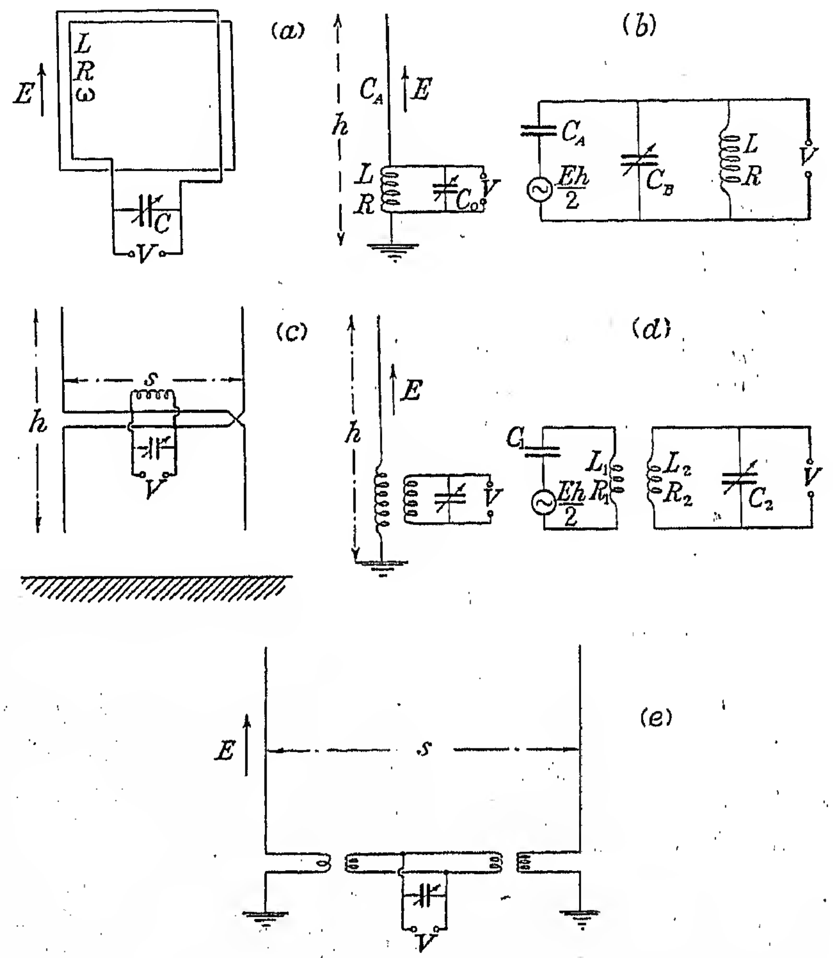


FIG. 18.

value as a figure of comparison of the relative effectiveness of the various types of aerial systems. All confusion is avoided, however, if it is remembered that, in calculating p, the effective height must be expressed in the same unit of length as that employed to specify the field strength. This will then be the unit in which p itself is expressed.

Pick-up Factor of Coil Aerial.

The case of a simple coil aerial (Fig. 18*a*) is of interest as constituting a standard of comparison for the other systems. The loop, of area-turns an, is assumed to be acted upon by a vertically-polarized ground wave of field strength E volts per metre and to be aligned in the direction of maximum pick-up. The output voltage (V) is measured across the tuning condenser. The capacitance to tune the system is C, which includes the self-

capacitance of the loop and the capacitance of the output leads, so that

$$V = I\omega L = -I/(\omega C)$$

The e.m.f., e, induced in the loop is given by

$$e = (2\pi an/\lambda)E$$

and $$I = e/R$$

Also $$V = [2\pi an\omega L/(\lambda R)]E$$

$$\therefore p = \frac{V}{E} = \frac{2\pi an}{\lambda}\frac{\omega L}{R} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If the loop is designed for maximum efficiency for the particular wavelength under consideration (i.e. so that C is only a little bigger than the self-capacitance of the loop) the value of $\omega L/R$ will not vary greatly with wavelength for a given area of loop on any wavelength between, say, 30 and 10 000 m. Further, for a constant area and tuning capacitance, the number of turns will be approximately proportional to the wavelength. It therefore appears that the pick-up factor of a coil aerial of given area and decrement will be of the same order for all wavelengths provided that the number of turns has approximately the optimum value for the wavelength in question.

Thus if $a = 1$ m^2, $\lambda = 300$ m, and $\omega L/R = 100$, the value of n to give the correct inductance for tuning with 200 $\mu\mu$F is 7, which gives, from equation (8), $p = 14 \cdot 6$. The pick-up factor of a 1-m square multi-turn coil aerial on its most efficient wavelength (minimum tuning capacitance) is thus of the order of 15 m for a very large range of wavelengths. A coil designed for a given wavelength will, however, not retain a constant pick-up factor when the wavelength is altered. Here we may still assume, however, that the value of $\omega L/R$ will not vary greatly, so that the pick-up factor will vary inversely with the wavelength. Thus a loop designed to have a pick-up factor of 10 m on a wavelength of 300 m will have a pick-up factor of 5 m on a wavelength of 600 m.

Pick-up Factor of Single Vertical Aerial.

The circuit as shown in Fig. 18(*b*) is assumed to consist of a vertical earthed aerial of effective height h_e tuned by means of a series inductance L shunted by a variable condenser C_B. The aerial is supposed to be under the influence of a vertical electric field of strength E volts per metre, and the output voltage (V) is measured across the terminals of the tuning condenser. The equivalent simplified circuit is also shown in Fig. 18(*b*), the capacitance of the aerial being represented by a condenser C_A and its inductance and resistance being neglected. The condenser C_B represents the combined capacitance of the tuning condenser, the leads, and the self-capacitance of the inductance. If the composite impedance formed by C_B, L, and R, be denoted by Z', the total current flowing through the generator in the equivalent circuit is given by

$$I_A = Eh_e/Z_e$$

where $Z_e = Z' + Z_A$

$$= \frac{R + j\omega L}{1 - \omega^2 C_B L + j\omega C_B R} - \frac{j}{\omega C_A}$$

$$= \frac{\omega R(C_A + C_B) + j(\omega^2 C_A L + \omega^2 C_B L - 1)}{\omega C_A(1 - \omega^2 C_B L + j\omega R C_B)}$$

Under the condition of resonance for which p is defined, Z_e must have its minimum value, which will occur very approximately when the imaginary part of the numerator is zero; that is, when

$$\omega^2 C_A L = 1 - \omega^2 C_B L$$

whence $$Z_e = \frac{R(C_A + C_B)}{C_A(\omega^2 C_A L + j\omega C_B R)}$$

The pick-up factor (p) is defined as

$$p = V/E = I_A Z'/E$$
$$= Z' h_e/Z_e$$

Substituting for Z' and Z_e, we have

$$p = \frac{h_e(R + j\omega L)}{1 - \omega^2 C_B L + j\omega C_B R} \cdot \frac{C_A(\omega^2 C_A L + j\omega C_B R)}{R(C_A + C_B)}$$
$$= \frac{C_A(R + j\omega L)h_e}{R(C_A + C_B)}$$

since $\omega^2 C_A L = 1 - \omega^2 C_B L$.

If $\omega^2 L^2 >> R^2$,

$$p = \frac{C_A}{C_A + C_B}\frac{\omega L}{R}h_e = \frac{(\omega L/R)h_e}{1 + (C_B/C_A)} \quad . \quad . \quad . \quad (9)$$

The further possible simplification of this to the form

$$p = \frac{C_A}{(C_A + C_B)^2}\frac{h_e}{\omega R}$$

which would appear to show that p, for a given aerial system, increases with the wavelength is misleading, since it would be impossible to load the aerial indefinitely

TABLE 5.

Pick-up Factor of Single Aerial.

λ	h	C_A	C_B	p
30	3	12	100	16
100	10	60	150	142
300	25	160	200	560
1 000	25	160	300	440
10 000	25	160	500	300

without increasing C_B as a result of the self-capacitance of L, and also increasing R as a result of non-uniform current in L: the loading coil would in fact tend to become a rejector circuit. A better simplification can be made by considering equation (9) only to apply to inductances which have small self-capacitance compared with C_B, for which the value $\omega L/R$ will remain sensibly constant. Taking, then, for conformity with the case of the coil aerial, a constant value of $\omega L/R$ of 100, it appears that if $C_B >> C_A$ the pick-up factor is inversely proportional to C_B/C_A and proportional to h_e.

Table 5 gives the value of p for $\omega L/R = 100$, at various wavelengths, with suitable aerial heights and spacing.

Vertical Aerial with Coupled Circuit.

This case, with its equivalent circuit, is shown in Fig. 18(*d*), and its consideration leads naturally to that of the coupled type of spaced-aerial system. To correspond

with the coupled systems described above, the aerial circuit will be regarded as untuned, with natural frequency above that of the signal, and the condenser of the secondary circuit will be assumed to be adjusted for the maximum output. This condition is sometimes referred to as "partial" resonance with tuned secondary. It will further be assumed that the horizontal feeders are sufficiently short compared with the working wavelength for any transmission-line effect to be neglected.

Let h_e = effective height of aerial,
M = mutual inductance between primary and secondary,
E = vertical component of electric field,
I_1, I_2 = primary and secondary currents,
C_1, C_2 = primary and secondary capacitances,
L_1, L_2 = primary and secondary inductances,
R_1, R_2 = primary and secondary resistances,
Z_1, Z_2 = primary and secondary impedances,
Z_1', X_1', R_1' = effective impedance, reactance, and resistance of primary,
and V = output potential difference across C_2.

Then
$$I_1 = Eh_e/Z_1'$$
$$I_2 = M\omega I_1/Z_2 = M\omega Eh_e/(Z_1'Z_2)$$
$$V = I_2\omega L_2$$

So that, for the pick-up factor of the system, we have

$$p = V/E = M\omega^2 h_e L_2/(Z_1'Z_2) \quad . \quad . \quad . \quad (10)$$

where, under partial resonance conditions,

$$Z_1' = R_1' = R_1 + (M\omega/Z_2)^2 R_2, \quad \text{since } X_1' = 0,$$

and $$Z_2 = R_2 + jX_2$$

where $$X_2 = (M\omega/Z_1)^2 X_1.$$

The formula can be considerably simplified by making assumptions likely to be tenable in practice about the aerial system, but in this case it is probably better to make the approximation at the stage where actual dimensions have been assigned.

Elevated, Balanced, and "U" Types.

To obtain a figure of any value for purposes of comparison it is necessary to consider the simplest circuit arrangement only and, in spite of the fact that untuned aerials are employed in practically all the systems described, to assume resonance conditions. The figure for p thus obtained will be too high for such cases, but it can be assumed that, with tightly coupled goniometers, where V is measured across a tuned search coil, the reduction in p will not be great. Also, since this factor will be the same for all systems, the figure will be correct for purposes of comparison. With this simplification, shown in Fig. 18(*c*), it is a simple matter to pass from the case of the vertical aerial to that of a spaced-aerial system of the elevated, balanced, or "U" type. The equivalent circuit remains essentially the same, but the induced e.m.f. for a given field (assuming orientation in the maximum direction) is reduced in the ratio $2\sin(\pi s/\lambda) : 1$. Hence, following on from equation (9), we may write for each of these cases, if $s << \lambda$,

$$p = \frac{2\pi s h_e}{\lambda} \cdot \frac{\omega L/R}{1 + (C_B/C_A)} \quad . \quad . \quad . \quad (11)$$

The further possible simplification of equation (11) to

$$p = \frac{sh_e}{cR} \cdot \frac{C_A}{(C_A + C_B)^2}$$

though attractively concise, must be ignored as misleading for the reasons given for the similar simplification in the case of the single aerial, but the simplification of assuming $\omega L/R$ sensibly constant may be again adopted. This enables the values of p to be calculated for a series of practical cases, and these are given in Table 6, where $\omega L/R$ is taken as 100 throughout. The aerial heights are taken as great as possible within the limits imposed by either natural wavelength or practical constructional features. The spacing is taken as large as possible within the limits of octantal error,* natural wavelength, undue increase of C_B, and constructional convenience. The value taken for C_A assumes a single aerial, 0·2 cm in diameter. The value of C_B is such as to allow for the probable capacitance of the feeders plus a convenient minimum value of variable condenser for adjustment.

TABLE 6.

λ	h	s	Pick-up factors†			
			(*a*)	(*b*)	(*c*)	(*d*)
30	3	5	16	16	5·0	15 (1 turn)
100	10	8	140	70	—	19 (3 turns)
300	25	15	560	174	72	15 (7 turns)
1 000	25	30	440	84	—	13 (20 turns)
10 000	25	100	300	19	—	11 (180 turns)

† (*a*) Single vertical aerial, $\omega L/R = 100$. (*b*) Elevated, "U," and balanced systems, $\omega L/R = 100$. (*c*) Coupled system. (*d*) Coil aerial, 1 metre square, $\omega L/R = 100$.

It will be noted that artificially increasing C_A by using devices such as cage aerials, increases p in proportion. There is, however, a limitation to this on account of the fact that the natural wavelength of the aerial is thereby increased, and, by reducing the loading inductance permissible, indirectly affects $\omega L/R$.

Coupled Type.

The case of the coupled spaced-aerial system, as shown in Fig. 18(*e*), may be arrived at from equation (10) as before, by multiplying by the spacing factor, $2\pi s/\lambda$. Hence we have, for this type,

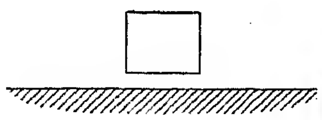
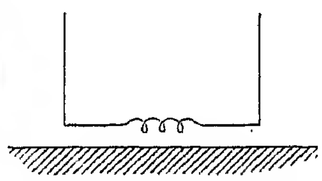
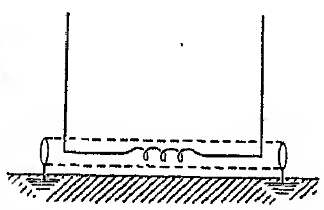
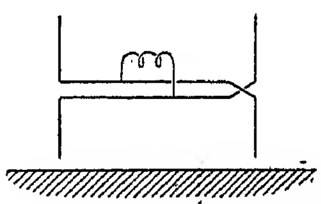
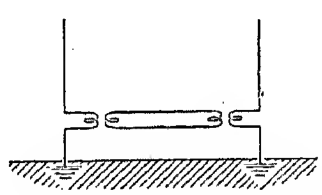
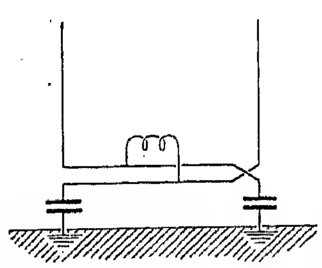
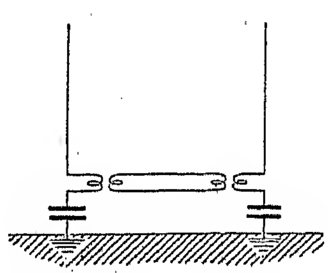
$$p = \frac{2\pi s h_e}{\lambda} \cdot \frac{M\omega^2 L_2}{Z_1'Z_2} \quad . \quad . \quad . \quad . \quad (12)$$

This formula has been applied to two cases closely resembling aerial systems actually experimented with at Slough, for which the appropriate values of the primary-

* An error in bearing of this type occurs when s is too large for the approximation $\sin(\pi s/\lambda) = \pi s/\lambda$ to be made.

TABLE 7.*

Comparison of Types Based on Experimental Measurements.

Type	Diagram	Dimensions h, h'† metres		Standard-wave error (degrees)	Pick-up factor, metres
Loop		Small compared with wavelength		35 (This value is not affected by changes in soil conductivity)	10 to 20 (1 metre square)
" U "		24	0·5	12	100 to 200
Screened " U "		24	0·5	6 (Reduction effect of screen taken is the most favourable of values found from all experiments)	—
Elevated or " H " type		24	15	2 (This value is not greatly affected by changes in soil conductivity)	100 to 200
Coupled		24	0·5	1 (Untuned aerial system. Inter-winding capacitance of transformer = 10 $\mu\mu$F)	50 to 100
Balanced		24	2·0	6 (Untuned aerial system)	100 to 200
Balanced-coupled		24	2·0	Less than 1 (Untuned aerial system)	50 to 100

* The value of the standard-wave error corresponds to a wavelength of 300 m and a soil conductivity of $1 \cdot 5 \times 10^8$ electrostatic units, and only holds for the aerial dimensions given. It will be realized from the matter contained in the paper that variations of any of the above quantities may very considerably alter the standard-wave error of the systems tabulated.

† h = height of aerial system, h' = height above ground of horizontal members.

and secondary-circuit constants could be given without difficulty. The values of the pick-up factor obtained are given, in their place, in Table 6, which, as will be seen, constitutes a comparison of the pick-up factors of all the types of aerial systems which have been dealt with in this section.

Summary of Results.

The pick-up factor of a coil aerial 1 m square, with turns to suit the wavelength as found by calculation and checked experimentally, is of the order of 15 m for all wavelengths large compared with the dimensions of the coil. Such an aerial is therefore useful as a standard of reference for the spaced-aerial direction-finder systems. The pick-up factor of the elevated, " U," and balanced types, when the dimensions and tuning capacitance have the optimum values for the working wavelength, has values ranging from equality with, to 20 decibels above, that of the standard coil, according to the wavelength. The maximum value occurs in the 300-m wavelength region.

The pick-up factor of the coupled type, of optimum dimensions and transformer constants for the working wavelength and assuming partial resonance conditions (untuned aerial, tuned secondary), ranges from 10 decibels below that of the coil to 14 decibels above it, the maximum again occurring in the 300-m wavelength region. There is thus, from the point of view of the pick-up factor, a most favourable wavelength band round about 300 m for all the spaced-aerial systems described in this paper. This results from the combination of practical considerations of spacing, aerial height, etc., with the limitations imposed on the systems by natural frequencies, feeder and tuning capacitances, etc.

The figures given in Tables 5 and 6 are optimum values, assuming a particular—most favourable—relationship between aerial constants, tuning capacitance, etc. In practice, most direction-finding systems are required to work over a range of wavelengths. The variations with frequency of a system having given dimensions are therefore of importance. Thus the pick-up factor of the coil aerial varies approximately with the first power of the frequency; that of the elevated, " U," and balanced types, as the cube of the frequency, and that of the coupled type as the fourth power. The rapid falling-off of p with λ is thus a common characteristic of all spaced-aerial systems. In all the systems, however, this can be considerably mitigated by coil-changing at suitable frequency intervals.

The calculations and measurements described above were all made with untuned aerial systems having the working wavelength considerably greater than the natural wavelength. The reverse case ($\lambda < \lambda_{nat.}$), which is less simple to calculate, is a possible one and is actually in use in connection with the coupled type. It has the advantage that the variation of pick-up factor with frequency is not so great.

Table 7 gives the values of standard-wave error and pick-up factor for the various systems which were tested. The results given are based on measured values in all cases. The dimensions of the aerial system and the wavelength are identical in each case.

(8) Conclusions.

The object of the work was to investigate, in a quantitative manner, the nature of the problems underlying the design of the most practical forms of spaced-aerial direction-finders. This has been done, and it has been shown that the subject is of some complexity. The performance of any given system is vitally affected by a large number of factors, including height of aerials, conductivity of soil, height above ground of horizontal members, etc., and a study of Tables 2, 3, 4, and 7, is probably the best way of arriving at an accurate estimate of the relative merits of each system. A few general statements can, however, be made which summarize the characteristics of each.

Thus the elevated type has the advantage that it can be used as a rotating single-aerial system. It has a high pick-up factor and its standard-wave error may be made low by raising it above the ground. Further, the standard-wave error is very little affected by changes in ground conductivity. It becomes less satisfactory in its fixed-aerial form, however, owing to the greater height above the ground to which the horizontal members must be raised in order to achieve a low standard-wave error and reasonably high pick-up factor.

The " U " type has the merit of simplicity and of high pick-up factor, but its standard-wave error, even when screened, remains high.

The coupled type, with which may be grouped the balanced-coupled type and the chain type, can be made to possess a very low standard-wave error. It has, however, a lower average pick-up factor than the others.

The balanced type, on the other hand, has a good pick-up factor but its standard-wave error has not yet been reduced much below 7°.

Development is possible in connection with all of the types to improve their all-round performance. The elevated and coupled types might be combined with advantage in respect of standard-wave error. The coupled type might be improved by introducing aerial tuning with matched transmission lines and again by studying the effect of inserting earthed screens between the transformer windings. The " U " type would give better results when worked with very high aerials on soil of high conductivity, possibly artificially improved over a large area in the locality of the aerial system.

(9) Acknowledgments.

The work was carried out as part of the programme of the Radio Research Board and is the result of close co-operation with the staff of the Radio Department of the National Physical Laboratory. A considerable part of the experimental work was carried out by Messrs. S. R. Chapman, A. F. Wilkins, and A. G. G. Thomas, whom the author wishes to thank for their assistance. The paper is published by permission of the Department of Scientific and Industrial Research.

APPENDIX.

Determination of Y/Z.

The components of the resultant electric field at the point under consideration are denoted in the usual way by X, Y, and Z, where Y is the horizontal component

at right angles to the direction of propagation and Z is the vertical component.

Considering the effect of these fields on an Adcock direction-finder, it is the Z component which produces the desired e.m.f., and the Y component which causes the undesired error-producing e.m.f. The X component can be neglected in most cases. Hence the ratio Y/Z is of great importance in determining the directional characteristics of the instrument. This factor is dependent,

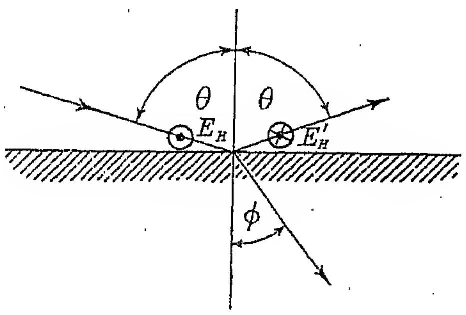


FIG. 19.

as the following analysis shows, on the conductivity (σ) and dielectric constant (κ) of the ground, the wave-length, the angle of incidence (θ) of the waves, and the height above the ground (h') of the point under consideration.

Value of Y at Boundary.

The component Y (Fig. 19) is produced by the horizontally-polarized component of the waves E_H and is the resultant of this force and that of the reflected wave E'_H. The reflection coefficient (ρ) is defined by the relation $\rho_H = E'_H/E_H$, and these forces are complex quantities (i.e. they are not necessarily in phase), so that ρ_H is also complex with respect to time.

It can be shown that the reflection coefficient is given by

$$\rho_H = \frac{\cos\theta - \sqrt{K'}}{\cos\theta + \sqrt{K'}} \quad . \quad . \quad . \quad . \quad (13)$$

where K' is the complex quantity $\kappa - (2j\sigma/f)$ and θ is the angle of incidence.

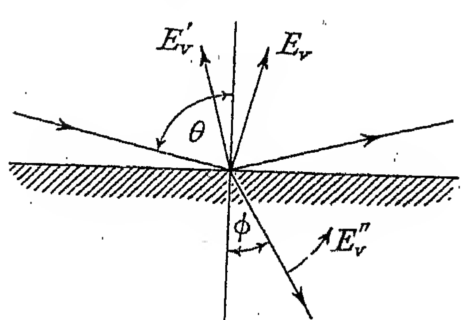


FIG. 20.

The component Y is then given by

$$Y = E_H(1 + \rho_H)$$

or, from (13),

$$\frac{Y}{E_H} = \frac{2\cos\theta}{\sqrt{K'} + \cos\theta} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Value of Z at Boundary.

The component Z is produced by the vertically-polarized component of the wave E_V (Fig. 20) and is the resultant of the combination of the vertical component of this ray with that of the reflected ray E'_V. The reflection in this case is defined by $\rho_V = E'_V/E_V$, where the forces are taken as positive when directed upwards and, as before, are complex.

It can be shown that

$$\rho_V = \frac{\cos\theta\sqrt{K'} - 1}{\cos\theta\sqrt{K'} + 1} \quad . \quad . \quad . \quad . \quad (15)$$

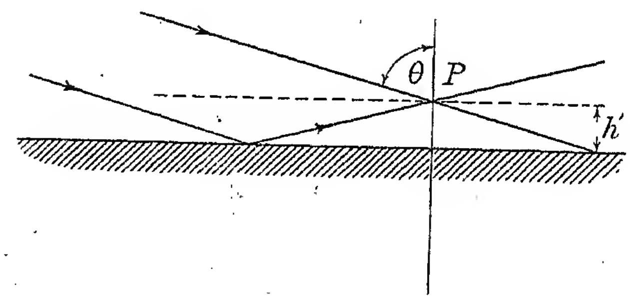


FIG. 21.

Hence, since from Fig. 20,

$$Z = E_V(1 + \rho_V)\sin\theta$$

we have

$$\frac{Z}{E_V} = \frac{2\cos\theta\sqrt{K'}}{\cos\theta\sqrt{K'} + 1}\sin\theta \quad . \quad . \quad . \quad (16)$$

Value of Y/Z at Boundary.

From (14) and (16),

$$\frac{Y}{Z} = \frac{E_H}{E_V}\,\frac{(\cos\theta\sqrt{K'} + 1)}{\sqrt{K'}(\cos\theta + \sqrt{K'})\sin\theta} \quad . \quad . \quad (17)$$

In the case of a standard wave, which has been defined in such a way that $E_H = E_V$ (in phase) and $\theta = 45°$, equation (17) becomes

$$\frac{Y}{Z} = \frac{\sqrt{(\frac{1}{2}K')} + 1}{\frac{1}{2}\sqrt{K'}\left[(1 + \sqrt{(2K')}\right]} \quad . \quad . \quad . \quad (18)$$

It is a great simplification in the numerical calculation for cases when $\sqrt{(\frac{1}{2}K')} > 1$, and therefore $\sqrt{(2K')} > 1$, to work with the approximation $Y/Z \doteq 1/\sqrt{K'}$.

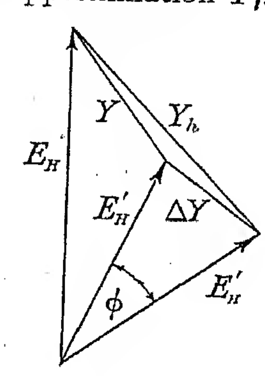


FIG. 22.

Value of Y/Z at a Given Height above Ground.

In this case (see Fig. 21) the phase relationship between E_H and E'_H is not that given by the expression (15), but the phase angle implicit in this expression has added to it the phase displacement ϕ between the two rays due to "path difference." This is given by

$$\phi = (4\pi h'\cos\theta)/\lambda \quad . \quad . \quad . \quad . \quad . \quad (19)$$

To obtain Y under these circumstances it is necessary to obtain the magnitude and phase angle of E'_H with respect to E_H, from (13), add to this angle the angle ϕ, and then calculate or obtain graphically the third side (Y_h) of the vector triangle (Fig. 22).

Expressed mathematically, this is

$$\frac{Y_h}{E_H} = \frac{2}{[1 + \sqrt{(2K')}]^2}[1 + 2K' + \sin\phi\,(1 - 2K')] \quad (20)$$

This is a somewhat complicated equation, but the graphical solution is an extremely simple one.

There is also a simple approximation, as may be seen from Fig. 22, namely

$$Y_h = Y + \Delta Y \text{ (algebraic sum)}$$

where Y is given by equation (14), and ΔY by the equation

$$\Delta Y = 2E'_H \sin \tfrac{1}{2}\phi \quad . \quad . \quad . \quad . \quad . \quad (21)$$

In the same way, in calculating the value of Z_h, allowance must be made for the phase lag due to path difference in this case, and it is necessary to add the two vectors $E_V \sin\theta$ and $E'_V \sin\theta$ with this extra phase angle ϕ as in Fig. 23. In most practical cases a very approximate value can be obtained by assuming $E'_V = E_V$ and that the total phase angle is ϕ; in other words that

$$Z_h/E_V = \sqrt{[2(1 + \cos\phi)]} \sin\theta \quad . \quad . \quad . \quad (22)$$

Value of Y/Z for a Given Aerial System.

It will have now been seen that the value of Y/Z depends on the wavelength, the soil constants, and the height above ground, but it must be noted that the height for which Y is calculated will be different from the height at which Z is calculated, or rather that Z must be integrated over the height of the vertical aerials.

If the height is small compared with the wavelength,

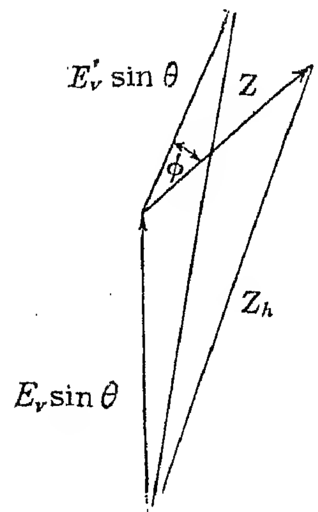


FIG. 23.

Z is almost constant over the whole height, and from (14), (21), and (22) we get the expression

$$\frac{Y_h}{Z_h} = \frac{E_H}{E_V} \cdot \frac{[2\cos\theta/(\sqrt{K'} + \cos\theta)] + 2\rho_H \sin\frac{1}{2}\phi}{\sqrt{[2(1 + \cos\phi)]}\sin\theta} \quad . \quad (23)$$

where $\phi = (4\pi h' \cos\theta)/\lambda$

h' being the height of the horizontal members. If the height is not small in comparison with the wavelength, it is necessary either to integrate Z over the height or to take a suitable weighted mean value which allows for the fact that the current distribution in the aerials is not uniform (taking, for example, the value of Z for one-third of the height of the aerial).

DISCUSSION BEFORE THE WIRELESS SECTION, 5TH DECEMBER, 1934.

Mr. C. E. Horton: I particularly like the author's conception of a standard-wave error, which enables one to obtain a figure of merit for alternative ways of embodying Adcock's basic principle. I have found transmissions from an aircraft extremely useful in testing the performance of an Adcock aerial. If an aeroplane with trailing aerial circles around a direction-finder first clockwise and then anti-clockwise, one generally finds, at any rate with a loop direction-finder, and at short ranges, that there is a substantial difference in bearing according to the direction of flight. At short ranges, and with an aeroplane flying at, say, 4 000 ft., one gets 20° or 30° of difference. A perfect Adcock aerial should of course show no difference. I hope the author will have an opportunity of trying this experiment on the balanced-coupled type which is shown in Table 7 as having a standard-wave error of less than 1°. I am sure it provides a searching test, and I think a valid one.

The existence of night errors has seriously limited the reliance placed on direction-finding bearings in ships for accurate navigational work; this is not to be wondered at, seeing that there is the possibility at night—especially when the range exceeds 100 miles—of getting errors of 20° or 30°. For that reason any device that tends to eliminate night errors is obviously of the greatest practical importance. Yet what do we find at the present time? Practically all ship direction-finders are still of the closed-loop type, in spite of the proved fact that they are liable to these substantial night errors. The inference is that there must be some great difficulty in producing a practical version of Adcock's principle. The difficulties, in my experience, are twofold. In the first place, it is extremely difficult to confine the pick-up to the aerial system itself. In other words, it is difficult to reduce spurious pick-ups to a sufficiently small magnitude. Take a very simple instance. In favourable circumstances in a ship one might get a separation between opposite aerials of 10 m; on a wavelength of 1 000 m (the normal beacon wavelength) the differential action of two aerials is about 6 per cent of the response of either aerial alone. To achieve an angular accuracy of 1° in these circumstances the total spurious pick-up, and likewise the difference between the pick-ups of opposite aerials, must not exceed about 2 per cent of this, which is 0·1 per cent of the pick-up of either aerial alone. This represents an extremely searching requirement, more

particularly when the angle of incidence of the wave is very steep.

As regards pick-up factor, I feel that Table 7 does not quite bring out the superiority of the loop direction-finder. In practice in a ship one is usually given a certain space in which to work; a 1-m cube is about the volume one would like a direction-finder to take up. In that space, with a rotating coil or loop direction-finder, one can in practice get all the sensitivity necessary. If one had to put up an Adcock aerial in the same space, however, the result would be, not a pick-up factor of 100 to 200 such as is referred to in the paper, but probably something less than 0·1. In other words, one must bear in mind that the area of the standard closed-loop aerial is 1 m × 1 m, whereas the area embraced by the Adcock system is 24 m × 15 m, which is several hundred times more. At the present time, we are not in practice getting the advantage of Adcock's invention that we ought to get. If only we could get that advantage the science of direction-finding would be placed on a new and more accurate footing. As an aid to navigation, direction-finders would then become comparable with other instruments of precision. At present they do not belong quite to that class.

Col. J. P. G. Worlledge: I entirely agree with Mr. Horton and with the author that the standard-wave error is a very convenient method of comparison between different types of aerials, but I should like to enter a *caveat* that it does not represent the maximum error which any system is capable of presenting. In actual fact, even the Adcock system may give an error of 90°. I qualify that by suggesting that the pick-up in the conditions of polarization which might produce the 90° error is likely to be so small that actual direction-finding would be impossible. In actual measurements taken on existing Adcock systems, designed 3 or 4 years ago, errors up to 35° have been noted; errors of 10°–15° are not very uncommon.

In the present state of the art I feel there is great necessity for the study of observations, taken with the most perfect direction-finding system that can be designed, from a statistical point of view. I believe it to be as yet entirely unproven that even when a wave is vertically polarized the plane of propagation is constant, particularly on short waves. In other words, even if the perfect direction-finder could be devised there is still a possibility of error owing to the fact that waves are not propagated on great circles. I think it is probable that the error is small, but I believe it is there. At present we do not possess a direction-finding system good enough to detect that possible error, but the pulse method will probably give the answer before long.

I should like to ask the author to reconcile his statement that the presence of an inter-winding screen was found to have no effect, with a statement which he makes later on in the paper that the effect of inserting earth screens between the transformer windings needs to be studied.

With regard to Mr. Horton's remarks about the unsuitability of direction-finding in its present state for navigational purposes, I think that the observations of Dr. Smith-Rose on long waves in 1922–23 show that, if a considerable number of observations is made over a period of about a quarter of an hour, the mean is very likely to be extremely close to the true mean. The position is not quite so bad when one need not depend on snap bearings. How true that remark of mine may be I do not think anybody knows at the present time, because we have only a few isolated cases of a frequency distribution of errors to go upon. I think its truth can only be tested by the application of statistical methods to direction-finding observation. Even when one is calibrating a station, these methods should be applied. There is also the very difficult question of deviations, i.e. site errors, which I do not think has yet been sufficiently fully investigated except on ships.

Mr. S. B. Smith: The author's reduction of the most important parameters of an Adcock aerial to a common quantitative basis is no mean achievement. By arriving at such figures for standard-wave error and pick-up factor I feel sure that improvements in aerial design will result.

The comparison of the various aerials given in Table 7 suggests that instrumentally the coupled system when suitably designed may possess great possibilities. In this regard it is interesting to note that the Bureau of Standards favour a somewhat similar system, called the "T.L." (transmission line) Adcock aerial. The Bureau of Standards do not appear to make special efforts to reduce the transformer inter-winding capacitances; from the paper it would seem that in this way the standard-wave error can be reduced from 8° to 1°.

When the aerial energy is fed to a central point via either 2-wire lines or concentric shielded lines, two degrees of freedom exist, namely, circulating currents in the lines, which convey the desired intelligence, and horizontal currents, which build up a field around the lines; these latter currents produce gradients along the feeder system, and discontinuities often result. The elimination of potential gradients in the concentric-feeder system will provide a low standard-wave error, while the chain-coupled system will impede the setting-up of potential gradients along the 2-wire lines if the impedance of this degree of freedom is made sufficiently high. The second method may often result in a serious loss in effective height of the system when the measurements are made across the input of the first amplifier. I have always realized that some form of radiation coupling between the aerial and a central point represents an ideal, but unfortunately many laboratory schemes designed to attain this end are not sufficiently stable.

In common with Mr. Eckersley I have had considerable experience of the shielded "U" Adcock aerial, but I am not at all certain that the conclusions arrived at in the paper justify the claim that the balanced-coupled system is 8 times superior to the best results obtained with the shielded "U" aerial. Our aim on medium-wave Adcock direction-finders has been to obtain high effective sensitivity, sharp minima, and extreme instrumental stability in service. Are these obtained in the author's system, or are some of these most important operating functions sacrificed in producing a very low standard-wave error? It may be of interest to give a short summary of recent experiences in connection with commercial installations of the shielded "U" type. (1) Higher short-period accuracy is desirable than at present appears possible with typical shielded "U"

aerials. (2) With high-grade narrow band-width receivers, extreme difficulty exists in obtaining first-class bearings on fields of 1 microvolt per metre on a wavelength of 900 metres. (This is a typical aircraft requirement.) It should be borne in mind that short-wave direction-finding has not reached a stage where the existing medium-wave organization can be displaced. (3) The standard deviation of the shielded " U " system, based upon many thousands of night bearings, is 2·37°, and under exactly similar conditions the loop standard deviation was 12·35°. During all of these observations the loop error frequently remained at ± 90° for long periods, while at other times no bearing was possible. In the Adcock case it was always possible to observe bearings, and the peak error did not exceed ± 23°. The ratio between loop and shielded " U " Adcock standard deviations is somewhat greater than the measured standard-wave errors given in Table 7. (4) The variations in performance of several similar installations can be due to poor earth conductivity, geographical location, and the nature of the strata. Forecasting the performance of any given site is an extremely hazardous venture, and the tendency is to test out sites before erection. I have in mind the Basle aerodrome station, where all our efforts failed to produce a precision superior to that of a loop, although at sites within 6 km results were obtained comparable with those of successful installations. (5) With any Adcock system the re-radiation from horizontal metal masses within $\frac{1}{4}$ wavelength may easily nullify the most precise setting-up of aerials. In such cases the daytime performance will not usually be jeopardized, but after sunset any Adcock type of installation may not be superior to a loop direction-finder.

I should be extremely interested in the author's practical experiences, particularly in regard to actual statistical results comparing the balanced-coupled system with the loop system; also with reference to effective sensitivity for ± 5° swing bearings. Has he made any measurements when local re-radiating sources are in close proximity to the aerials?

Mr. J. F. Coales: The author has made a very thorough investigation of several varieties of direction-finders of the Adcock type. Unfortunately, he seems to condemn them all, since for small values of the inclination of the wave to the vertical the standard-wave error has to be multiplied by a large factor.

With a loop-aerial direction-finder situated above a perfectly reflecting surface, one outstanding feature is that if the atmospheric wave is either elliptically polarized with the major axis in the plane of incidence, or circularly polarized, there is no error if no ground wave is present. For the higher frequencies there is some indication that the atmospheric wave is usually approximately circularly polarized and so large errors using a frame coil are uncommon on these frequencies, a result which has been borne out in long-range trials. These trials have, however, led us to believe that for this to be true a very perfect frame-coil system is required. The essential conditions for a perfect loop-aerial direction-finder are: (*a*) Only the frame coil shall pick up radiant energy. (*b*) The two halves of the coil shall be exactly balanced in regard to pick-up factor, and there shall be no inequality in the feeder systems. (*c*) In the case of the Bellini–Tosi system there shall be no interaction between the two loops. (*d*) The frame coil shall be situated above a perfectly reflecting surface. It is obvious that of these conditions (*a*) to (*c*) must also apply to Adcock direction-finders, and from the paper it appears desirable that (*d*) should also be satisfied.

Taking condition (*a*), the author had little success as a result of screening the horizontal leads, but I would suggest that the leads were never perfectly screened. A tube is only an effective screen for a single wire when it is at earth potential along its whole length, and I suggest that the leads should be run underneath an earth mat; then, provided that the earth mat is large enough, they must be entirely screened. This method has been used and found to be successful with regard to screening.

As regards (*b*), with a small Adcock aerial system used on medium waves it has been found essential to have the pick-up factors of the four vertical aerials exactly equal. It is obvious that when the aerial spacing is small (only one-hundredth of the wavelength) a small difference of pick-up factor will swamp the phase effect. It is therefore necessary to use some balancing device to ensure that the responses of the four aerials are equal.

Condition (*c*), i.e. no interaction between the two circuits at right angles, has also been found to be most important. Incidentally, it is difficult to achieve when the aerial system is small owing to the residual effect being small in comparison with the pick-up in one aerial, but by exceedingly careful screening of each aerial conductor it has been possible to reduce the interaction to a negligible amount. In satisfying this condition I think that an earth mat is again a help, but of course it can only be used to the best advantage with the " U " type system. The outstanding point with regard to an earth mat is what size and shape it must have so as not to produce any harmful effect. There must be some shape and size which is not injurious, since an infinitely large earth mat can obviously do no harm.

On this account I think that the " U " type of Adcock direction-finder system is the most promising; and when we know what size of earth mat it is necessary to have, provided we satisfy the conditions previously mentioned, I think we shall have a perfect Adcock aerial system.

Air-Commodore J. B. Bowen: I have one brief question to ask. In meteorological work it is necessary for an aeroplane to go up daily to a height of some 20 000 ft., and it is desirable that the aeroplane should remain substantially over its own aerodrome. The practical difficulties of the pilot are very considerable. He is generally, and particularly at this time of year, flying in cloud, and he has to take a large number of observations. I should be grateful if the author could indicate whether, by means of any form of spaced-aerial direction-finder, it would be possible to tell the pilot that he was within some cone above the aerodrome; and, if so, the approximate angle subtended at the apex of that cone. It would be necessary to allow for the case of the aeroplane being drifted away by high wind to some distance laterally, so that the aerial system would require to have a good pick-up. Other general factors would be that the wavelength would be short and the work would be done by day.

Mr. J. F. Herd: Previous speakers have expressed pessimism regarding the present position of direction-finding. Much of this is justified, but a realization of the existing difficulties is perhaps the best possible means of leading to improvement.

A number of important problems are being investigated by the Radio Research Board, including further work in connection with the coupled system described by the author, as well as other methods of seeking immunity from abnormally-polarized downcoming fields. The problem is largely a propagational one and it assumes increasing importance in relation to shorter waves, where it is effectively "night time all day." As an example of the effects of propagation, however, one can quote measurements over such a relatively short distance as that from Teddington to Slough, where, at a frequency of 3·5 megacycles per sec. on a spring afternoon, the ray returned from the ionosphere had a strength about half that of the ground ray, and was abnormally polarized, so that a large error would have been produced in a closed-coil direction-finder. An important part of the investigation of direction-finding conditions is therefore that of the relative intensities of the ionospheric and ground rays on different frequencies and at different distances. Other important items are the relatively immediate environment of the direction-finding station contributing to what is described as "site error," as well as the more wide-scale effects known broadly as lateral deviation, interpreted in respect of both the ground ray and the ionospheric ray.

The author tells us that the Adcock aerial system is not necessarily going to give us a perfect result, and Col. Worlledge has already mentioned, in the discussion, that with an Adcock system it is still possible to have an error of 90°. He added, however, that in conditions of such an error the amplitude of the signal would be so small as to be useless for practical direction-finding. This fact is very well illustrated with cathode-ray direction-finding apparatus on medium broadcast waves, using the aerial system illustrated in the author's last slide. Watching this on a fairly distant signal (from Langenberg) under conditions of twilight fluctuation, I have been impressed by the fact that while the signal remained of sensible amplitude—certainly of practical working amplitude—the variation of apparent direction was surprisingly small. Large variations of apparent direction were invariably accompanied by a fall of signal strength to extremely low values with violent phase-changes, which, even at small amplitudes, were well revealed on the indicating oscillograph. In practical observations, however, these conditions were so clearly marked and so exceptional in their incidence that an operator of even slight experience would have unhesitatingly discarded the observations as invalid.

Mr. A. J. Gill: The question of devising a direction-finder which will not be subject to night errors is a very important one, and in the Post Office we have been interested in the use of such a direction-finder at those stations which provide direction-finding bearings to ships. We have recently installed an Adcock-type aerial at Cullercoats, but it is too early to express an opinion on the results obtained.

With regard to the question of site errors, we have met with rather an interesting case in connection with the Bellini–Tosi installation at Niton wireless station, Isle of Wight. The station is erected at approximately sea level in what is more or less a "built-up" area, with rapidly rising ground immediately behind it. We found great difficulty in using a direction-finder on the station site, partly owing to the physical characteristics of the site itself and also owing to reflections from the transmitting aerial into the direction-finding aerial; any slight alteration to the transmitting-aerial system was usually accompanied by some change in the characteristics of the direction-finder receiver necessitating re-calibration. We decided to try the effect of putting the direction-finder aerial about a mile from the station, up on the higher ground, and bringing the signals in by means of two lead-covered cables, one to each Bellini–Tosi loop, while the vertical-aerial signal required to determine sense was transmitted over a phantom circuit composed of the two cables. The results obtained with this arrangement have exceeded expectations. No error appears to have been introduced by the cables and transformers, while the site errors previously existing have been reduced to such an extent that the station can now give reliable bearings over a sector double that previously possible.

Mr. R. H. Barfield (*in reply*): Mr. Horton's method of calibrating a direction-finder by means of downcoming waves from a circulating aeroplane should prove extremely valuable and I hope to be able to employ it on some future occasion. It occurs to me, however, that though there should be no difficulty in obtaining the angle of incidence, the angle of polarization of the wave might be extremely uncertain unless it can be assumed that the whole of the radiation emanates from the trailing aerial. This difficulty might, however, be overcome by making simultaneous measurements on a loop direction-finder.

I agree that the pick-up factor of spaced-aerial systems in general leaves much to be desired in comparison with that of coil systems, but I think that Table 7 shows this clearly enough, since the dimensions of each system are clearly given and it only needs a very simple calculation, such as Mr. Horton has himself provided, to obtain the ratio for systems of equal area.

Col. Worlledge's remarks on the possibilities of very large errors on the most perfect of spaced-aerial systems require certain qualifications which have been satisfactorily supplied by Mr. Herd and therefore need not be further dealt with. A statistical examination of large numbers of observations would undoubtedly provide very useful guides to the practical users of direction-finding systems liable to variable errors. The apparent contradiction, pointed out by Col. Worlledge, with regard to the effect of an inter-winding screen in the coupled system has been removed in the final form of the paper; as now explained, the screen was connected entirely differently in the two cases. Site errors, ionospheric scattering, and other forms of lateral deviation, do not come within the scope of the paper, except that the extent to which they commonly occur may determine the limit to which it is useful to carry the reduction of instrumental error in the design of Adcock systems.

Mr. Smith foresees future improvement in design as a

result of the method of measurement described in the paper. This is extremely gratifying, as it was with this final end in view that so much attention has been given to the principles underlying the performance of existing imperfect Adcock systems. I share this speaker's interest in the "T.L." Adcock aerial system of the Bureau of Standards, and have also noted that the inter-winding capacitance of the coupling transformers receives no special attention in this case. It seems almost certain that a low-impedance path between the earths of the opposed aerials must give rise to residual errors of some magnitude.

Mr. Smith clearly sums up the essential difference in the two lines of attack on the problem of eliminating polarization error which are respectively represented by the screened "U" and coupled types. The conclusions drawn from my experiments of the general superiority of the latter over the former with regard to polarization can only be answered in the course of time, after comparisons have been made of the practical working of the two systems on different wavelengths and distances and types of downcoming waves. The question is to some extent linked up with that of scattering and lateral deviation.

Mr. Smith gives an extremely interesting and valuable summary of his recent practical experiences with the screened "U" system and asks for information of the same kind about the coupled type. No long series of tests have as yet been carried out on the balanced coupled system of a similar nature to those described by Mr. Smith. The coupled type (unbalanced), balanced type, and "U" types, have, however, been tested in this way and the results are given in previous publications* to which Mr. Smith is referred. The results, in general, showed error reduction factors with regard to the loop direction-finder of the same order as those obtained by the experimentally measured values of the standard-wave error.

The effect of masses of metal, short lengths of wire, and other conductors, between the aerials of an Adcock system or anywhere in its neighbourhood has already received some attention both experimentally and theoretically. In the case of the rotating elevated type, for example, it was definitely ascertained that the box containing the amplifier was not contributing appreciably to the standard-wave error as determined with the system near the ground. In a later system of this type with a considerably larger central conducting mass a contribution amounting to about 20 per cent was detected. Such an effect will, of course, become of great importance as the more serious causes of polarization error are removed, and it will be necessary to investigate this phenomenon more fully in future.

Mr. Smith suggests that pick-up factor, instrumental accuracy, and stability, may have to be sacrificed to obtain low standard-wave errors. The first of these has, I think, been adequately dealt with in the paper; as regards the others, it is inevitable that a more complex system should be more vulnerable in this respect than a simple type, and I would agree with him that Adcock direction-finders are in general more difficult to design for a given instrumental accuracy and stability than the single-coil or Bellini–Tosi types.

It is true, as Mr. Coales points out, that I have shown that all existing Adcock systems are subject to large errors with waves having small angles of incidence and large angles of polarization. This is because the expression for the errors involves the tangent of these angles, which go to infinity for their extreme values. I have also shown, however, methods by which the elevated, coupled, and balanced types can, by careful design, have their errors reduced to almost any degree of smallness desired. I am further inclined to agree with Mr. Coales that the "U" type, by careful study of different sizes of earth mat and screening arrangements, might also be made to possess a much lower standard-wave error. It is indeed hoped that the work has cleared the ground for a further all-round development of Adcock systems.

In answer to Air-Commodore Bowen, since an aircraft circling round an Adcock direction-finder at a height of 20 000 ft. in a circle of 2 miles radius would radiate waves having an angle of incidence of 25°, a suitably designed Adcock direction-finder should be able to follow the craft without difficulty and be able to detect the direction of drift and correct the aircraft by telephony, so as to keep it within 2 or 3 miles of its base. If the craft should drift further than this, there would be no difficulty in guiding it back again from distances of the order of 50 miles, given sufficient power from the ground and aeroplane transmitters to maintain communication with each other over the range.

Mr. Gill's description of the successful installation of a remote-aerial Bellini–Tosi direction-finder is of great interest. The application of this method to Adcock systems is a promising field of development and, in particular, should enable the elevated type to be exploited to the full extent of its possibilities. A certain amount of experimental work, however, will be necessary to ascertain to what extent the long transmission line reacts on the standard-wave error of the system.

* *Journal I.E.E.*, 1930, vol. 68, p. 1052; also Radio Research Board Annual Report, 1931, p. 41.

ANNUAL DINNER, 1935.

The Annual Dinner of the Institution was held at Grosvenor House, Park Lane, London, on Thursday, 7th February, 1935, when the President, Prof. W. M. Thornton, O.B.E., D.Sc., D.Eng., presided over a gathering numbering more than 810 persons. Among those present were: The Rt. Hon. Sir Kingsley Wood, M.P. (*Postmaster-General*); The Rt. Hon. the Earl Poulett; The Rt. Hon. the Viscount Elibank; The Rt. Hon. the Viscount Falmouth; The Rt. Hon. Lord Stonehaven, P.C., G.C.M.G., D.S.O., LL.D. (*President, Institution of Naval Architects*); The Rt. Hon. Lord Pentland; The Rt. Hon. Lord Eltisley, K.B.E.; Major the Hon. O. M. Guest; Mr. John W. Dulanty, C.B., C.B.E. (*High Commissioner for the Irish Free State*); Col. Sir Donald Banks, K.C.B., D.S.O., M.C. (*Director-General, General Post Office*); Sir Henry Pelham, K.C.B. (*Permanent Secretary, Board of Education*); Sir Richard A. S. Redmayne, K.C.B. (*President, Institution of Civil Engineers*); Sir Frank E. Smith, K.C.B., C.B.E., D.Sc., LL.D., F.R.S. (*Secretary to the Council, Department of Scientific and Industrial Research; Secretary, Royal Society*); The Very Rev. Sir W. Foxley Norris, K.C.V.O., D.D., F.S.A. (*Dean of Westminster*); Sir William Bragg, O.M., K.B.E., F.R.S. (*Honorary Member, I.E.E.; Fullerian Professor of Chemistry, Royal Institution*); Sir Arnold B. Gridley, K.B.E.; Sir Cyril W. Hurcomb, K.B.E., C.B. (*Permanent Secretary, Ministry of Transport*); Sir E. M. Hughman; Col. Sir Henry G. Lyons, Sc.D., D.Sc., F.R.S. (*President, Institute of Physics*); Sir William Noble; Col. Sir Thomas F. Purves, O.B.E. (*Past President*); Sir M. G. Simpson; Mr. W. H. Ansell, M.C. (*Vice-President, Royal Institute of British Architects*); Mr. N. Ashbridge, B.Sc.(Eng.) (*Member of Council*); Mr. Ll. B. Atkinson (*Past President; Honorary Member*); Mr. J. R. Beard, M.Sc. (*Member of Council*); Mr. H. R. Beasant (*Hon. Secretary, Western Centre*); Mr. C. Valon Bennett (*President, Institution of Gas Engineers*); Mr. W. S. Burge (*Member of Council*); Mr. W. Burton, M.Eng. (*Chairman, South Midland Centre*); Prof. J. K. Catterson-Smith, M.Eng. (*Member of Council*); Mr. R. A. Chattock (*Past President*); Mr. G. A. Cheetham (*Chairman, North-Western Centre*); Mr. F. W. Crawter (*Honorary Treasurer*); Col. R. E. B. Crompton, C.B., F.R.S. (*Past President; Honorary Member*); Mr. Ewart G. Culpin, F.R.I.B.A., M.T.P.I., J.P. (*Vice-Chairman, London County Council*); Mr. A. E. Cutforth, C.B.E., F.C.A. (*President, Institute of Chartered Accountants*); Mr. M. H. Damme (*President, Koninklijk Instituut van Ingenieurs*); Mr. Charles Day, M.Sc.Tech. (*President, Institution of Mechanical Engineers*); Mr. R. G. Devey [*Chairman, Mersey and North Wales (Liverpool) Centre*]; Mr. J. M. Donaldson, M.C. (*Past President*); Dr. S. F. Dorey (*Chief Engineer-Surveyor, Lloyd's Register of Shipping*); Dr. P. Dunsheath, O.B.E., M.A. (*Member of Council*); Mr. R. N. Eaton (*Hon. Secretary, Irish Centre*); Mr. W. C. Eaton, C.B. (*Principal Assistant Secretary, Technological Branch, Board of Education*); Lieut.-Col. K. Edgcumbe, T.D. (*Past President*); Mr. V. Z. de Ferranti (*Member of Council*); Mr. A. P. M. Fleming, C.B.E., M.Sc. (*Member of Council*); Mr. W. H. Fuller (*Hon. Secretary, Sheffield Sub-Centre*); Mr. J. Grosselin (*Secretary, Société Française des Électriciens*); Mr. J. S. Highfield (*Past President*); Mr. A. G. Hiscock (*Hon. Secretary, Hampshire Sub-Centre*); Mr. E. E. Hoadley (*President, Incorporated Municipal Electrical Association*); Mr. R. Hodge (*Chairman, Western Centre*); Mr. Frank Hodges (*Member, Central Electricity Board*); Mr. H. Hooper (*Hon. Secretary, South Midland Centre*); Mr. P. V. Hunter, C.B.E. (*Past President*); Mr. A. E. Jepson (*Hon. Secretary, North-Western Centre*); Lieut.-Col. A. G. Lee, O.B.E., M.C. (*Vice-President, I.E.E.; Engineer-in-Chief, General Post Office*); Brig.-Gen. R. F. Legge, C.B.E., D.S.O. (*Member of Council*); Mr. W. McClelland, C.B., O.B.E. (*Member of Council*); Mr. R. Borlase Matthews (*Chairman, Transmission Section*); Mr. L. E. Mold, O.B.E. (*Chairman, North-Eastern Centre*); Mr. A. Nichols Moore (*Past Chairman, Western Centre*); Mr. W. M. Mordey (*Past President; Honorary Member*); Mr. E. W. Moss (*Member of Council*); Brig.-Gen. Magnus Mowat, C.B.E. (*Secretary, Institution of Mechanical Engineers*); Mr. S. R. Mullard, M.B.E. (*Chairman, Wireless Section*); Mr. C. G. Morley New (*Member of Council*); Mr. Clifford C. Paterson, O.B.E. (*Past President*); Mr. H. B. Poynder (*Hon. Secretary, North-Eastern Centre*); Mr. H. W. H. Richards (*Member of Council*); Mr. C. Rodgers, O.B.E., B.Sc., B.Eng. (*Member of Council*); Dr. A. Russell, M.A., LL.D., F.R.S. (*Past President*); Mr. T. S. G. Seaward (*Hon. Secretary, Tees-side Sub-Centre*); Mr. H. C. Siddeley (*Past Chairman, Argentine Centre*); Mr. R. P. Sloan, C.B.E. (*Vice-President*); Mr. Roger T. Smith (*Past President*); Mr. H. Hepworth Thompson (*President, Illuminating Engineering Society*); Mr. V. Watlington, M.B.E. (*Member of Council*); Mr. John D. Watson (*Chairman, Association of Consulting Engineers*); Mr. O. C. Waygood [*Hon. Secretary, Mersey and North Wales (Liverpool) Centre*]; Mr. W. J. H. Wood (*Member of Council*); Mr. C. S. Wright, O.B.E., M.C., M.A. (*Director of Scientific Research, Admiralty*); Mr. Johnstone Wright (*Member of Council*); Mr. H. T. Young (*Vice-President*); and Mr. P. F. Rowell (*Secretary*).

The following messages were received from kindred institutions abroad:—

From the Société Française des Électriciens:

"At a time when our two nations have just made a fresh effort to assure lasting and fruitful peace in the world, the Société Française des Électriciens is particularly happy to offer congratulations to the Institution and to its President, together with best wishes for close collaboration between the two societies."

From the Société Belge des Électriciens:

"On the occasion of the Annual Dinner of their British colleagues, the members of the Société Belge des Électriciens send all best wishes for the Institution's prosperity."

The toasts of "His Majesty the King" and "Her Majesty the Queen, His Royal Highness the Prince of Wales, and the other members of the Royal Family," were proposed by the President and were loyally received.

The Rt. Hon. Sir Howard Kingsley Wood, M.P. (*Postmaster-General*), in proposing the toast of "The Institution of Electrical Engineers," said: "I wish first to thank the President for inviting me to be present to-night at this wonderful gathering. I think I can claim that the Post Office is intimately associated with the electrical industry in engineering problems, and I hope that you who are present to-night recognize everything that the Post Office is doing for the electrical industry and that you will bear in mind the miles of telegraph and telephone wires that have to be maintained overhead, underground, and under the sea.

"One part of the Post Office of which I am very proud is the fine research station we possess, one of the best equipped in the country. It is constantly employed on the latest developments in the science of electrical communication. Another matter which I rank of equal importance, and on which I think you will agree with me, is that the Post Office engineers have won for themselves a high and honoured place in the electrical world. Recent developments in connection with the telephone service will result not only in an increase in the orders for new plant but also in considerable achievements in the development of communication itself. To a large extent I can speak only as an outsider, but I do say that we are living in an age of miracles. Only a few days ago in the House of Commons I made an announcement of the opening stages of television in this country; and at a luncheon this afternoon we were able to listen to conversations which were heard both in South Africa and in London and which were very remarkable in their character and marked an important development in our communications. I therefore venture to suggest that at no time in our history were the science and the practice of communication advancing more rapidly than at present.

"So far as your own Institution is concerned, you have recently made a grant for radio research in the Polar regions. The Post Office, as a large user of world-wide radio, is keenly interested in a project of that kind. Another matter which comes within my official knowledge is in connection with broadcasting. The broadcasting public is certainly indebted to your Institution for your investigations into the problem of electrical interference with broadcasting, and I hope that a constructive and satisfactory solution of the difficulties will be found without resort to legislation. Nevertheless, if legislation is considered to be necessary, I will do my best to help.

"In conclusion, let me say how glad I am, having regard to the office which I now hold, to be able to propose to-night the toast of 'The Institution' and to couple with it the name of your President. I wish the Institution every possible success and every member of it health, prosperity, and good luck in the coming year."

The President, in responding to the toast, said: "I thank you, Sir Kingsley, on behalf of the Institution, for the generous way in which you have proposed our health. We are, I believe, both healthy and vigorous, as may be gathered from this wonderful gathering to-night, and our numbers grow steadily. The department over which you rule is worthily represented on our Council by our Past President, Sir Thomas Purves, and our Vice-President, Colonel Lee, his successor. We are in fact in the direct line of descent from the pioneers of telegraphy, and if, in the process of evolution, we have come to deal with immense machines and vast powers, with megawatts rather than microwatts, the methods by which they are controlled are in their delicacy and precision a legacy of the old telegraph days.

"In 20 years, if not earlier, every corner of the land will thank the political insight that made the national electric power scheme possible, and the engineers who have carried it out. I believe that in time the grid will be as much a part of the nation's life as the telephone or telegraph is now, though not, of course, so intimate and personal a part. Of course some experimenting is necessary, and we have not yet reached finality in our methods of distribution, but in this we have had the advantage of the experience gained in the operation during many years of the great power companies, and incidentally in the great national power scheme of the government of New Zealand. Our power sales engineers might well consider the methods adopted in that progressive country, and may even take a leaf out of your own book, by offering a universal special rate after 7 p.m. of one-third of a penny per unit retail for all purposes.

"One of the most interesting events of the session has been the establishment of the Transmission Section, for efficient distribution is the immediate problem of the future. Only those who have served on the Council know the extent of the work that this new Section entails. Every Section is in fact an Institution in itself, and we are grateful to Mr. Rowell and his staff for the splendid manner in which the many and varied activities of the Institution are welded into a coherent whole.

"Like every other body we must be fed—in our case fed with ideas. The essential vitamins of our profession are the new ideas of pure science—the fresh facts or laws that are discovered by physical research or in the borderland between physics and engineering. I hope that we shall never lose sight of that fact. It is one of the causes of pride in our race that we have been able to make, every 50 years or so, such vitalizing discoveries.

"Some years ago, when in Germany, I remarked to a director of a great manufacturing company on the outstanding intelligence and ability of one of his foremen. He replied: 'I know England well, and I believe that our workmen are, on the whole, better educated than yours, but you as a nation have the fortunate gift of throwing up from time to time men of supreme genius—Newton, Faraday, Maxwell, Rutherford—who have started new epochs in science.'

"What is the secret of it? A capacity for intense and prolonged thought and work, profound logical common

sense, a power of associating facts and ideas, and, like a hound, an acute sense of the trail. This is not the time or place to carry such analysis farther, but it may not be inappropriate to recall that Faraday had also a great power of inspiring affection. When the front wing of Armstrong College was opened in 1906 by King Edward it fell to me to show him round. I called his attention to a fine portrait of Faraday. He stood, with Queen Alexandra, looking at it and then said with deep feeling: 'Ah, yes, I knew him very well, he used to teach me.' Such a memory of the two great men is worth placing on record. Science, as a source of endless curiosity, is a true social solvent, in which a king and his people are one.

"I venture to think that the great technical societies, such as ours, are the modern counterpart of the medieval guilds, to which England owes so much. They were chiefly concerned with the maintenance of a good standard of work, with the education of apprentices, and with the help of members in distress. That would not be a bad definition of our own aims. By the reading of scientific and technical papers and the supervision of wiring regulations—which are much more than the title implies—we fulfil the first. We examine, partly in co-operation with the Board of Education, about 3 000 students a year, and our Benevolent Fund is always in action. In order to deal with urgent cases we wish to increase this fund, and we hope that there will be a generous response to the appeal recently issued for more subscribing members, even if in each case the amount is small.

"I have only one more thing to say. I think we may want yet another Section or Sub-Section of the Institution, namely that of Electro-Sonics. This remark is born of a series of experiences when visiting our Local Centres and Sub-Centres, in some of which my speeches were audible, and in others inaudible. But I think you will all agree that the present loud-speaking arrangements are excellent. To one who is accustomed, as I am, to judge from the faces of his audience whether his points are going home, the microphone is too impersonal. Even if it were camouflaged, in the Greek manner, to resemble a face, it could not even yawn. Imagination pales at the thought of what the speeches at such a dinner as this would be without the microphone; but the perfect microphone system, suitable for all the queer-shaped rooms in which we dine and speak, has yet to be devised. I commend it to Sir Frank Smith and his department, and we shall be only too glad to co-operate in the discovery of the principles of perfect stentorian speech.

"I have a very interesting announcement to make before I sit down. One of our kindred societies, the Société Française des Électriciens, is represented here to-night by its director, Mr. Grosselin, who has attended in place of its President, Mr. Henri Milon, the latter having been prevented from making the journey. Mr. Grosselin has just informed me that the French Government have appointed our Secretary, Mr. Rowell, to be an Officer of the Legion of Honour."

Mr. J. Grosselin then asked the President's permission to make, on behalf of his Government and his Société, a gesture of compliment to their English colleagues. He detached from the lapel of his coat the Cross of Officer of the Legion of Honour and presented it to Mr. Rowell, remarking that the latter, by the long and valuable services he had rendered to the electrical engineers of both countries, was an exceptionally worthy recipient of the Order. He was happy to be able to signify in this way his own personal appreciation of Mr. Rowell's services, and to signify the cordial feelings of the French Government, and the Société Française des Électriciens, towards the Institution.

Mr. P. F. Rowell, who received this mark of distinction amid loud applause, replied in French that he was unable to express adequately his recognition of the great honour which had been conferred upon him by the French Government and the Institution's kindred society in France. All his colleagues of the Institution would feel that in making the award to him the French Government had honoured the Institution and each individual member. On behalf of himself and them, he thanked Mr. Grosselin very heartily.

Mr. R. P. Sloan, C.B.E. (Vice-President), then proposed the toast of "Our Guests," to which **Sir Richard A. S. Redmayne, K.C.B.** (*President, Institution of Civil Engineers*), and **Sir Henry Pelham, K.C.B.** (*Permanent Secretary, Board of Education*), responded.

A reunion was subsequently held.

ARGENTINE CENTRE: CHAIRMAN'S ADDRESS

By K. N. Eckhard, Member.

(Abstract *of Address delivered at* Buenos Aires, 30*th May*, 1934.)

After describing in detail the development of electric traction on the Buenos Aires suburban lines of the Central Argentine Railway, I propose to give a few notes on the possible future developments of electric traction, emphasizing the more important points on which, in my opinion, manufacturers and designers should concentrate.

Although the railway system described is supplied with power from its own power house, I do not propose to dwell on the future developments of power plant except to remark that, when considering equipment of power stations which have to deal with a load made up almost entirely of heavy electric traction, very special characteristics are required. Only those who have had experience of a similar load are in a position to realize what it means in practice.

A carefully designed time-table, whereby trains are scheduled in such a manner as to limit the number starting at any particular moment, is all very well in theory, but when dealing with a service in which the scheduled running-time between stations is in many cases under 2 minutes, it does not work out in practice. The intercalation of main-line trains over the suburban tracks is also bound to upset the most carefully designed schedules.

In order, therefore, to reduce the capacity of the plant in commission to that required for the normal service only, it is essential that the plant should have a remarkably high capacity for dealing with sudden peaks of comparatively short duration. This characteristic is necessary not only for the turbo-alternator plant but also for the boilers.

The present reliability of high-tension alternators, whereby the necessity of using step-up transformers is obviated and a continuous saving of 2–3 per cent made possible, is an advance towards economical operation which should help towards further development.

As regards substations, the development of the mercury-arc rectifier complete with the supervisory control system has, to my mind, been one of the greatest advances in favour of the low-tension third-rail direct-current system of electrification. The limitations, from the economic standpoint, that previously existed with this system of electrification no longer hold good.

A substation building to house rectifier plant—the switchgear and transformers being installed out of doors—can be cheaply constructed as compared with the cost of a building for housing rotary convertors, etc. This means that it is now economically possible to place substations at short distances apart and all connected to a supervisory control system, thus avoiding attendants and consequently high operating costs. By this means excellent electrical conditions are maintained on the tracks and troubles from stray return currents are avoided.

As regards the electrical equipment of rolling stock, there is, in my opinion, a large field for further improvement in design and performance. Although the cost of overhauling electrical equipment is not a big item, the constant need for inspection and revision should be avoided. While the actual work called for in revising the electrical equipment of coaches is not of great magnitude, very careful revision is essential in order to avoid defects arising in service. An equipment has yet to be designed which can be locked and sealed and remain so, only being looked at when a coach is in the shops for its periodical overhaul, say, after 80 000 miles' running.

There is very little doubt in my mind that such equipments will eventually be developed. The electric traction industry might profit by taking a lesson from the performance expected and obtained from the now comparatively complicated electrical equipment of an automobile. In many cases such an equipment is in the hands of an amateur as far as engineering knowledge is concerned, and yet failures are few.

That progress has been made, I do not doubt, and by employing higher-grade staff for carrying out the revision, the periods between revisions can be lengthened; but there is still room for improvement.

The importance of this subject has, I feel, been somewhat overlooked. The capital cost of rolling-stock is a very big item of expenditure for an electrification scheme. It is therefore necessary to reduce to a minimum the number of coaches required. The necessity of increasing this number and, therefore, the capital expenditure, in order to allow for revision requirements, should be avoided.

Another very important development which is anxiously awaited is a system whereby regenerative braking can be applied to multiple-unit stock such as is generally used in suburban services.

The design of motors has been perfected so that we can obtain, within reason, whatever acceleration we desire. While this is of course necessary in order to obtain high schedule speeds, we are faced immediately with the difficulty of frequently dissipating energy in order to stop the trains at stations. To do this with the present methods of braking, using a cast-iron block pressing on the steel tyres, it becomes economically necessary to limit the speed at which the brakes have normally to be applied.

The developments as regards regenerative braking have mostly been applied to electric locomotives operating in hilly country. It appears to me, however, that

there is a far more fruitful field for this development in suburban electric traction, especially on systems where there are a large number of trains continually starting and stopping. Points of considerable importance are also the saving of energy and the avoidance of the continual distribution of cast iron about the track, of the heating and wear of tyres, and of the spoiling of the exterior paint and cleanliness of rolling stock by cast-iron dust—all of which call for additional labour, not only to replace the cast iron worn away but also to maintain the rolling stock in an attractive condition.

On the Central Argentine Railway suburban electric lines, in spite of the great improvements that have been made in the design of brake blocks, 450 tons of cast iron are annually distributed on the track. The importance of this subject will readily be realized, especially at this time when other methods of railway traction, designed to operate more economically than steam locomotives, are progressing rapidly and, possibly, to the detriment of the more extensive electrification of steam lines.

As a matter of interest the following figures are submitted, which readily show the importance of the cost of brake shoes:—

	Direct operating costs	
	Per cent	
Electric current	33	not including contributions to renewals
Motormen	15	
Maintenance of rolling stock ..	27	
Renewal of brake shoes ..	10	
All other expenses	15	
	100	

The above costs do not include traffic or permanent-way expenses.

The use of non-ferrous brake blocks is advancing, but the cost of such blocks and the large number required—all of which would have to be imported—do not make them an economical proposition at the present time for use in the Argentine.

Progress is also necessary in coach design, with a view to a reduction of weight. This is somewhat difficult when the existing conditions as regards seating capacity and other amenities in force in the Argentine are complied with. The necessity for louvre shutters as well as windows which have to open and close does much to add to the weight of the coach bodies.

Whilst roller bearings, as applied to axle-boxes, appear to be a favourable proposition, it is somewhat doubtful whether the economies to be gained therefrom are sufficient to pay for the first cost of such bearings. The makers claim that the following savings can be effected by their use:—

	Per cent
Current	8 (coal only)
Oil	76
Oilers	61

These savings are estimated to be approximately £5 000 per annum on the Central Argentine suburban lines. In order to obtain this saving, 2 960 roller-bearing axle-boxes would have to be fitted, and each axle-box would cost something like £20.

The advantage claimed by the makers—that troubles and expenses from hot axle-boxes can be eliminated—should not be taken into account, as even with waste-lubricated axle-boxes these troubles should be non-existent.

Roller bearings, however, call for very careful fitting and adjustment, and the cost of this has to be offset against the possible saving in operating expenses.

The use of roller bearings for traction-motor armatures has become very general in recent years. The advantages claimed are the saving in energy and in lubricating costs, also the avoidance of trouble from hot bearings. The fitting of roller bearings is, however, a costly matter, not only when originally erected but when the motors are being overhauled.

With waste-packed armature bearings, as long as cotton waste is used—not wool waste as generally recommended by the manufacturers—and the bearings are fitted and lubricated with normal care, bearing troubles disappear and it is doubtful whether real economies are to be gained from roller bearings.

There is no doubt that straight electrification has many advantages over any other form of rail transport, provided always that the traffic density has reached a certain pitch. It is therefore essential, if further schemes are to be carried out or extensions made to existing systems, for further progress to be made as regards reduction of capital costs and operating expenses.

In regard to operating costs, excluding traffic and permanent-way costs which are more or less constant for any form of traction, the cost of operating in the Argentine an electric service having similar characteristics to those of the Central Argentine suburban lines, can be taken, with exchange at par, at 2s. 6d. per train-mile, including contributions to the renewals fund. These costs may be divided as follows:—

	Pence per train-mile*
Electric current (coal at 30s. per ton) ..	12
Motormen	3
Maintenance of rolling stock	13
Revision of rolling stock	2

* Exchange at par.

I suggest that the firms who manufacture equipment for electric traction are somewhat backward in obtaining first-hand experience of their products, and I submit that it would be advantageous to all concerned if they were more inclined to lend railway companies improved equipment or plant for trial. The operating companies might then learn by experience the value of these improvements, and the manufacturers would obtain reliable information as to the behaviour of the equipment under service conditions, since even the best-equipped factories have not the facilities at hand for trying out new apparatus under service conditions.

In view of the present financial conditions, it would appear that manufacturers, after developing and thoroughly trying out new equipment or plant under operating conditions, should be in a position to sell

such plant and equipment on the basis of accepting as payment a proportion of the resultant economies.

As regards future progress in electric traction on the Central Argentine Railway, owing to world financial conditions it is difficult at the present to prophesy any development.

The development of urban districts within 50 km of Buenos Aires would no doubt be stimulated if improvements were undertaken in the method of traction and in the service given. The traffic density justifying electrification would not be reached for many years and, owing to the necessity of not only electrifying the lines but also transmitting power, comparatively heavy capital expenditure would be involved.

The severe and uneconomical competition from alternative methods of traction within the existing suburban area, however, may eventually be the deciding factor as regards the advisability of developing the outlying districts, thus utilizing rolling stock and equipment which are not required owing to the falling-off of traffic to and from the suburbs near Buenos Aires.

There are also advantages to be obtained from the haulage of main-line passenger and cargo trains by electric locomotives over the suburban lines, but the existing financial position must necessarily delay further development in this direction, as well as the possibility of main-line electrification.

TAPPING THE HIGH-TENSION GRID.*

By T. D. Oswald, B.Sc., Graduate.

(Abstract *of paper read before the* North-Eastern Students' Section, 16*th February*, 1934.)

During the last seven years there has been designed and constructed a national system of high-voltage transmission lines linking the main power stations of this country. The chief essential of this scheme lies in the ease with which the voltage may be stepped up or down by means of highly efficient static transformers. These transformers are of large size and of relatively simple construction, being the application of well-known design principles to somewhat higher powers and voltages than have previously been considered standard practice in this country.

A much more difficult problem, however, is now confronting the electrical engineer. This is the supply of electricity to small rural communities and farms direct from the grid. These loads will probably not exceed a maximum demand of 50 kW at 240 volts. Agriculturalists throughout the country have been keenly interested by electrical propaganda, and it is clear that all possible methods of tapping the grid for this type of load should be investigated.

If the load is to be supplied by a single-phase transformer connected between the line and the neutral, the winding ratio will be of the order of 300 to 1. The difficulty to be overcome here is how to manufacture an efficiently insulated transformer at an economic price for such a small output. With the continual raising of line voltage, and consequent reduction in current values, there comes a time when the engineer is led to consider whether a device depending on the periodic variation of the magnetic field, necessarily produced by the current, is the best possible solution. It would appear more logical to utilize the electrostatic field, the efficacy of which is increased by the augmentation of the voltage. An attempt to put this theory into practice is seen in the capacitance divider which has been used to supply current to aerial beacons on grid towers. This method suffers from the defect that its output is too small under present conditions; but as the output increases with the voltage, it would appear that the capacitance divider may have an increased application in the future as the line voltage rises.

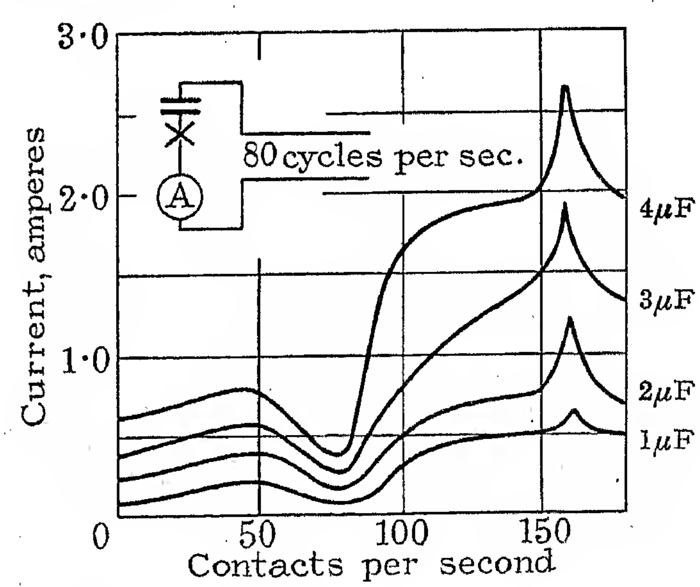


Fig. 1.

Prof. Thornton had previously found that an interrupter might be used for the multiplication of small currents in a capacitance circuit; it was therefore decided to investigate the laws governing this type of circuit, the conditions necessary for optimum effect, and the possibility of applying this method to tapping the grid. A series of experiments was carried out with circuits containing a condenser, an ammeter, a rheostat, and a mechanical interrupter, in series on an alternating-current supply.

Fig. 1 shows the current obtained, plotted against the frequency of interruption when the circuit was connected to an 80-cycle supply.

* A Students' Premium was awarded by the Council for this paper, and it is the practice of the Council in such cases to publish the paper, in full or in abstract, in the *Journal.*

At synchronous frequency of interruption a marked decrease in current is recorded. This is accounted for by the fact that the condenser is charged up to the contact voltage and, as subsequent contacts occur on the same point of the supply-voltage wave, no change can take place in the condenser's state of charge, a reduction of current being the result. The other peculiarity is the peak of current when the contact frequency is twice the supply frequency; in this case the voltage on the condenser suffers a reversal at each contact, and a large current therefore flows.

A further experiment was performed at this double frequency, in which the instant of contact was varied progressively over the whole voltage wave. Fig. 2 indicates how the current varies with the point of contact.

As a first approximation, the voltage on the condenser

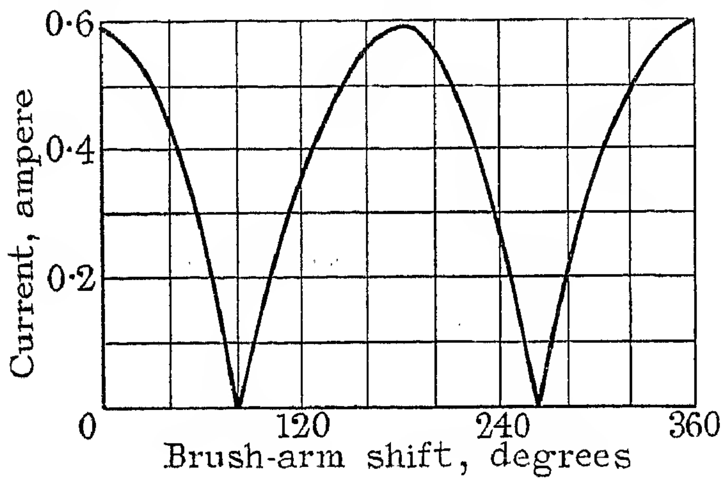


FIG. 2.

may be assumed to be of rectangular wave-form and may be written as

$$e_c = \frac{4E}{\pi}(\sin \omega t + \tfrac{1}{3}\sin 3\omega t + \tfrac{1}{5}\sin 5\omega t + \ldots)$$

where E = supply-voltage peak and $\omega = 2\pi f$. As the condenser current is $C(de/dt)$, where C = capacitance,

$$i_c = \frac{4E\omega C}{\pi}(\cos \omega t + \cos 3\omega t + \cos 5\omega t + \ldots)$$

but as the normal condenser current is $\omega CE \cos \omega t$ it is apparent that some multiplication should be expected from the interrupter.

When a capacitance C is charged by switching on a supply through a resistance R, the current obeys an exponential law.

With the assumption of a small condenser (of the order of 1 μF) it is possible to develop a more exact formula for the current produced in the interrupted circuit. This is

$$i_{r.m.s.} = E\left[\cos\theta + \cos(\theta + \omega T)\right]\sqrt{\left[\frac{fC}{R}\left\{1 - \epsilon^{-2T/(RC)}\right\}\right]}$$

where θ = phase angle of contact, f = supply frequency, and T = period of contact in seconds.

It will be noticed that the variation of the period of contact acts in two different ways; by increasing the period, the term $\cos(\theta + \omega T)$ is decreased, whereas $\left[1 - \epsilon^{-2T/(RC)}\right]$ is increased. An experiment was performed in which the period T was varied over the full range, and it was shown therefrom that, with practical values of R and C, the value of T should be as short as possible, i.e. the effect of the exponential term was negligible. With this assumption the current may be expressed as

$$i_{r.m.s.} = 2E\cos\theta\sqrt{(fC/R)}$$

A series of experiments was arranged to test this formula, and it was shown to be correct in its general form. Also, the formula for power in a resistive circuit, i.e. watts $= 4fCE^2\cos^2\theta$, was shown to give approximately correct values for the circuit used.

These experiments and deductions indicate that by the inclusion of an interrupter it should be possible to pass sufficient power through a relatively small condenser to serve small loads. For example, a maximum load of 50 kW could be supplied from a 132-kV grid line through a condenser of about 21 mμF.

A suitable control factor is suggested by the term $\cos\theta$, it being only necessary to install a relay operated by the output voltage and moving a brush-arm device so as to vary the contact angle θ. This arrangement will maintain the output voltage correctly, irrespective of the load handled and, moreover, independent of the exact value of the line voltage. This latter is an important point, as it is possible to hold the sending and receiving voltages of long transmission lines at the required values, although at the intermediate tapping points the voltage varies with the load.

The results show that the tapping of high-tension lines for small powers is a commercial possibility, given a suitable form of interrupter. Of this there are many possible types, dynamic or static. Of the latter the Marx air-blast valve will serve as an example.

The advantages claimed for this system are:—

(1) Probability of cheapness.

(2) Increasing application as the line voltage rises, whereas the use of transformers becomes more difficult.

(3) The smallness of the capacitance required enables the simplest form of condenser to be used—the bushing itself will probably suffice. The factor of safety of the insulation is therefore high.

(4) Constant-voltage output.

(5) Simplicity of voltage control.

In conclusion I wish to thank Prof. W. M. Thornton, of Armstrong College, for giving me the opportunity of carrying out the investigation.

DISCUSSION ON

"ELECTROMAGNETIC FORCES SET UP BETWEEN CURRENT-CARRYING CONDUCTORS DURING SHORT-CIRCUIT." *

Messrs. J. W. Bayles and **A. Donkin** (*communicated*): The author has produced an interesting paper on the application of the known force expressions to practical arrangements of conductors. We should, however, like to draw further attention to certain points raised therein.

For the case of conductors meeting at right angles it is stated that if the integration is carried out along one conductor right up to the centre line of the second, the force reaches an infinite value. It is, of course, not legitimate to do this since conditions change for points within the radius of the second conductor, and only part currents must then be considered. This point has been investigated by Dunton,† who has suggested approximate allowances. Similar considerations do not, however, apply to the case of two conductors mutually disposed at right angles but lying in different planes. This fact is rather obscured in expression (vi) of Fig. 4, where r is taken as the radius of one conductor. Actually r need not be the conductor radius, but may be any distance, down to zero, and the force still has a finite value, since the shape of the force distribution curve is different in this case from that corresponding to arrangement (v) of Fig. 4. Unfortunately, the force distribution curve is not shown in Fig. 3, although the other arrangements treated are all represented. The actual distribution of force on non-intersecting conductors mutually disposed at right angles is approximately as shown in Fig. A. This case has been treated in detail by us in a short paper.‡ Incidentally, in connection with Fig. 3, it is presumed that arrangement (c) refers to displaced parallel conductors, and not to conductors in different planes as stated, since parallel conductors must lie in one plane.

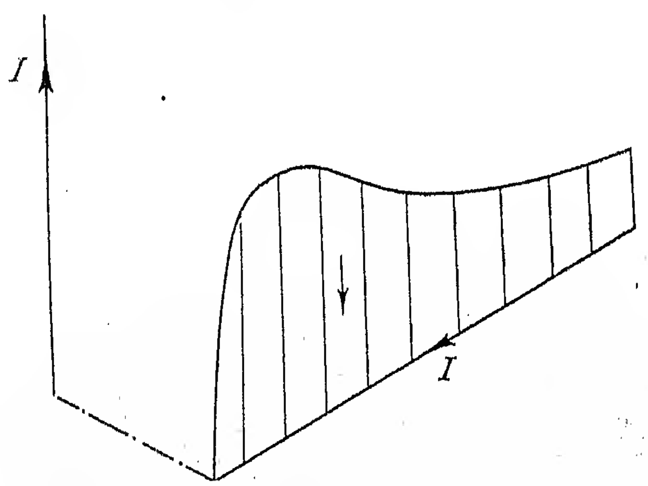


FIG. A.—Force distribution on non-intersecting conductors at right angles.

Certain discrepancies would appear to have occurred in the force expressions given in Fig. 4. Thus, expressions (i) and (iii) should, according to Dunton, be given as follows:—

$$\text{(i)}\ F = \frac{4 \cdot 5 i i_1}{10^8 s}\left[\sqrt{(l^2 + s^2)} - s\right] \text{ lb.}$$

$$\text{(iii)}\ F = \frac{2 \cdot 25 i i_1}{10^8 s}\left\{\sqrt{\left[(l + k)^2 + s^2\right]} - \sqrt{\left[(l + q)^2 + s^2\right]} - \sqrt{(k^2 + s^2)} + \sqrt{(q^2 + s^2)}\right\} \text{ lb.}$$

Furthermore, the factor 10^7 appears to be misplaced in expression (iv); also expression (vi) appears to be inconsistent with expression (v), to which it should reduce if $s = p = 0$, allowing for the fact that expressions (v) and (vii) are approximations in which it is assumed that k is large compared with the conductor radius. It should also be pointed out that although force arrows are shown on three sides of each of the diagrams (vii) and (viii), the corresponding expressions apply only to the horizontal conductors in each case, k and l being interchanged in (vii).

With reference to the author's remarks on the treatment of current distribution and skin and proximity effects, interesting suggestions have also been put forward by Beetz† for analysing the effects of these phenomena on the electromagnetic forces.

It is appreciated that the author's treatment of conductors in the vicinity of inductive material is of a descriptive or qualitative nature. It is not clear, however, from a comparison of the lower diagrams in Fig. 11 with Fig. 19(b), how the horizontal forces are reduced. Fig. 19(b) appears to show that the horizontal forces due to the iron balance out. Apparently also the phase barriers completely suppress the normal forces between the phase conductors. Fig. 11 shows, however, that the effect of inductive barriers is merely to reduce the horizontal forces between conductors, and the lateral forces due to the barriers are again apparently balanced out. It would be very interesting, in this connection, to know to what extent eddy-current shielding would occur in a circuit-breaker tank with phase barriers, and whether the author has any information on the behaviour of non-magnetic conducting barriers, e.g. copper sheet or plate.

The experiments on the reactions of structures to pulsating and alternating force waves are interesting and instructive. It would be useful to know whether the author has made tests on the natural frequencies of the usual forms of switch stems and cross-bars, since the possibility of resonance occurring would evidently be likely to produce dangerous conditions, although in the usual case the short-circuit currents flow for only a few half-cycles before interruption, and the inertia of such conductors might prevent maximum conditions being

* Paper by Mr. G. L. E. METZ (see vol. 75, p. 527).
† *Journal of Scientific Instruments*, 1927, vol. 4, p. 440.
‡ *World Power*, 1933, vol. 19, p. 216.

† W. BEETZ: *Elektrotechnik und Maschinenbau*, 1930, vol. 48, p. 761.

attained. It appears to be necessary in any case at least to take forces corresponding to peak currents, as the author points out.

Turning to the analysis of contact forces in Fig. 14, it would appear that certain factors have been overlooked. Thus in the cases of finger contacts and plug-and-socket contacts, although it is true that there are forces tending to open the contacts there may be also considerable attractive or closing forces between the fingers or socket contacts when carrying short-circuit currents in parallel. These forces, which are not indicated in the diagrams, would in many cases more than counterbalance the opening force. It is quite possible in practice, of course, to have current flowing down only one finger or one socket contact, in which case the opening force would be unbalanced. This, however, is not the case illustrated.

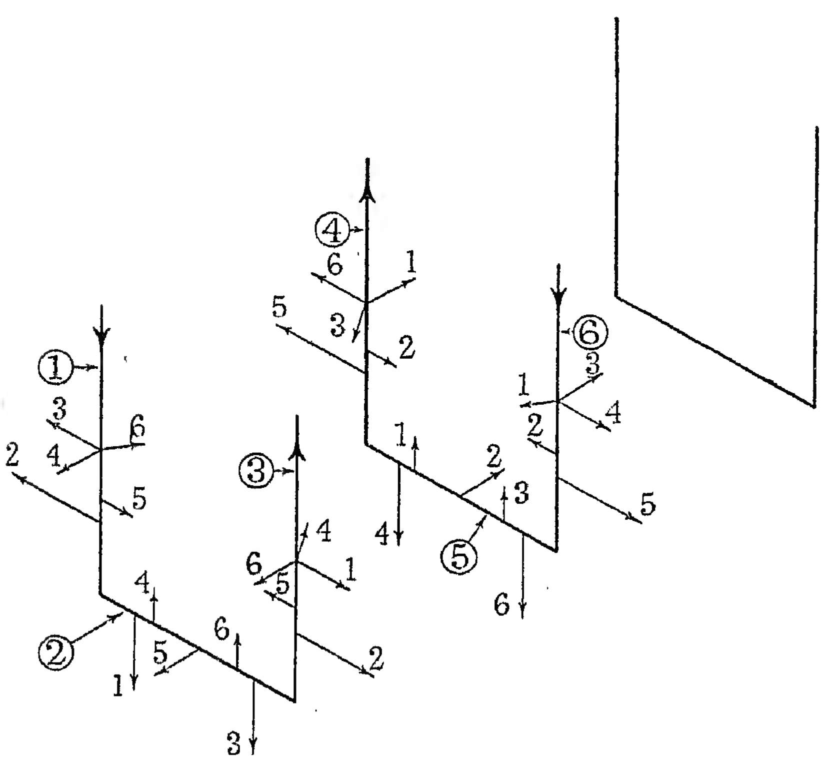


FIG. B.—Forces due to single-phase short-circuit.

It is believed, furthermore, that in addition to the electromagnetic forces considerable gas-pressure forces may occur within the contacts owing to the initial arc set up when closing on short-circuit, or by burning at the faces of the contacts; and that contacts may be opened by this means. Due attention must therefore be paid to all the factors involved if a reliable and balanced design is to be produced.

In connection with Fig. 16(*a*) it should be mentioned that although this arrangement gives maximum blow-out effect upon the arcs, it also intensifies the blow-out force on the switch cross-bar, and since this force is sometimes a source of considerable trouble when a circuit breaker closes on short-circuit the arrangement of Fig. 16(*a*) is not necessarily always the best to use. Clothier* has described an ingenious form of make-and-break discriminating contacts, the make being on separate dashpot-controlled contacts at close centres and the break on ordinary heavy contacts at wide centres. This serves to ease the closing load whilst still utilizing the electromagnetic forces to the full for breaking circuit.

* *Journal I.E.E.*, 1932, vol. 71, p. 366, Fig. 5.

Considering now the diagrams of Fig. 18, it appears to us that some confusion has arisen regarding the number and directions of the forces acting. Furthermore, small forces are produced on the switch stems owing to current flowing in the adjacent cross-bars, this being a case of non-intersecting conductors mutually disposed at right angles. A more accurate version of the case is shown in Fig. B. The deflection diagrams are not readily understood as drawn, and do not serve any very useful purpose. The deflections will obviously be influenced by force distribution and by the shapes and strengths of the various switch members. Deflections and stresses due to electromagnetic forces are very important in the design of circuit-breaker members, and must be checked by careful analysis of bending moments and the strengths of the members. Frequently the members under stress are composite beams or cantilevers consisting of conductors with surrounding or supporting insulators and clamps. Due regard must then be paid to the stresses set up in each component of the switch member.

Fig. 19 should also show, for completeness, the small inward forces on the conducting stems caused by current in the adjacent cross-bars.

We should like to point out that other cases of forces between straight coplanar conductors which are inclined to one another, and also between skew conductors, are sometimes required. Some interesting work has been done on the former case by Dunton.* Both cases are, however, capable of treatment by graphical means; this is often an advantage in practical work.

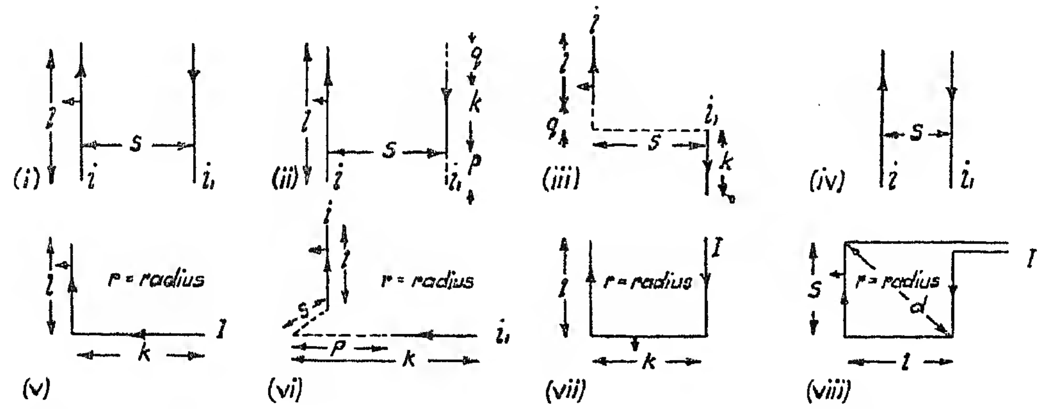
Mr. H. C. Fox (*communicated*): The paper contains many errors. In particular, attention may be drawn to Fig. 4 on page 528 (only two formulæ even approximately accurate out of eight given), formula for M and formula (4) for F on page 529 (both incorrect), and Table 2 on page 535 (almost wholly erroneous). Other data are very definitely open to question. The author has sent me a revised set of formulæ applying to Fig. 4, but even these are in places approximate only. In my opinion the subject matter of the paper is of great importance and has for so long been neglected by the Institution that the paper should have been circulated in proof form and discussed before publication in the *Journal*. To print it in its present form was really not fair to the author.

Mr. G. L. E. Metz (*in reply*): I am indebted to Messrs. Bayles and Donkin for directing attention to Fig. 4 of my paper. Certain inaccuracies have occurred in the formulæ given in this figure, and the correct equations, together with the direction of current-flow and the particular conductor upon which the force is calculated, are given in Fig. C.

Equations (v) and (vi) in Fig. C, relating to conductors at right angles, ignore the effects of currents and fluxes inside the conductors themselves, and in the case of equation (v) a further simplification is made which is permissible only when r^2 is small compared with k. These particular equations and their derivatives are therefore only strictly applicable to problems where conductors of small section are used.

The currents and fluxes inside the conductors themselves and the radii of conductors do not appear to

* *Loc. cit.*



i) $F = \frac{4 \cdot 5\, i i_1}{10^8 s}\left\{\sqrt{(l^2+s^2)} - s\right\}$ lb.	ii) $F = \frac{2 \cdot 25\, i i_1}{10^8 s}\left\{\sqrt{(k+p)^2+s^2} - \sqrt{(p^2+s^2)} - \sqrt{(q^2+s^2)} + \sqrt{(l-p)^2+s^2}\right\}$ lb. (Force on l)
iii) $F = \frac{2 \cdot 25\, i i_1}{10^8 s}\left\{\sqrt{(l+q+k)^2+s^2} - \sqrt{(l+q)^2+s^2} - \sqrt{(q+k)^2+s^2} + \sqrt{q^2+s^2}\right\}$ lb. (Force on l)	iv) $F = \frac{5 \cdot 4\, i i_1}{10^7 s}$ lb./ft. Where s is in inches (for long conductors)
v) $F = \frac{2 \cdot 25 I^2}{10^8}\left\{\log_e \frac{2k}{r} - \log_e \frac{k+\sqrt{k^2+l^2}}{l}\right\}$ lb. (Force on l)	vi) $F = \frac{2 \cdot 25\, i i_1}{10^8}\left\{-\log_e \frac{\sqrt{(k^2+l^2+s^2)}+k}{\sqrt{p^2+l^2+s^2)}+p} + \log_e \frac{\sqrt{(k^2+s^2+r^2)}+k}{\sqrt{(p^2+s^2+r^2)}+p}\right\}$ lb. (Force on l)
vii) $F = \frac{4 \cdot 5 I^2}{10^8}\left\{\log_e \frac{2k}{r} - \log_e \frac{l+\sqrt{k^2+l^2}}{l}\right\}$ lb. (Force on k)	viii) $F = \frac{4 \cdot 5 I^2}{10^8}\left\{\log_e\left[\frac{2ls}{r(l+d)}\right] + \frac{d}{l} - 1\right\}$ lb. (Force on s)

FIG. C.—Equations giving electromagnetic force on various arrangements of straight conductors, assuming:—

(*a*) Current measured in amps. (d.c.).
(*b*) Current concentrated on centre.
(*c*) Conductors located in a medium whose permeability is unity.
(*d*) Conductors of small section (r^2 small compared with conductor length).

affect the forces to a great extent in many practical problems. For this reason the simplified forms of the equations for conductors at right angles, and their derivatives, are given in Fig. C.

There are, however, certain rather extreme cases, such as the calculation of the throw-off force on the cross-bar of a large-current low-voltage oil circuit-breaker where r^2 is not small compared with k and where the effect of part currents in the conductors might be expected to be appreciable. The more complete forms of equations (v)–(viii) in Fig. C, for cases where r^2 cannot be ignored and where the effects of currents and fluxes inside the conductors are taken into account, are given below.

In most practical cases the simplified formulæ in Fig. C will give results quite near enough for practical purposes. When considering the throw-off force on the cross-bar of an oil circuit-breaker, the results given by the application of Equation (vii*a*) will be found to be quite accurate enough for most purposes.

The lower diagram of Fig. 11 is intended to show that the forces under the conditions indicated will tend to counteract one another; this tendency is again referred to in Fig. 19.

The effect of the inductive barriers intended to reduce the horizontal forces is considered to be important, and the figures in question were prepared with the object of directing attention to this point. I have no informa-

Equation (v*a*)

$$F = \frac{2 \cdot 25 I^2}{10^8}\left[\log_e \frac{l}{r} - \log_e\left\{\frac{\sqrt{(k^2+l^2)}+k}{\sqrt{(k^2+r^2)}+k}\right\} + 0 \cdot 25\right] \text{lb.} \quad \text{(Force on } l\text{)}$$

Equation (vii*a*)

$$F = \frac{4 \cdot 5 I^2}{10^8}\left[\log_e \frac{k}{r} - \log_e\left\{\frac{\sqrt{(k^2+l^2)}+l}{\sqrt{(r^2+l^2)}+l}\right\} + 0 \cdot 25\right] \text{lbs.} \quad \text{(Force on } k\text{)}$$

Equation (viii*a*)

$$F = \frac{4 \cdot 5 I^2}{10^8}\left[\log_e \frac{s}{r} - \log_e\left\{\frac{\sqrt{(s^2+l^2)}+l}{\sqrt{(r^2+l^2)}+l}\right\} + \sqrt{\left(1+\frac{s^2}{l^2}\right)} - 0 \cdot 75\right] \text{lb.} \quad \text{(Force on } s\text{)}$$

Equation (vi) already takes the value of r^2 into account, but for large conductors the fluxes inside the conductor must also be included so that the limits of the final integration are extended from r and l to 0 and l, and the force becomes

tion relating to tests giving the behaviour of non-magnetic and conducting phase barriers, and I should be interested to learn if any such information is available. The condition of resonance referred to by Messrs. Bayles and Donkin is very difficult to investigate, but

Equation (vi*a*)

$$F = \frac{2 \cdot 25\, i i_1}{10^8}\left[-\log_e\left\{\frac{\sqrt{(k^2+l^2+s^2)}+k}{\sqrt{(p^2+l^2+s^2)}+p}\right\} + \log_e\left\{\frac{\sqrt{(k^2+s^2)}+k}{\sqrt{(p^2+s^2)}+p}\right\}\right] \text{lb.} \quad \text{(Force on } l\text{)}$$

it is possibly more concerned with the insulator supports than with the conductors themselves. The conductors may reach an elevated temperature with a few cycles of the short-circuit current, resulting in a reduction of stiffness. The stiffness of the conductor tends to decrease rapidly with increase of temperature and there is a tendency for the majority of the force to be passed on to the supports.

It is clearly stated in the text that part of the forces only are shown in Fig. 14. This figure is intended to direct attention to the existence of certain forces of repulsion which are rarely referred to. It is true that other forces are also produced which may be in the reverse direction and in certain circumstances may be larger than the forces shown, and attention is directed to this fact in the text of the paper. I am particularly pleased to find that Messrs. Bayles and Donkin agree as to the presence of the particular components shown, as these are often overlooked. The example shown in Fig. 16 was selected in order to refer to electromagnetic effects which, as far as I am aware, have only received detailed attention in Germany and which merit consideration.

Fig. 18 appears to agree with Fig. B of Messrs. Bayles and Donkin when the small force due to the current in the adjacent cross-bar is added. This force is counterbalanced by the remaining forces of large magnitude, and for this reason is not shown.

INSTITUTION NOTES.

Kelvin Medal.

The Kelvin Medal Award Committee of the Institution of Civil Engineers have awarded the Medal for 1935 to Sir Ambrose Fleming, M.A., D.Sc., F.R.S., in recognition of his services to electrical science, and particularly of his invention of the thermionic valve.

International E.H.T. Conference, Paris.

The next session of the above Conference will be held from the 27th June to the 6th July, instead of from the 6th to the 15th June as announced in the December, 1934, issue of the *Journal* [vol. 75, page 830].

Overseas Activities.

Argentina.

At a meeting of the Argentine Centre held on the 27th September, 1934, at Buenos Aires, a paper by Mr. R. J. Musto entitled "Electric Welding" was read and discussed.

The Annual General Meeting of the Centre was held on the 6th December and was followed by the Annual Dinner, at which 36 members and guests were present. Mr. K. N. Eckhard, Chairman of the Centre, presided.

Bombay.

A meeting, arranged by the Local Committee and attended by 43 members and guests, was held at Bombay on the 12th December, 1934, when a paper by Mr. B. J. J. Nicora on "The Kandó Phase Convertor System" was read and discussed.

Calcutta.

A paper by Mr. W. H. Adcock, entitled "Street-Lighting Practice," was read and discussed at a meeting organized by the Calcutta Local Committee, which was held on the 14th December, 1934.

At a meeting held on the 22nd January, 1935, at which 42 members were present, a discussion took place on "The Development of the Use of Electricity in Sparsely-Populated Districts."

A discussion (opened by Mr. K. G. Sillar) on the Bengal Electricity Duty Bill, 1935, took place at a meeting held on the 19th February, 1935, at which members of the Bengal Government and officers who will be responsible for the collection of the Electricity Duty were present as guests.

China.

The Annual General Meeting and Dinner of the China Centre were held on the 18th January, 1935, at Shanghai. Mr. A. J. Percival, Chairman of the Centre, presided, and the attendance numbered 69 members and guests.

A meeting of the Centre was also held at Shanghai on the 1st February, 1935, when a paper by Mr. C. D. Du Mars was read and discussed. Twenty-three members and guests were present.

Members from Overseas.

The Secretary will be obliged if members coming home from overseas will inform him of their addresses in this country, even if they do not desire a change of address recorded in the Institution register.

The object of this request is to enable the Secretary to advise such members of the various meetings, etc., of the Institution and its Local Centres, and, when occasion arises, to put them into touch with other members.

Communications from Overseas Members.

Overseas members are especially invited to submit for publication in the *Journal* written communications on papers read before the Institution or published in the *Journal* without being read. The contributor's country of residence will be indicated in the *Journal*. In this connection a number of advance copies of all papers read before the Institution are sent to each Local Hon. Secretary abroad to enable him to supply copies to members likely to be in a position to submit communications.

Elections and Transfers.

At the Ordinary Meeting of the Institution held on the 14th February, 1935, the following elections and transfers were effected:—

Elections.

Member.

Fairburn, Charles Edward, M.A.

Associate Members.

Bishop, Francis Arthur.
Gallé, Geoffrey Alexand.
Green, Alfred Leonard.
Jackson, Tudor.
Lawn, Edmund Henry.
Lowe, John.
Olgin, Constantin, B.Sc.
Partington, Harold Lees.

Associates.

Barham, John Huntley.
Davies, George.
Rigby, George.
Salter, Arthur George.

Graduates.

Davey, Claude, B.Sc.
Drake-Brockman, Alan Godfrey, Capt., R.E.
Fisher, Bernard.
Gaze, Philip Elliot, B.A.
Gray, Harold James, B.Sc.
Mackley, Jack, B.Sc.(Eng.)
Page, Maurice.
Rudge, Thomas Richard.
Segel, John.
Thomas, Geoffrey Bolton, B.Sc.(Eng.).

Students.

Andrews, John Charles.
Atkinson, Leslie Frederick.
Balean, Richard Masters.
Ballantyne, George Kello.
Bayes, Geoffrey Gilbert.
Beadle, David Gilbert.
Bryant, Victor Owen E.
Bugg, Ronald.
Carding, Joseph Thom.
Clark, Stuart Vincent E.
Cobb, Patrick Roydon A.
Cozens, William James, B.Sc.
Croshaw, Raymond Meadows.
Crumblehulme, Leslie Ashton.
Desyllas, Panos Lawrence.
Driscoll, Louis, Junr., B.Sc.
Ellis, Alec William.
Field, Dennis Cromwell.
Goodliffe, John.
Gourgey, Reginald Elias.
Gunston, Arthur Percival.
Hale, Mervyn.
Hammond, Harold Frank.
Hanhart, Paul.
Hatchard, Albert William.
Hatton, Kenneth Norman.
Hendriks, Samuel Louis.
Hilton, Peter.
Holland, Charles John A.
Hopwood, Robert.
Howie, Thomas, B.Sc.
Hume, John McIntosh.
Hunt, Robert Jacob.
Hunter, William Beatty.
Johns, John William.
Jones, Leslie Gordon.
King, Frederick Harold.
Kittel, Claude Gerard A.
Lingard, George Harold.
Lockett, Norman, B.Sc.
Lunnon, Ronald David.
Macdonald, George John.
Mackenzie, Nigel Morell.
McLaren, David Hall.
MacLean, James Alexander.
McLoughland, Thomas.
Martin, Evelyn John B.
Mason, David Herman.
Menon, Kottiezeth Raman.
Milne, John Dawson.
Osborn, Cyril Francis.
Peterson, Colin Charles K.
Phillips, Ernest Beynon.
Pringle, Terence Alexander, B.Sc.
Purcell, Geoffrey Stuart.
Raman, R. Sundara.
Read, Richard Alfred.
Sandercock, Reginald Gilbert.
Sarma, Rama Kanta.
Sauer, Edward James.
Scott, Robert Howard.
Smith, William John.
Stokes, Bernard.
Tait, James Sharp.
Taitt, Stephen Edmund.
Tipping, Leslie.
Watson, Sylvester Neville.
Watters, William Melville S.

Transfers.

Associate Member to Member.

Cutbush, Charles George.
deWardt, Robert George.
Smith, Frederick Daniel, D.Sc.

Associate to Associate Member.

Troup, George.

Graduate to Associate Member.

Austin, Cyril.
Bennett, James.
Cliffe, Allen William F., B.Sc.(Eng.).
Cunuder, Felix John.
Hands, Howard Frederic, B.Sc.(Eng.).
Hards, Thomas Marshall, B.Sc.(Eng.).
Harris, Robert William L., B.Sc.
Johnson, Edward Harrison, B.Sc.
Mason, Cyril Arthur, B.Sc. (Eng.).
May, George Egerton.
Ray, Satyendra Nath, M.Sc., B.Sc.(Eng.).
Ryburn, Raymond Steadman.
Smith, Douglas MacLeod.
Thurner, Walter Maurice F., B.A.
Uscinski, Alexander, B.E.

Student to Associate.

Williams, Alfred Thomas.

In addition, the following transfers have been effected by the Council:—

Student to Graduate.

Amos, Kenneth John.
Bancroft, Charles Norman.
Barton, Raymond Chetwode, B.Sc.
Barton, Richard James.
Bate, Rex.
Beattie, Herbert.
Bedford, Geoffrey Francis, B.Sc.
Billington, William Ernest.
Bradley, John Harvard.
Buckingham, George Ronald, B.Sc.
Bunyan, Thomas Walter, B.Sc.(Eng.).
Burton, Leslie.
Byford, Max Arnsby.
Calvert, Raymond.
Carette, Alan Desmond.
Chant, Eric Roy P.
Clark, William Brindley.
Coupe, Kenneth Wilson.
Dixon, Irving Blaiklock.
Dolan, Walter Henry.
Dossor, Frederick.
Dudley, Clarence Richard.
Dunkel, Christian Peter.
Ferrier, James Alexander, B.Sc.(Eng.).
Finucane, Paul Condon.
Fowlie, William Stephen.
Francis, William Charles.
Gill, John Barnes.
Goodall, William Ernest.
Grey, John George.
Griffiths, Trevor, B.Sc. (Eng.).
Guthrie, Andrew.
Haigh, Roland William.
Handley, Derek.
Hawker-Smith, Richard Elmhirst.
Hendry, Norman.
Hickman, Michael Hoste, B.Sc.(Eng.).
Hind, Douglas McGechan.
Holmes, Douglas.
Holt, Peter.
Holt, Richard George S.
Howard, Frank Maurice.
Irvine, Felix James.
Jackson, Stalvies.
Jamison, Alexander.
Johnson, James Lewis H.
Jones, Allan Newton, B.Sc. (Eng.).
Jones, Reginald Hargreaves.
Laycock, Dudley.
Lewis, Kenneth Rupert, B.Sc.(Eng.).
Lulla, Pribhasing Sewasing, B.Sc.Tech.
Macaulay, Arthur John.

Student to Graduate—continued.

McInnes, Richard Edward, B.E.
McKeon, Douglas Percy, B.Sc.
Marr, Robert.
Mehta, Hans Raj.
Milburn, Thomas Pease, B.Sc.(Eng.).
Miller, Albert Arthur.
Moore, Thomas Stuart, B.Sc.(Eng.).
Morcom, William John, B.Sc.(Eng.).
Morris, Edward Hollingworth, B.Sc.(Eng.).
Neave, David Peter B.
Neave, James William C.
Needham, Arthur Wheeldon.
New, Charles Morley.
Noble, George Saint.
Parker, William Frederick.
Pearson, Lewis Albert.
Philps, Geoffrey Allwright.
Piggott, Leslie Sylvester.
Plyer, Donald Challis.
Potts, Russell, B.Sc.
Rand, Herbert Francis.
Rankin, George David.
Rye, Arthur Roger.
Sayers, Thomas Campbell, B.Sc.
Searle, Anthony Roger I.
Slingsby, Stanley.
Smeatham, Thomas Roy.
Smith, Robert Main A., B.Sc.(Eng.).
Smith, Wilfred Frank.
Stevenson, Geoffrey Lancaster.
Tantawi, Mohammed Kamel M.
Thomas, Eric Charles.
Towers, Richard Walter.
Tucker, Joseph Harold L.
Wilcox, Thomas Webster.
Williams, John Cambray.
Williams, Lewis Wilton de P.
Yells, Harold Mervyn.
Yoh, Joseph, B.Sc.

Accessions to the Reference Library.

[NOTE.—The books cannot be purchased at the Institution; the names of the publishers and the prices are given only for the convenience of members; (*) denotes that the book is also in the Lending Library.]

NEWTON, *Sir* I. Mathematical principles of natural philosophy and System of the world. Translated by A. Motte, 1729. The translation revised, and supplied with an historical and explanatory appendix by F. Cajori. la. 8vo. xxxv + 680 pp. (Cambridge: University Press, 1934.) 35*s.*

OLSON, H. F., *Ph.D.*, and MASSA, F., *M.Sc.* Applied acoustics. 8vo. xiv + 430 pp. (Philadelphia: P. Blakiston's Son and Co., 1934.) $4.50. (*)

PAPIN, M. D. Pour se préserver des dangers de la foudre et de l'électricité. sm. 8vo. 208 pp. (Paris: G. Doin et Cie., 1934.) 18 *francs*.

PHILLIPS, H. B., *Ph.D.* Differential equations. 3rd ed. 8vo. vi + 125 pp. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1934.) 10*s.* 6*d.*

PLANCK, M. Introduction to theoretical physics. Translated by H. L. Brose. 5 vols. 8vo. (London: Macmillan and Co., 1932–33.) (*)
vol. 1, General mechanics. ix + 272 pp. 12*s.*
vol. 2, The mechanics of deformable bodies. 234 pp. 10*s.* 6*d.*
vol. 3, Theory of electricity and magnetism. xii + 247 pp. 10*s.* 6*d.*
vol. 4, Theory of light. vi + 216 pp. 10*s.* 6*d.*
vol. 5, Theory of heat. viii + 301 pp. 12*s.*

POWELL, J. E. Payment by results: introduction—organization—rate-fixing. 8vo. viii + 411 pp. (London: Longmans, Green and Co., 1934.) 10*s.* 6*d.* (*)

RANDALL, J. T., *M.Sc.* The diffraction of X-rays and electrons by amorphous solids, liquids and gases. 8vo. xii + 290 pp. (London: Chapman and Hall, Ltd., 1934.) 21*s.*

REIMANN, A. L., *Ph.D.* Thermionic emission. 8vo. xi + 324 pp (London: Chapman and Hall, Ltd., 1934.) 21*s.* (*)

REYNER, J. H. Television: theory and practice. 8vo. x + 196 pp. (London: Chapman and Hall, Ltd., 1934.) 12*s.* 6*d.* (*)

RICHTMYER, F. K. Introduction to modern physics. 2nd ed. 8vo. xviii + 747 pp. (New York, London: McGraw-Hill Book Co., Inc., 1934.) 30*s.* (*)

RICKER, C. W., and TUCKER, C. E. Electrical engineering laboratory experiments. 3rd ed. 8vo. xvii + 404 pp. (New York; London: McGraw-Hill Book Co., Inc., 1934.) 15*s.* (*)

ROSE, T. G. Higher control; a manual for company directors, secretaries and accountants. With a foreword by A. H. Pollen. 8vo. xvi + 269 pp. (London: Sir Isaac Pitman and Sons, Ltd., 1934.) 12*s.* 6*d.* (*)

ROWBOTHAM, J. L., and ALTHAM, T. F. Voltage regulation on a.c. systems. 8vo. 151 pp. (Walton-on-Thames: Hackbridge Electric Construction Co., Ltd., 1934.) 5*s.* (*)

SCOTT-TAGGART, J. The manual of modern radio. 8vo. 384 pp. (London: The Amalgamated Press, Ltd., 1933.)

SMITH, A. W., *Ph.D.* Electrical measurements in theory and application. 8vo. xviii + 413 pp. (New York; London: McGraw-Hill Book Co., Inc., 1934.) 18*s.* (*)

SMITH, F. C. Hot water and steam supply by electricity. A treatise on electric water heating and systems of supply, steam and electric steam boilers. 8vo. viii + 142 pp. (London: E. and F. N. Spon, Ltd., 1934.) 7*s.* 6*d.* (*)

—— Warming buildings by electricity. 8vo. vii + 162 pp. (London: E. and F. N. Spon, Ltd., 1934.) 8*s.* 6*d.* (*)

SMITH, S. P., *D.Sc.*, and SAY, M. G., *Ph.D.*, *M.S.* Electrical engineering design-class manual. sm. 8vo. xv + 221 pp. (London: Oxford University Press, 1934.) 5*s.* (*)

SMITH, W. H. A guide to draughtsmanship. 2nd ed. sm. 8vo. vii + 107 pp. (London: E. and F. N. Spon, Ltd., 1934.) 4*s.* 6*d.*

SOKOLNIKOFF, I. S., *Ph.D.*, and SOKOLNIKOFF, E. S., *Ph.D.* Higher mathematics for engineers and physicists. 8vo. xiii + 482 pp. (New York; London: McGraw-Hill Book Co., Inc., 1934.) 24*s.* (*)

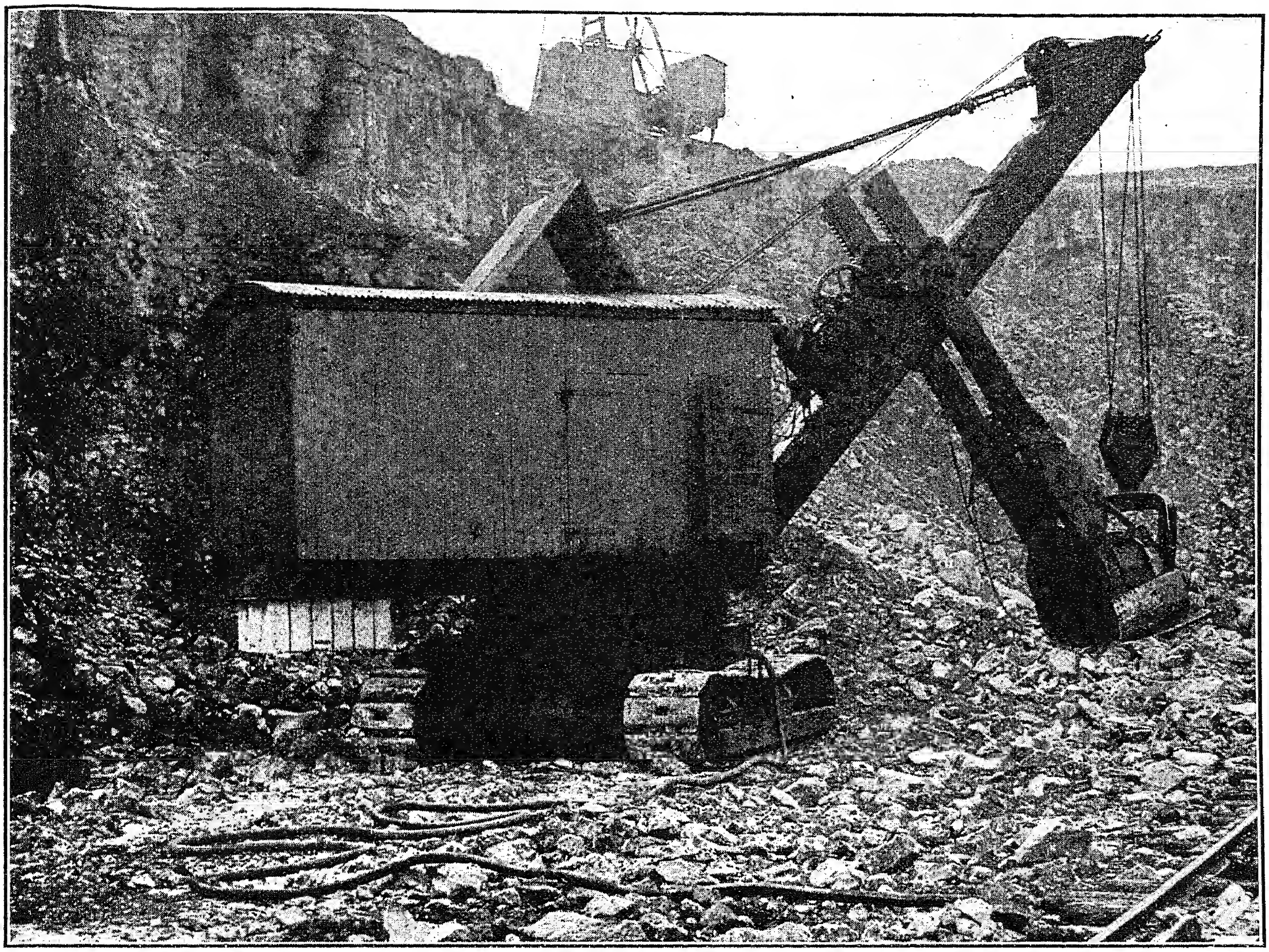
SOMMERFELD, A. Atomic structure and spectral lines. Translated from the 5th German edition by H. L. Brose. 3rd ed., vol. 1. xi + 675 pp. (London: Methuen and Co., Ltd., 1934.) 35*s.*

STARR, A. T., *M.A.* Electric circuits and wave filters. 8vo. xiv + 375 pp. (London: Sir Isaac Pitman and Sons, Ltd., 1934.) 21*s.* (*)

STEVENS, H. P., *M.A.*, *Ph.D.*, and STEVENS, W. H. Rubber latex. [2nd ed.] 8vo. 156 pp. (London: The Rubber Growers' Association, Inc., 1933.)

STONER, E. C., *Ph.D.* Magnetism and matter. 8vo. xv + 575 pp. (London: Methuen and Co., Ltd., 1934.) 21*s.* (*)

STUBBINGS, G. W. Automatic protection of A.C. circuits. 8vo. viii + 293 pp. (London: Chapman and Hall, Ltd., 1934.) 15*s.* (*)

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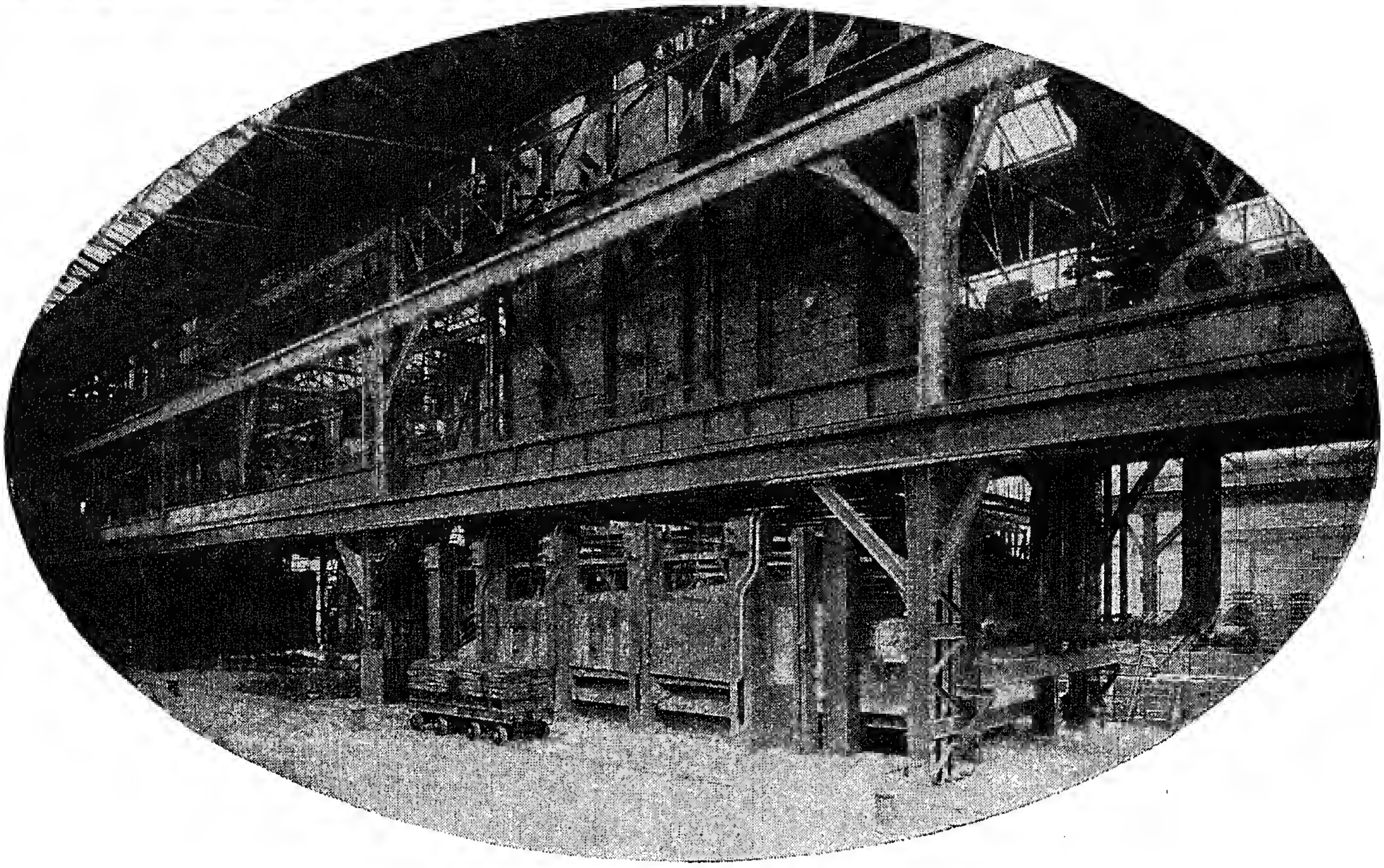


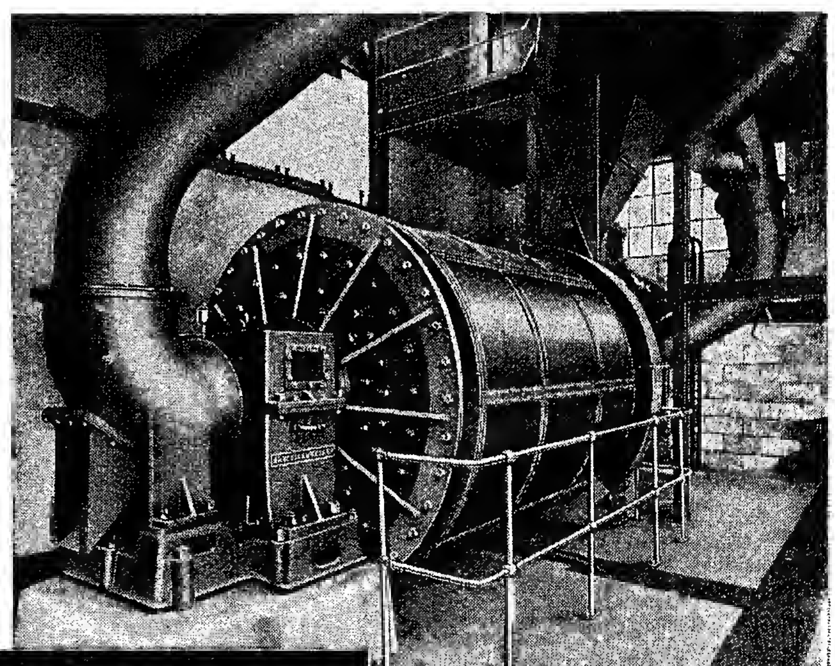
A BABCOCK PULVERISED FUEL FIRED METALLURGICAL INSTALLATION WITH B. & W. WASTE HEAT BOILERS

AT the Prescot plant of British Copper Refiners Ltd. are two reverberatory furnaces each capable of producing more than 1,000 tons per week of copper with a purity of 99.95%. Both these furnaces are fired with pulverised fuel from storage bins through B. & W. screw feeders and Babcock Fitch Burners, and both furnaces are equipped with a B. & W. waste heat boiler designed for a maximum evaporation of 24,000 lbs. of steam per hour. The second furnace, more recently installed is shown above, with the original furnace on the left in the same picture.

As the requirements for steam in this plant are small, the bulk of the steam generated is sold to the parent company, British Insulated Cables Ltd., where it is used for the generation of power as well as for various process heating requirements.

A Babcock Tube Mill of 5 tons per hour capacity is installed, which is shown on the right, as well as a Fuller-Kinyon Transport System for the safe conveyance of the pulverised fuel from the mill to the bunkers at the boilers.

The Babcock Fitch Burner is specially designed for metallurgical work, a feature being close manual control of the length of flame.



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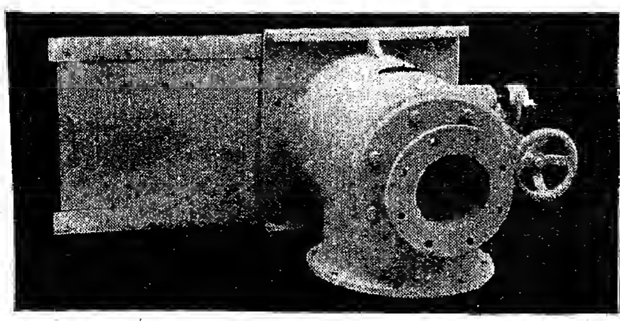

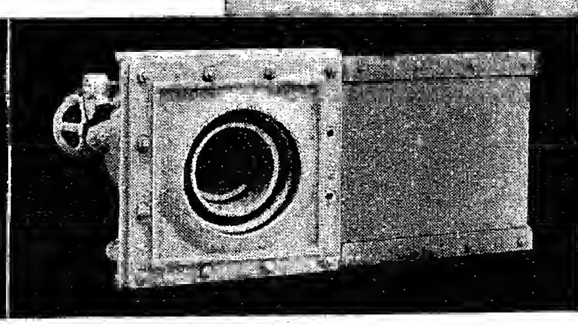

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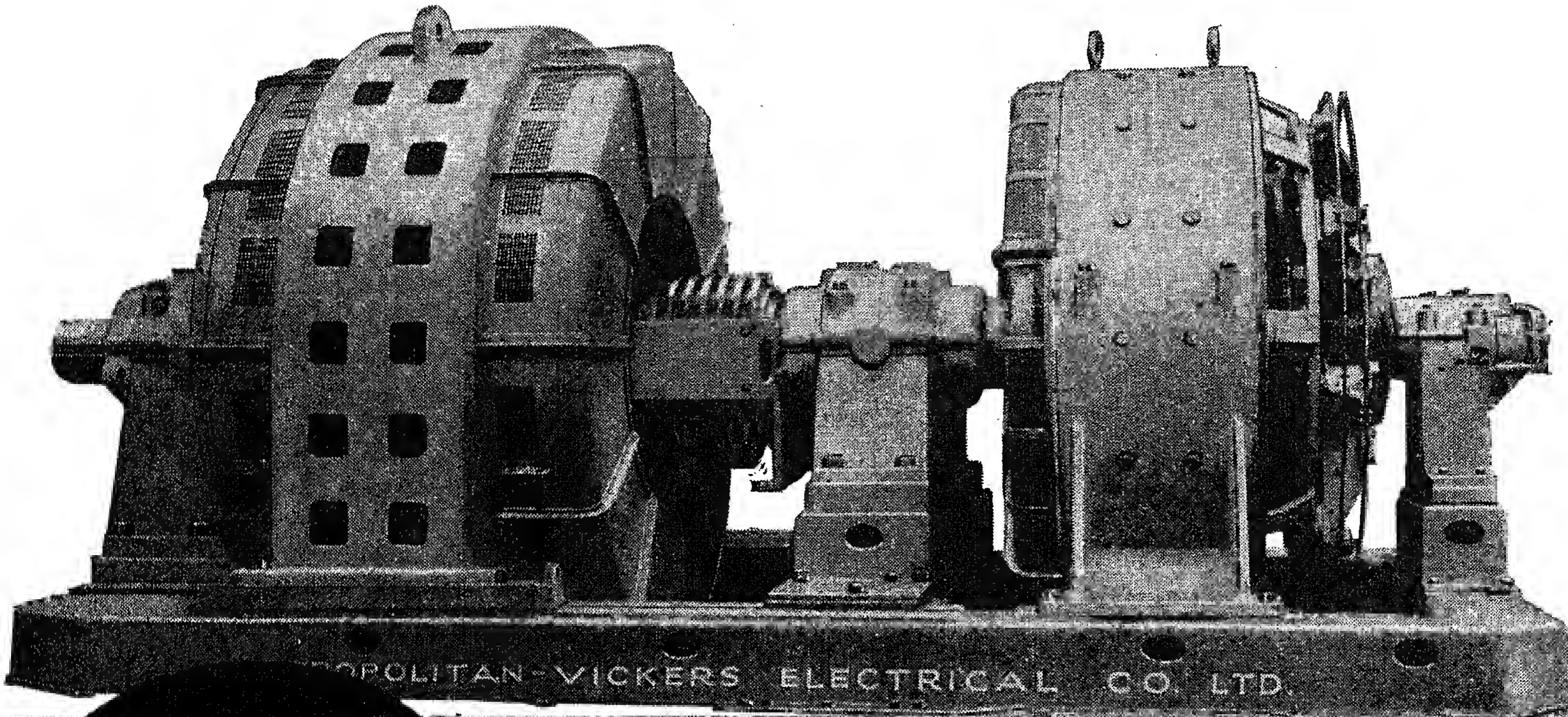

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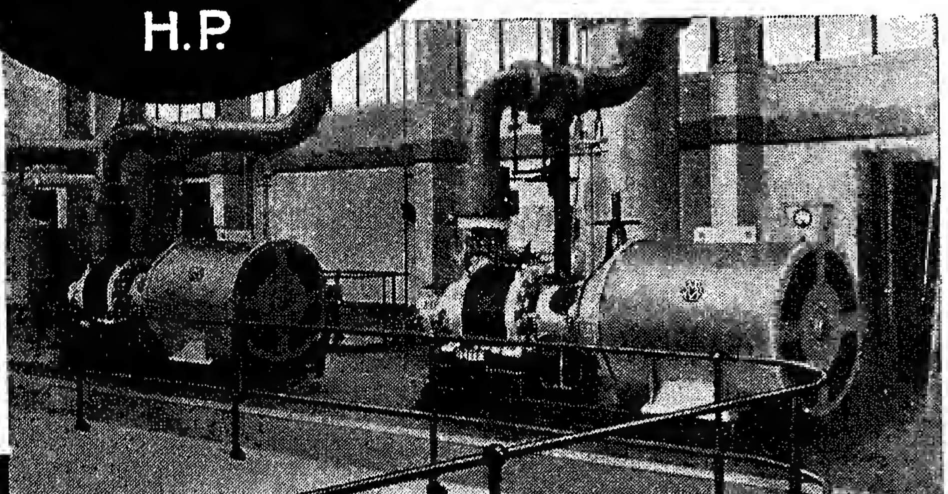

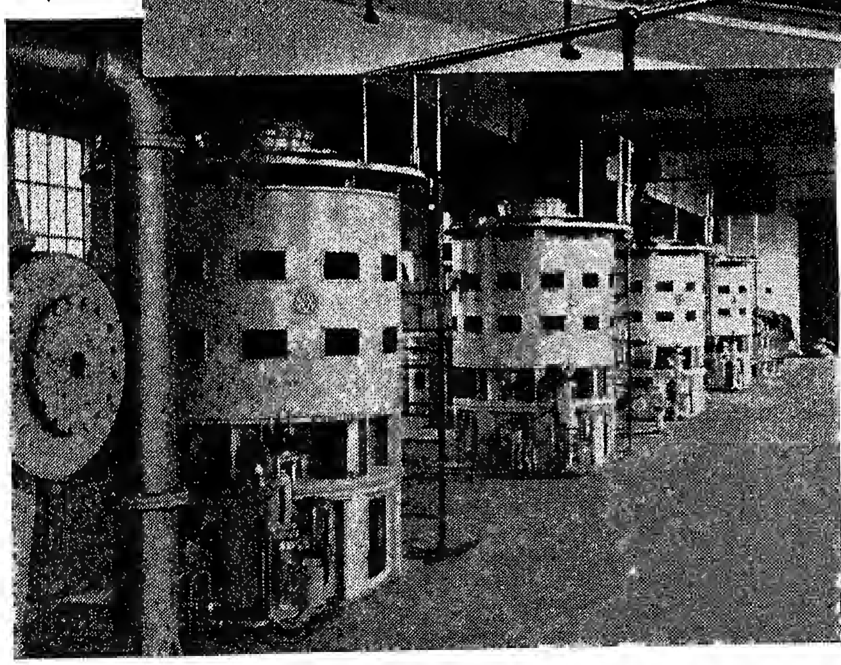

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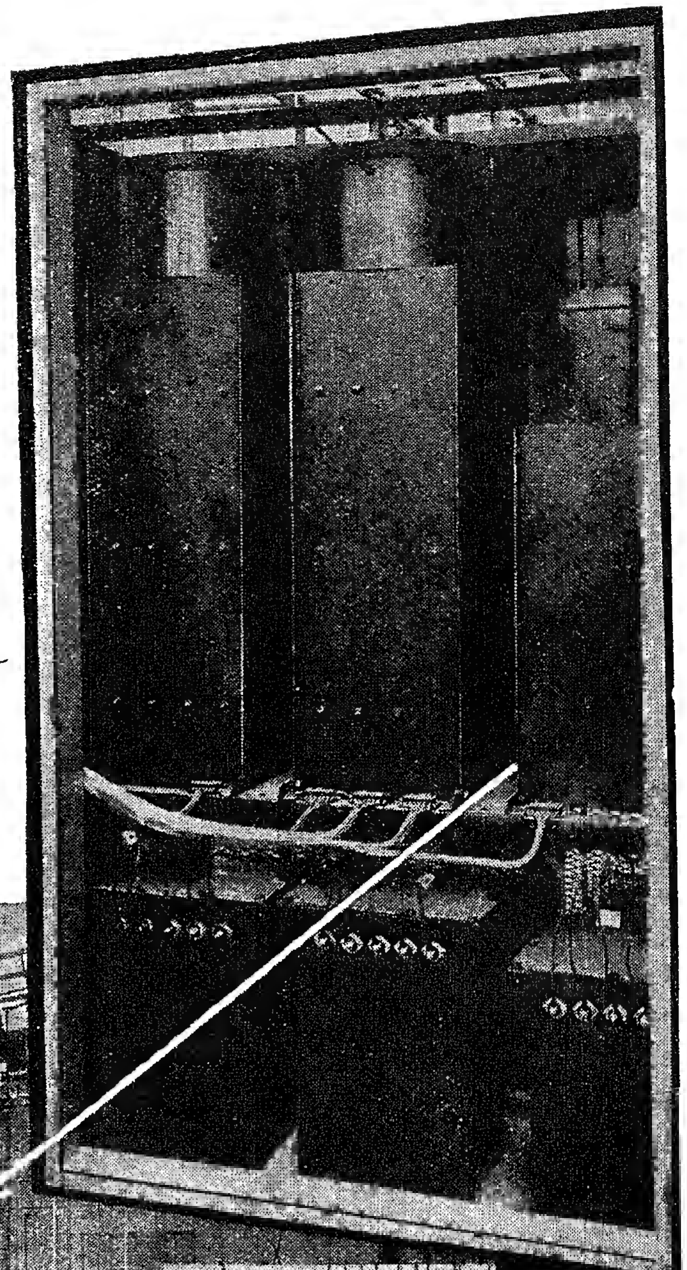

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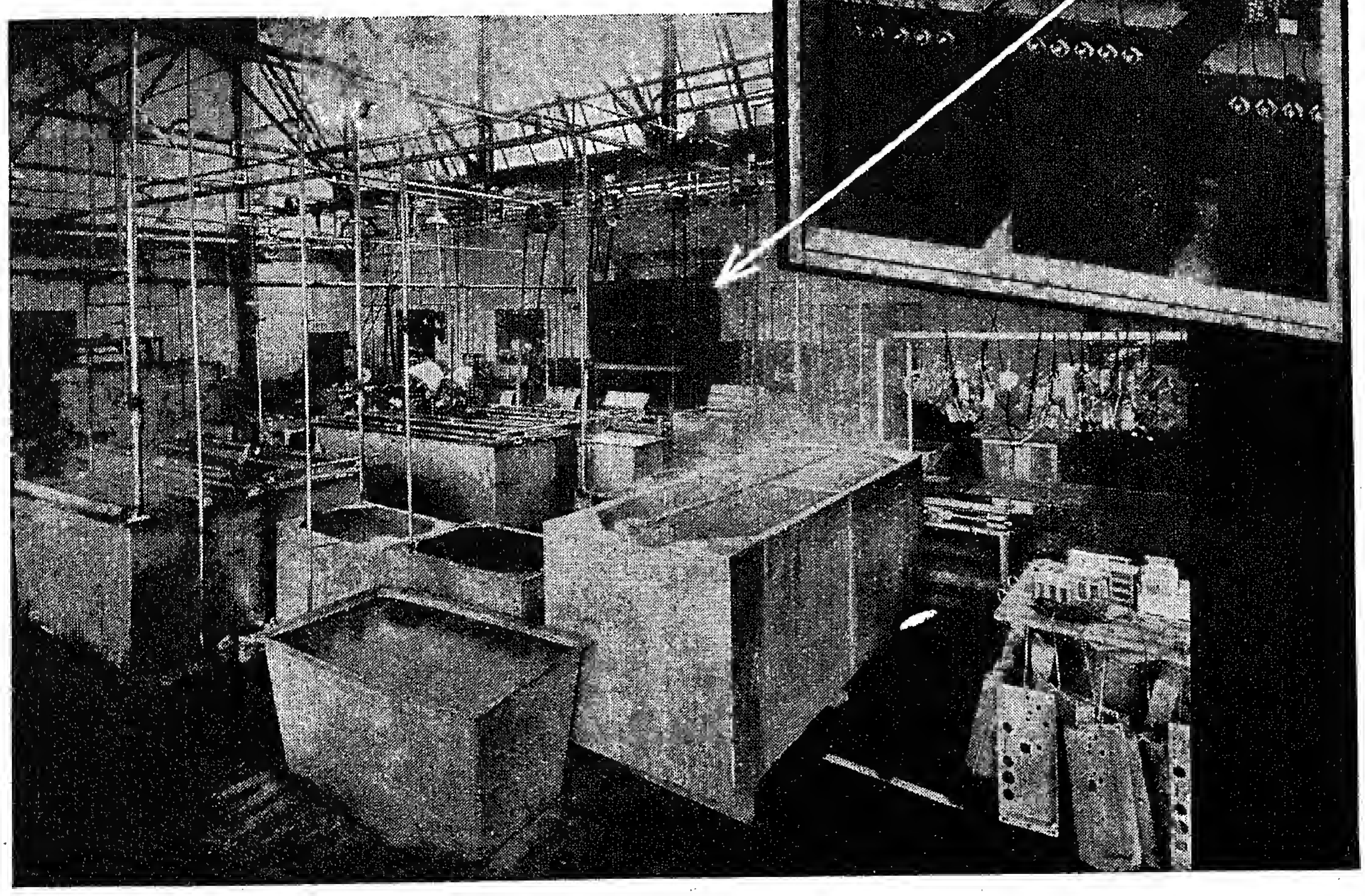

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